

10.1 Square Root Functions

- Square root Function – *Square root of Variable \sqrt{x}*

- Radicand – *express under radical sign*

Recall with Quadratics:

$$y = a(x - h)^2 + k$$

↑ stretch/shrink
↑ rt/left *↑ up/dawn*

$$y = a\sqrt{x} + h + k$$

↑ rt/left
↑ stretch/shrink *↑ up/dawn*

Examples

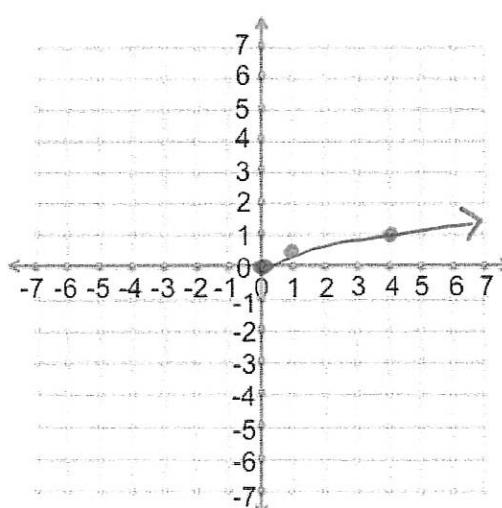
Graph each function and compare to the parent function. Also state the domain and range.

$$y = \frac{1}{2}\sqrt{x}$$

shrink

$$D: x \geq 0$$

$$\text{range: } y \geq 0$$



x	y
0	0
1	0.5
4	1

$$y = -\frac{1}{2}\sqrt{x}$$

Shrink
& flip

$$D: x \geq 0$$

$$R: y \leq 0$$

$$y = \sqrt{x} - 1$$

Down 1

$$D: x \geq 0$$

$$R: y \geq -1$$

$$y = \sqrt{x + 1}$$

left 1

$$D: x \geq -1$$

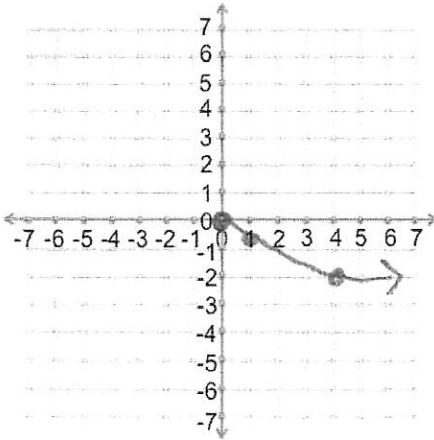
$$R: y \geq 0$$

$$y = 3\sqrt{x - 2}$$

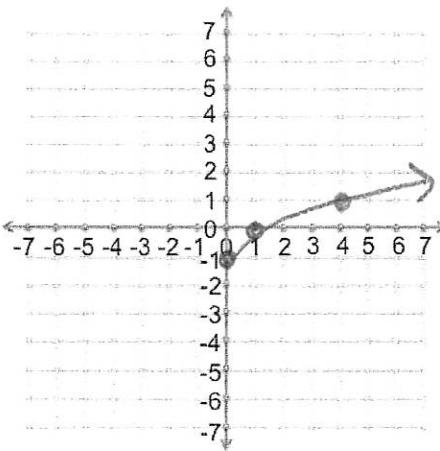
stretch \nearrow
Rt 2

$$D: x \geq 2$$

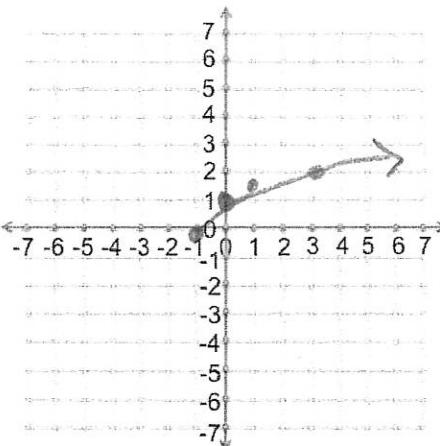
$$R: y \geq 0$$



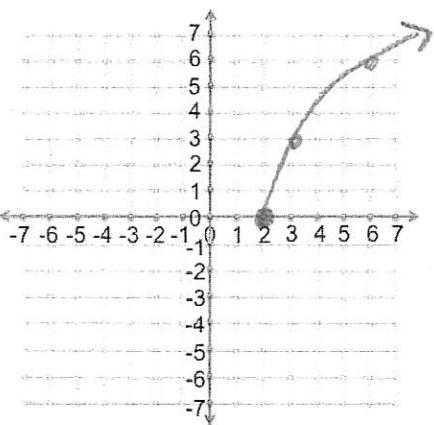
x	y
0	0
1	-1/2
4	-2



x	y
0	-1
1	0
4	1



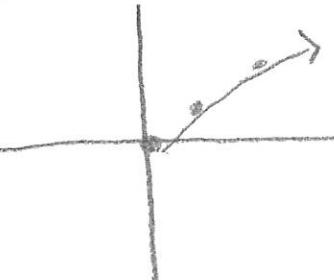
x	y
0	1
1	sqrt(2)
3	2



x	y
0	DNE
2	0
3	3
6	6

Examples – Real World

The speed s of a tsunami, in meters per second is given by $s = 3.1\sqrt{d}$, where d is the depth of the ocean water in meters. Graph the function. If a tsunami is traveling in water 26 meters deep, what is its speed?



$$s = 3.1\sqrt{26}$$

$$15.8$$

d	s
0	0
1	3.1
4	6.2

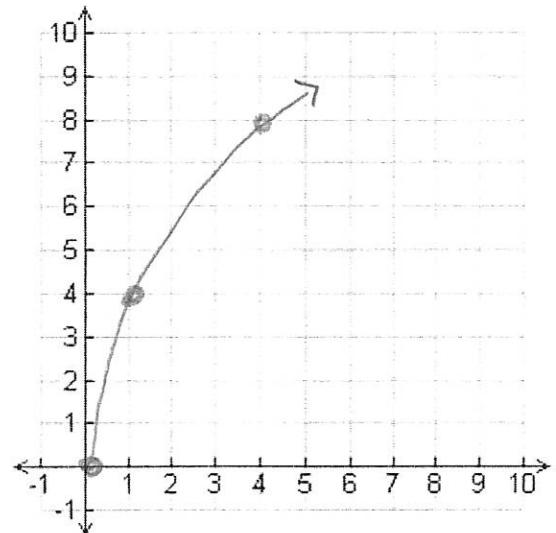
The perimeter of a square is given by the function $P = 4\sqrt{A}$, where A is the area of the square.

- a) Graph the function.

A	P
0	0
1	4
4	8

- b) Determine the perimeter of a square with an area of 225 square meters.

$$\begin{aligned} A &= \underline{\quad} \\ P &= 4\sqrt{225} \\ &= 4\sqrt{225} \\ &= 4 \cdot 15 \\ &= 60 \end{aligned}$$



- c) When will the perimeter and the area be the same value?

$$s = \underline{\quad}$$

$$\frac{P}{4} = \sqrt{A}$$

$$\frac{P^2}{16} = A$$

10.2 Simplifying Radical Expressions

Explore

Values of a and b	Value of $\sqrt{a} \bullet \sqrt{b}$	Value of \sqrt{ab}
$a = 4, b = 9$	$\begin{array}{l} \sqrt{4} \cdot \sqrt{9} \\ 2 \cdot 3 \end{array}$	$\begin{array}{l} \sqrt{36} \\ 6 \end{array}$
$a = 9, b = 16$	$3 \cdot 4$	12
$a = 25, b = 4$	$5 \cdot 2$	10
$a = 16, b = 36$	$4 \cdot 6$	24

Values of a and b	Value of $\frac{\sqrt{a}}{\sqrt{b}}$	Value of $\sqrt{\frac{a}{b}}$
$a = 4, b = 16$	$\frac{2}{4}$	$\sqrt{\frac{4}{16}} = \frac{1}{2}$
$a = 9, b = 25$	$\frac{3}{5}$	$\frac{3}{5}$
$a = 36, b = 4$	$\frac{6}{2}$	3
$a = 4, b = 49$	$\frac{2}{7}$	$\frac{2}{7}$

Summarize:

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{\frac{a}{b}}}{\sqrt{1}}$$

Examples

Simplify the following exactly. (Answers should not be decimals)

$$\sqrt{54} \quad \sqrt{16 \cdot 4}$$

4 $\sqrt{4}$

$$\sqrt{2} \cdot \sqrt{24} = \sqrt{48}$$

\nwarrow

$$\sqrt{16 \cdot 3}$$

4\sqrt{3}

$$\sqrt{180}$$

$$\sqrt{36 \cdot 5}$$

Ex $\sqrt{20}$

$$\frac{\wedge}{\sqrt{4.5}}$$

$$\frac{\sqrt{32}}{\sqrt{16 \cdot 2}}$$

452

<u>Perfect Squares</u>	
4	64
9	81
16	100
25	
36	
49	

$$\sqrt{2} \cdot \sqrt{24} = \sqrt{48}$$

\wedge

$$\sqrt{16 \cdot 3}$$

4\sqrt{3}

$$\sqrt{52}$$

$$\sqrt{4 \cdot 13}$$

$$2\sqrt{13}$$

$$\sqrt{x} = x^{\frac{1}{2}}$$

$$\sqrt{x^8} = (x^8)^{\frac{1}{2}} = x^4$$

You can also simplify variables under the square root!

Original

Simplified

Original

Simplified

$$\sqrt{x^2}$$

$$x$$

$$\sqrt{x^3} = \sqrt{x^2 \cdot x}$$

$$x\sqrt{x}$$

$$\sqrt{x^4}$$

$$x^2$$

$$\sqrt{x^5} = \sqrt{x^2 \cdot x^2 \cdot x}$$

$$x^2\sqrt{x}$$

$$\sqrt{x^6}$$

$$x^3$$

$$\sqrt{x^7}$$

$$x^3\sqrt{x}$$

Examples

$$\sqrt{x^4y^5} \rightarrow \sqrt{y^4 \cdot y}$$

$$x^2y^2\sqrt{y}$$

$$\sqrt{x^9y^7z^{10}}$$

$$\sqrt{x^8 \cdot x \cdot y^6 \cdot y \cdot z^{10}}$$

$$x^4y^3z^5\sqrt{xy}$$

$$\sqrt{45x^4y^5z^6}$$

$$\frac{9 \cdot 5}{3\sqrt{5}} x^2y^2\sqrt{y} z^3$$

$$(3x^2y^2z^3\sqrt{y})$$

$$\sqrt{32r^2k^4t^5}$$

$$\frac{16 \cdot 2}{4\sqrt{2}}$$

$$4\sqrt{2} rk^2t^2\sqrt{t}$$

$$4rk^2t^2\sqrt{2t}$$

- Rationalize the denominator - removing square root from denominator.

Ex

$$\frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\cancel{\sqrt{3}}}{2}$$

\uparrow
mult
by $\frac{\text{denom}}{\text{denom}}$

Ex

$$\sqrt{\frac{2}{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{10}}{5}$$

Examples

Simplify each expression.

$$1) \sqrt{\frac{3k}{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}} = \frac{\sqrt{24k}}{8}$$

$$\frac{\sqrt{24k}}{8} = \frac{\cancel{2}\sqrt{6k}}{8} = \frac{\sqrt{6k}}{4}$$

$$2) \frac{\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{3}$$

$$3) \sqrt{24a^2b^7}$$

$$\frac{4\sqrt{6}ab^3\sqrt{b}}{2\sqrt{6}}$$

$$2ab^3\sqrt{6b}$$

$$4) \sqrt{36x^6y^8}$$

$$6x^3y^4$$

$$5) \frac{\sqrt{3x}}{\sqrt{y^5}} \cdot \frac{\sqrt{y^5}}{\sqrt{y^5}} = \frac{\sqrt{3xy^5}}{y^5}$$

$$\frac{y^2\sqrt{3xy}}{y^5} = \frac{\sqrt{3xy}}{y^3}$$

10.3 Operations with Radical Expressions

- You can add or subtract radical expressions similar to how you add or subtract like terms.

Examples

a) $\underline{6\sqrt{5}} + \underline{2\sqrt{5}} - \underline{5\sqrt{5}}$ $3\sqrt{5}$

b) $9\sqrt{11} + 3\sqrt{11} - 5\sqrt{11}$ $7\sqrt{11}$

c) $\underline{7\sqrt{2}} + \underline{8\sqrt{11}} - \underline{4\sqrt{11}} - \underline{6\sqrt{2}}$ $-1\sqrt{2} + 4\sqrt{11}$

d) $2\sqrt{18} + 2\sqrt{32} + \sqrt{72}$

$$\begin{array}{ccc} 9 \cdot 2 & 16 \cdot 2 & 36 \cdot 2 \\ 2 \cdot 3\sqrt{2} & 2 \cdot 4\sqrt{2} & (6\sqrt{2}) \\ (6\sqrt{2}) & (8\sqrt{2}) & (20\sqrt{2}) \end{array}$$

e) $6\sqrt{27} + 8\sqrt{12} + 2\sqrt{75}$

$$\begin{array}{ccc} 9 \cdot 3 & 4 \cdot 3 & 25 \cdot 3 \\ 6 \cdot 3\sqrt{3} & 8 \cdot 2\sqrt{3} & 2 \cdot 5\sqrt{3} \\ (18\sqrt{3}) & (16\sqrt{3}) & (10\sqrt{3}) \\ (44\sqrt{3}) \end{array}$$

You can also multiply radicands.

mult front parts, then mult radicals

Example

Simplify each expression.

$$\text{a) } 2\sqrt{3} \cdot 4\sqrt{6}$$

$$8 \sqrt{18}$$

8.352
2452

$$b) 4\sqrt{2}(3\sqrt{2} + 2\sqrt{6})$$

$$12\sqrt{4} + 8\sqrt{12}$$

$$24 + 16\sqrt{3}$$

c) $9\sqrt{5} \cdot 11\sqrt{15}$

$$99 \sqrt{75}$$

49553

d) $5\sqrt{3}(3\sqrt{2} - \sqrt{3})$

1556-5.3⁴

1556-15

1556-15

Example

Find the area of a rectangle in simplest form with a width of $4\sqrt{6} - 2\sqrt{10}$ and a length of $5\sqrt{3} + 7\sqrt{5}$.

$$(4\sqrt{6} - 2\sqrt{10})(5\sqrt{3} + 7\sqrt{5})$$

$$20\sqrt{8} + 28\sqrt{30} - 10\sqrt{30} - 14\sqrt{50}$$

$$20\sqrt{18} + 18\sqrt{30} - 14\sqrt{50}$$

$\overset{A}{9.2}$ $\overset{A}{14.2}$ $\overset{A}{25.2}$
 $2\sqrt{3}$ $14\sqrt{5}$

$$60\sqrt{2} + 18\sqrt{30} - 70\sqrt{2}$$

10.4 Radical Equations

You can also solve equations with radicals.

$$\text{Solve } \sqrt{x+2} - 4 = 3$$

$$\begin{array}{r}
 +4 \quad +4 \\
 \hline
 \sqrt{x+2} = 7 \\
 \end{array}$$

$$x+2 = 49$$

$$\begin{array}{r}
 -2 \quad -2 \\
 \hline
 x = 47
 \end{array}$$

1st get radical alone

$$\text{Solve } (\sqrt{x+1} = x-1)^2$$

$$x+1 = (x-1)(x-1)$$

$$x+1 = x^2 - 2x + 2$$

$$0 = x^2 - 3x + 1$$

Factor, graph,
Quad

$$\frac{3 \pm \sqrt{5}}{2}$$

- Extraneous Solutions –

Solutions that don't work when you plug them back in.

Examples

$$\text{Solve } \sqrt{x-3} + 8 = 15$$

$$\begin{array}{r}
 -8 \quad -8 \\
 \hline
 \sqrt{x-3} = 7 \\
 \end{array}$$

$$x-3 = 49$$

$$\begin{array}{r}
 x = 52
 \end{array}$$

Check

$$\sqrt{52-3} + 8$$

$$\sqrt{49} + 8$$

$$7 + 8 = 15 \checkmark$$

$$\text{Solve } \sqrt{2-y} = y$$

$$2-y = y^2$$

$$y^2 + y - 2 = 0$$

$$(y+2)(y-1) = 0$$

$$-2 \neq 1$$

Check

$$\sqrt{2+2} = -2$$

$$\sqrt{0} = -2$$

No!

So -2 extraneous

$$\sqrt{2-1} = 1$$

$$\sqrt{1} = 1 \checkmark$$

Yes!

$$\text{Solve } (x-3 = \sqrt{x-1})^2$$

$$(x-3)(x-3)$$

$$x^2 - 6x + 9 = x - 1$$

$$x^2 - 7x + 10 = 0$$

$$(x-5)(x-2) = 0$$

$$5 \text{ and } 2$$

Check

$$5-3 = \sqrt{5-1}$$

$$2 = \sqrt{4}$$

$$2 = 2 \checkmark$$

Yes

$$5$$

$$2-3 = \sqrt{2-1}$$

$$-1 = \sqrt{1}$$

$$-1 = 1$$

No

2 Extraneous

Example – Real World

An object is dropped from an unknown height and reaches the ground in 5 seconds.

Use the equation $t = \frac{\sqrt{h}}{4}$, where t is time in seconds and h is height in feet to find the height from which the object was dropped.

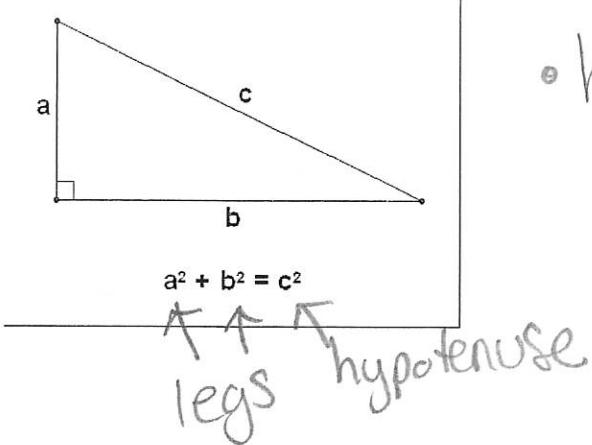
$$5 = \frac{\sqrt{h}}{4}$$

$$20 = \sqrt{h}$$

$$400 = h$$

400 ft

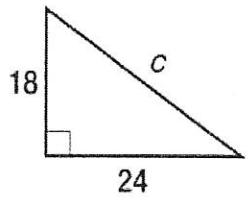
10.5 The Pythagorean Theorem



• hypotenuse - longest side & across from rt angle

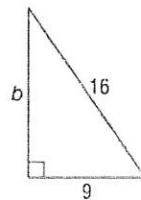
Examples

Find the lengths of the missing sides.



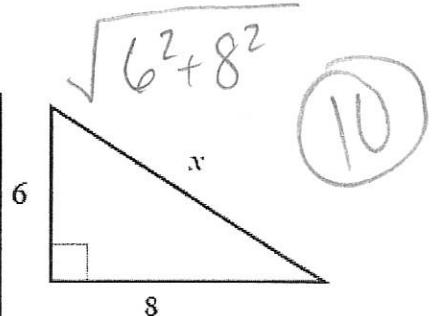
$$\sqrt{18^2 + 24^2}$$

30

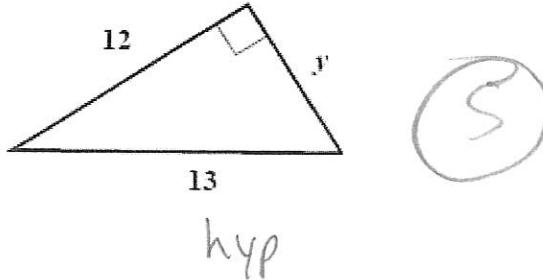


$$\sqrt{16^2 - 9^2}$$

13.23

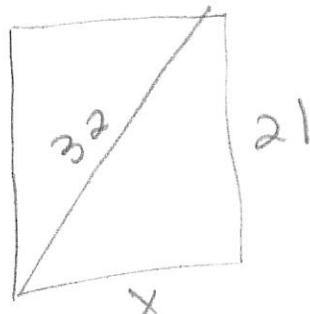


$$\sqrt{13^2 - 12^2}$$



Example – Real World

The diagonal of a television screen is 32 inches. The width of the screen is 21 inches. Find the height of the screen.



$$\sqrt{32^2 - 21^2}$$

24.15

- Pythagorean Triple – 3 whole numbers that satisfy pyth.
thm.
↑
no decimals or fractions

Examples

Determine if the following measures can be the lengths of a right triangle.

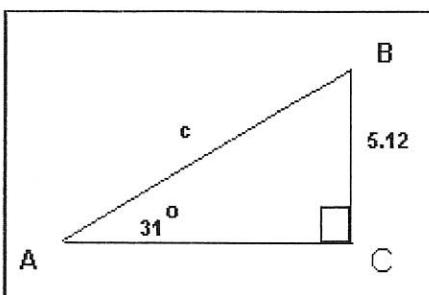
a) 7, 12, 15 $7^2 + 12^2 = 15^2$
 $49 + 144 = 225$ No.

b) 30, 40, 50 $30^2 + 40^2 = 50^2$
 $900 + 1600 = 2500$ Yes

c) 6, 12, 18 $6^2 + 12^2 = 18^2$
 $36 + 144 = 296$ No.

10.6 Trigonometric Ratios

Think about how you would solve the following triangle

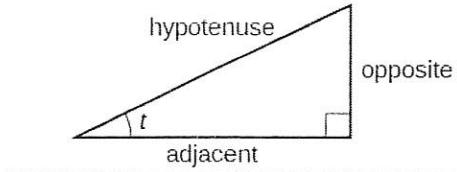


• Can't use Pyth. Thm. not enough information!

- Trigonometry - ratios of sides in rt. Δs
- Trigonometric Ratios

SOH-CAH-TOA

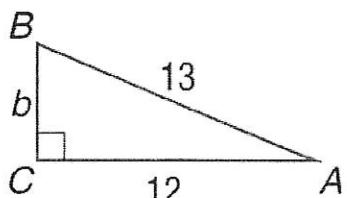
Sine	$\frac{\text{opp}}{\text{hyp.}}$
Cosine	$\frac{\text{adj.}}{\text{hyp.}}$
Tangent	$\frac{\text{opp}}{\text{adj.}}$



trig word LAngle = _____

Examples – Non Calculator

Find the values of the three trigonometric ratios for angle B.

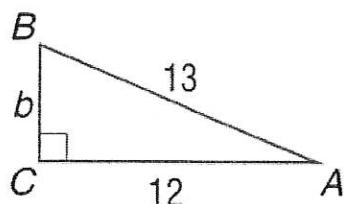


$$\sin B = \frac{12}{13} \quad \frac{\text{opp}}{\text{hyp}}$$

$$\cos B = \frac{b}{13} \quad \frac{\text{adj}}{\text{hyp}}$$

$$\tan B = \frac{12}{b} \quad \frac{\text{opp}}{\text{adj}}$$

Use the triangle below to find the values of the three trigonometric ratios for angle A.



$$\sin A = \frac{b}{13}$$

$$\cos A = \frac{12}{13}$$

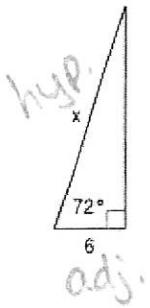
$$\tan A = \frac{b}{12}$$

You can also use your rules of Algebra to solve for missing sides of right triangles!

Steps:

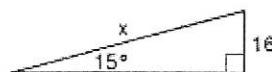
- 1) Look at how what you have relates to the angle. opp, adj, hyp.
- 2) Decide which trig ratio to use SOH CAH TOA
- 3) Set up the ratio
- 4) Solve the equation

Examples



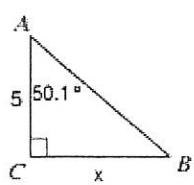
$$\cos 72 = \frac{6}{x}$$

19.4



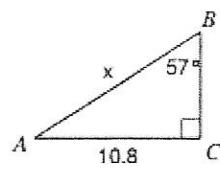
$$\sin 15 = \frac{16}{x}$$

61.8



$$\tan 50.1 = \frac{x}{5}$$

5.98

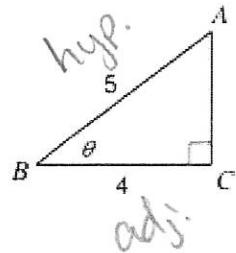


$$\sin 57 = \frac{10.8}{x}$$

12.88

Day 2 – Solving for Angles

You can also solve for angles in right triangles!



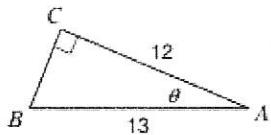
$$\cos \theta = \frac{4}{5}$$

$\Theta =$

make sure
degree mode!

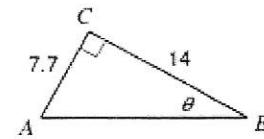
Examples

Solve for the missing angles.



$$\cos \theta = \frac{12}{13}$$

22.6°

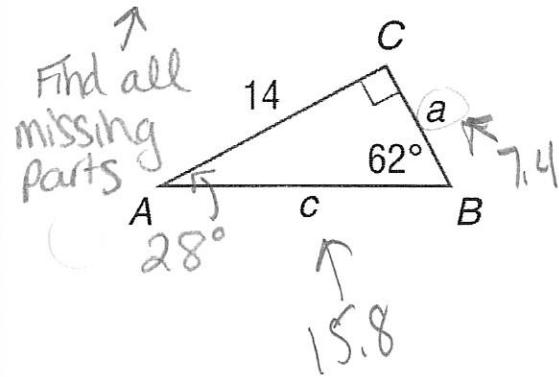


$$\tan \theta = \frac{7.7}{14}$$

28.8°

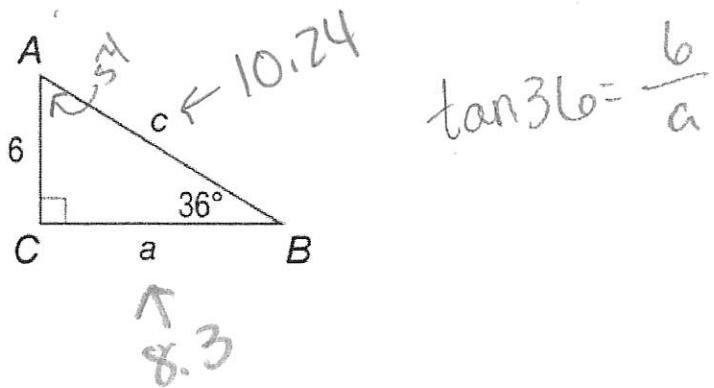
Examples

Solve the right triangle. Round each side to the nearest tenth.



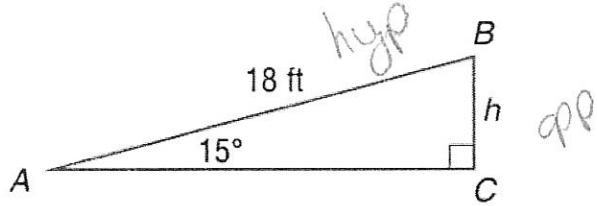
$$\tan 62^\circ = \frac{14}{a}$$

15.8



Example

CONVEYOR BELTS A conveyor belt moves recycled materials from Station A to Station B. The angle the conveyor belt makes with the floor of the first station is 15° . The conveyor belt is 18 feet long. What is the approximate height of the floor of Station B relative to Station A?



$$18 \cdot \sin 15^\circ = \frac{h}{18} \cdot 18$$

$$4.66$$