11.1 Inverse Variation

Recall:

- Direct Variation – when $x\uparrow, y\uparrow$
  $x\downarrow, y\downarrow$
- Inverse Variation – when $x\uparrow, y\downarrow$
  $y=\frac{K}{x}$ $xy=K$

Examples

Determine whether each table or equation represents an inverse or a direct variation and justify your thinking.

a)

X	6	8	10
Y	3	4	5

Direct $x\uparrow, y\uparrow$ $y=\frac{1}{2}x$

b)

X	1	2	3
Y	12	6	4

Inverse $x\uparrow, y\downarrow$ $xy=K$

c) $-2xy = 20$

$y=\frac{20}{-2x}$ $y=\frac{-10}{x}$ Inverse

d) $x = -0.5y$

$y=\frac{x}{-.5}$ Direct, $-2x$

e) $-3x = y$ Direct Linear

Examples

$y = \frac{K}{x}$

Assume that y varies inversely as x. If y=5 when x = 3, write an inverse variation equation that relates x and y.

$y = \frac{K}{x}$

$5 = \frac{K}{3}$

$K = 15$

So $\boxed{y = \frac{15}{x}}$

Assume that y varies inversely as x. If y = 5 when x = 12, find x when y = 15.

$y = \frac{K}{x}$

$5 = \frac{K}{12}$

$60 = K$

So $15 = \frac{60}{x}$

4

- Product Rule If (x_1, y_1) & (x_2, y_2) are sol. to inverse problem then $x_1 y_1 = x_2 y_2$

Example – Real World

When two people are balance on a seesaw, their distances from the center of the seesaw are inversely proportional to their weights. How far should a 105-pound person sit from the center of the seesaw to balance a 63-pound person sitting 3.5 feet from the center?

$105x = 63 \cdot 3.5$

2.1 ft

The Smith family can drive from their home in Wisconsin to Chicago in 4 hours if they drive an average of 45 miles per hour. How long would it take them if they increased their speed to 50 miles per hour?

$$4 \cdot 45 = x \cdot 50$$

$$180 = 50x$$

$$3.6 \text{ hours}$$

Example – No Calculator

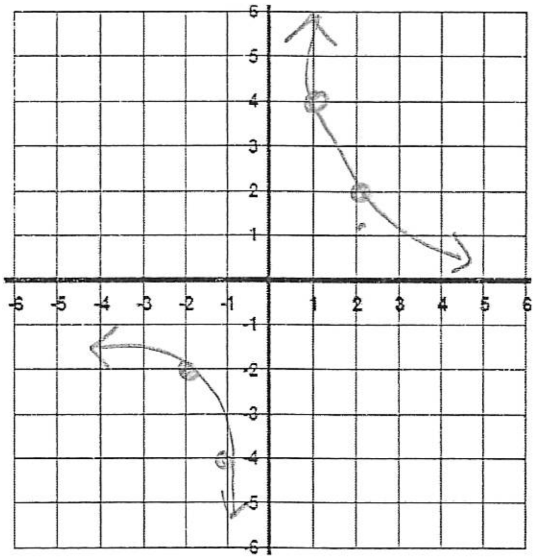
Graph an inverse variation in which y = 1 when x = 4.

$$y = \frac{K}{x}$$

x	y
-2	-2
-1	-4
0	—
1	4
2	2

$$1 = \frac{K}{4}$$

$$K = 4 \qquad y = \frac{4}{x}$$



Graph an inverse variation in which y = 16 when x = 4.

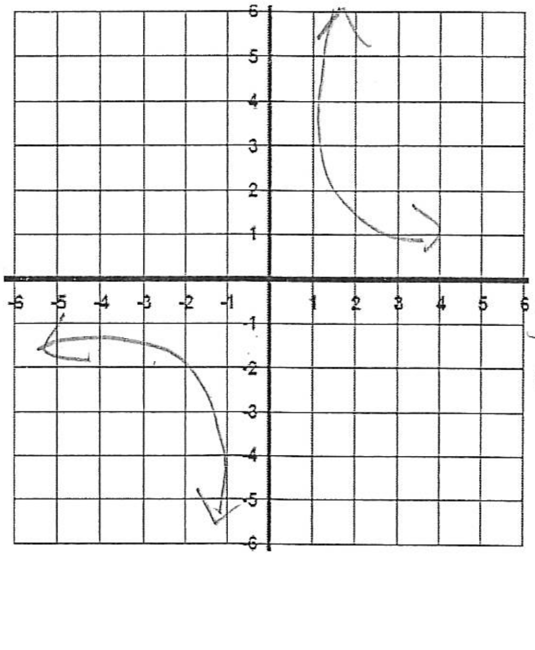
$$y = \frac{K}{x}$$

$$16 = \frac{K}{4}$$

$$K = 64$$

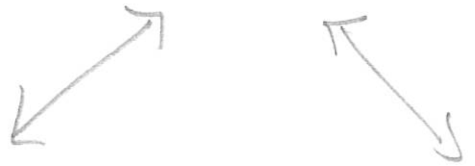
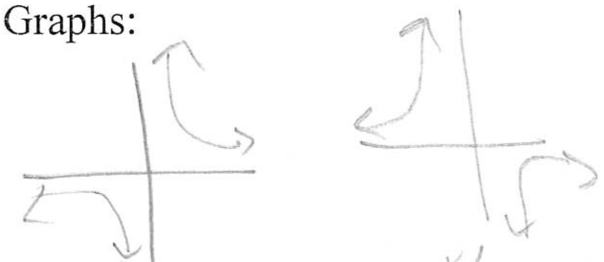
x	y
-2	-32
-1	-64
0	—
1	64
2	32

$$y = \frac{64}{x}$$



Scale wrong

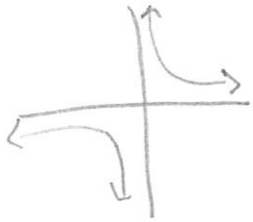
Summarize:

Direct Variation / Linear / Arithmetic	Inverse Variation
Graphs: 	Graphs:  $y=k/x$ k pos $y=k/x$ k neg
• $y=Kx$ • Directly • $\frac{y}{x}$ constant	• $y=\frac{K}{x}$ • Inversely • xy constant

11.2 Rational Functions

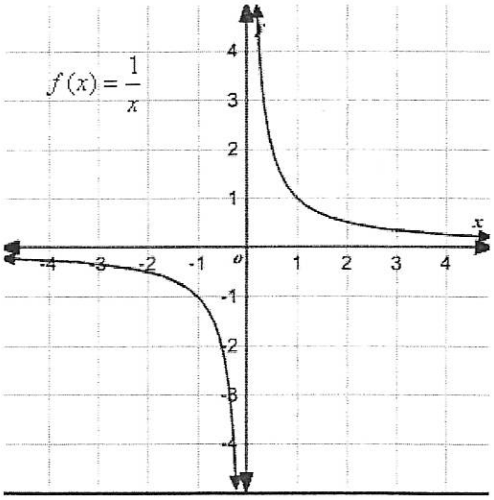
If you have to read a 300 page book for English class and you procrastinated and waited until x days before it is due to begin reading, how many pages would you have to read per day?

$$y=\frac{300}{x}$$



- Rational Functions

x	y
-4	-.25
-2	-.5
0	Error ← Asymptote $y=0$ $x=0$
2	.5
4	.25



Explore

Graph each of the following functions on your calculator. Also complete the tables.

$y = \frac{7}{x}$	$y = \frac{7}{x-1}$	$y = \frac{7}{x+2}$

X	Y
-2	-3.5
-1	-7
0	—
1	7
2	3.5

X	Y
-2	-2.33
-1	-3.5
0	-7
1	—
2	7

X	Y
-2	—
-1	7
0	3.5
1	2.33
2	1.75

How does the number where the table says ERROR, relate to the original equation?

zero of denominator

$x \neq 0$ $\qquad$ $x-1 \neq 0$ $\qquad$ $x+2 \neq 0$

$\qquad\qquad\quad$ $x \neq 1$ $\qquad$ $x \neq -2$

Examples – No Calculator

State the excluded value for each function.

a) $y = \frac{3}{x}$ $\qquad x \neq 0$

b) $y = \frac{3}{x+2}$ $\qquad x \neq -2$

c) $y = \frac{8}{2x+1}$ $\qquad 2x+1 \neq 0$

$2x \neq -1$

$x \neq -\frac{1}{2}$

d) $y = \frac{4}{x-3}$ $\qquad x \neq 3$

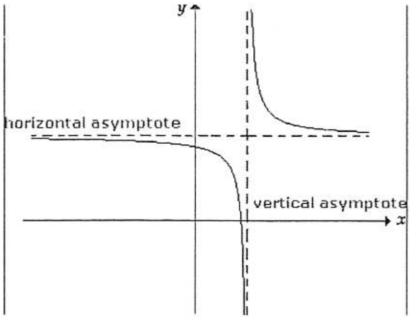
Example – Real World

If x students will compete in a talent show lasting 100 minutes, the function $y = \frac{100}{x}$ represents the number of minutes available for each act.

a) If there are 5 acts, how long can each perform? $\frac{100}{5} = 20$

b) If there are 25 acts how long can each perform? $\frac{100}{25} = 4$

- Asymptotes – line the graph of a function approaches. In table (Error)



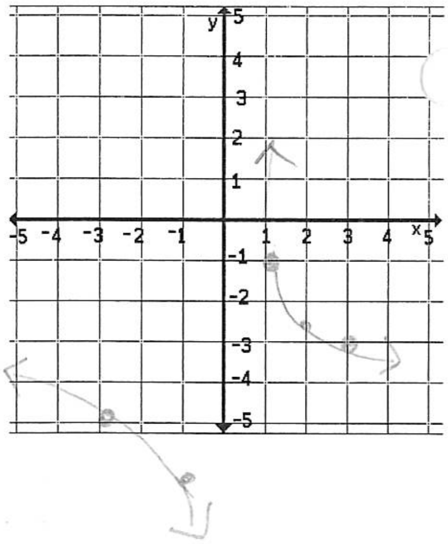
Examples

Identify the asymptotes for each function and graph the function.

a) $y = \frac{3}{x} - 4$

x	-3	-1	1	2	3
y	-5	-7	-1	-2.5	-3

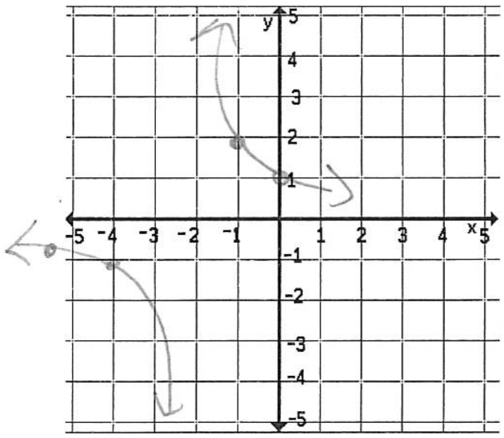
$\boxed{\begin{array}{l} x = 0 \\ y = -4 \end{array}}$



b) $y = \frac{2}{x+2}$

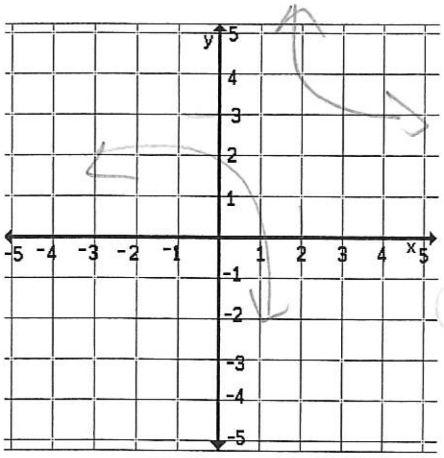
$\boxed{\begin{array}{l} x = -2 \\ y = 0 \end{array}}$

x	-6	-4	-1
y	-1/2	-1	2



c) $y = \frac{1}{x-2} + 3$

$\boxed{\begin{array}{l} x = 2 \\ y = 3 \end{array}}$



11.3 Simplifying Rational Expressions

Remember you cannot divide by zero!

Examples

State the excluded values for each expression.

a) $\frac{3b-2}{b+7}$ $b \neq -7$

b) $\frac{5z^2+2}{z^2-z-12}$ Quad $z \neq 4$

$z \neq -3$

c) $\frac{8}{2x+1}$ $2x+1 \neq 0$

$2x \neq -1$

$x \neq -\frac{1}{2}$

Example

Suppose a cylinder has a volume of 770 cubic inches and a diameter of 12 inches. Find the height of the cylinder. $r=6$

$V=\pi r^2 h$

$770=\pi 6^2 h$

$770=36\pi h$

$\boxed{h=6.8}$

Recall:

$x^a \cdot x^b$	x^{a+b}
$\frac{x^a}{x^b}$	x^{a-b}
$(x^a)^b$	x^{ab}
$(xy)^a$	$x^a y^a$
$(\frac{x}{y})^a$	$\frac{x^a}{y^a}$

Example

Simplify the following expressions using the rules of exponents.

a) $\frac{32x^5y^2}{4xy^7}$ $\frac{8x^4}{y^5}$

b) $\frac{(6x^2)(3y^6)}{36x^5y^3}$ $\frac{18x^2y^6}{36x^5y^3}$ $\boxed{\frac{1y^3}{2x^3}}$

c) $\frac{(3x^2)^3(2xy^4z)}{4x^3z^6}$ $\frac{27x^6 \cdot 2xy^4z}{4x^3z^6}$ $\frac{54x^7y^4z}{4x^3z^6} = \frac{27x^4y^4}{2z^5}$

d) $\frac{20x^8y^2}{4xy^3}$ $\frac{5x^7}{y}$

11.4 Multiplying and Dividing Rational Exponents

Multiplying Rational Functions is the same as multiplying fractions.

$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$

$\frac{3x}{5} \cdot \frac{2x}{x+1} = \frac{6x^2}{5x+5}$

Examples

Find each product and simplify.

a) $\frac{7x^2y}{12z^3} \cdot \frac{14z}{49xy^4} = \frac{98x^2yz}{588xz^3y^4} = \frac{1x}{6y^3z^2}$

b) $\frac{3x}{16x^2} \cdot \frac{8x^2}{3} = \frac{24x^3}{48x^2} = \frac{x}{2}$

c) $\frac{(\cancel{b+3})}{(4b-12)} \cdot \frac{(b^2-4b+3)}{(b^2-7b-30)}$ $\frac{(\cancel{b-3})(b-1)}{4(\cancel{b-3})\,(\cancel{b+3})(b-10)} = \frac{b-1}{4(b-10)} = \frac{b-1}{4b-40}$

d) $\frac{\cancel{x+3}}{x} \cdot \frac{5}{x^2+7x+12}$ $\frac{5}{x(\cancel{x+3})(x+4)} = \frac{5}{x^2+4x}$

Examples

The velocity that a spacecraft must have in order to escape Earth's gravitational pull is called the escape velocity. The escape velocity for a spacecraft leaving Earth is about 40,320 kilometers per hour. What is this speed in meters per second?

$$\frac{40{,}320\,\cancel{km}}{1\,\cancel{hr}} \cdot \frac{1000m}{1\,\cancel{km}} \cdot \frac{1\,\cancel{hr}}{3600sec} = \frac{40320000\,m}{3600\ sec}$$

11,200m/sec

The slowest land mammal is a three-toed sloth. It travels 0.07 mile per hour on the ground. What is this speed in feet per second?

$$\frac{.07\,\cancel{mi}}{\cancel{hr}} \cdot \frac{5280ft}{1\,\cancel{mi}} \cdot \frac{1\,\cancel{hr}}{3600sec} \qquad \frac{369.6ft}{3600sec}$$

.1026 ft/sec

Day 2 – Dividing

$$\frac{2}{3} \div \frac{5}{4}$$

Keep Chng Flip

$$\frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$$

Examples

Find each quotient.

a) $\frac{8}{20x^3} \div \frac{44}{25x^2}$

$$\frac{8}{20x^3} \cdot \frac{25x^2}{44} = \frac{200x^2}{880x^3} \qquad \frac{5}{22x}$$

b) $\frac{3x+9}{x^2} \div \frac{(x+3)}{1}$

$$\frac{\overset{3\cancel{(x+3)}}{3x+9}}{x^2} \cdot \frac{1}{\cancel{x+3}} = \frac{3}{x^2}$$

c) $\frac{y-6}{2y+6} \div \frac{y+2}{y+3}$

$$\frac{y-6}{2\cancel{(y+3)}} \cdot \frac{\cancel{y+3}}{y+2} = \frac{y-6}{2y+4}$$

11.6 Adding and Subtracting Rational Expressions

Remember to add or subtract fractions you need a common denom. !

$\frac{2}{3} + \frac{4}{5}$

$\frac{10}{15} + \frac{12}{15} = \frac{22}{15}$

$\frac{3}{4} - \frac{1}{2}$

$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

Examples (a-c No Calculator)

Simplify the following.

a) $\frac{4b}{15} + \frac{16b}{15}$ $\quad \frac{20b}{15}$ $\quad \frac{4b}{3}$

b) $\frac{7x+9}{x-3} - \frac{x-5}{x-3} = \frac{6x+14}{x-3}$

c) $\frac{3x}{11-x} + \frac{-5x}{x-11}$ $\quad \frac{8x}{11-x}$

$\frac{5x}{11-x}$

d) $\frac{x+7}{x^2+6x+9} + \frac{x+3}{x-3}$

$\cancel{x^2} + \cancel{4x} - 21 + \cancel{x^3} + \cancel{6x^2} + \cancel{9x} + 3x^2 + \cancel{18x} + 27$

$\boxed{\frac{x^3+10x^2+31x+6}{x^3+3x^2-9x-27}}$

$\cancel{x^3} + \cancel{6x^2} + 9x - \cancel{3x^2} - \cancel{18x} - 27$

e) $\frac{4d^2}{d} + \frac{d+2}{d^2}$

$\frac{4d^3}{d^2} + \frac{d+2}{d^2}$

$\boxed{\frac{4d^3+d+2}{d^2}}$

Example

For the first 15 miles, a biker travels at x miles per hour. Then, due to a downhill slope, the biker travels 2 miles at a speed that is 2 times as fast.

a) Write an expression to represent how much time the biker is bicycling.

$$\frac{2}{2}\left(\frac{15}{x}\right)+\frac{2}{2x} \qquad \frac{30}{2x}+\frac{2}{2x}=\frac{32}{2x}=\frac{16}{x}$$

b) If the biker is bicycling at a rate of 8 miles per hour for the first 15 ~~minutes~~ miles, find the total amount of time the biker is bicycling.

$$\frac{16}{8}=2 \text{ hours}$$

A train travels 5 miles from Lynbrook to Long Beach and then back. The train travels about 1.2 times as fast as returning from Long Beach, If r is the train's speed from Lynbrook to Long Beach, write and simplify an expression for the total time of the round trip.

$$\frac{1.2}{1.2}\left(\frac{5}{r}\right)+\frac{5}{1.2r}$$

$$\frac{6}{1.2r}+\frac{5}{1.2r}=\frac{11}{1.2r}$$

11.8 Rational Equations

Explore

Find the cross product of the following expressions. What do you notice?

$\frac{2}{5} = \frac{4}{10}$	$\frac{3}{7} = \frac{9}{21}$	$\frac{1}{5} = \frac{4}{20}$	$\frac{7}{3} = \frac{14}{6}$
$20=20$	$63=63$	$20=20$	$42=42$

Solve $\frac{7}{x} = \frac{5}{x-4}$

$7(x-4)=x5$

$7x-28=5x$

$-28=-2x$

$x=14$

Example

Callie can run 3 miles an hour faster than Michael. Callie can run 5 miles in the same time it takes Michael to run 3 miles. Solve $\frac{5}{x+3} = \frac{3}{x}$ to find how fast Michael can run. Check the solution.

$5x=3x+9$

$2x=9$

4.5

Examples

Solve the following.

a) $\frac{7}{y-3} = \frac{3}{y+1}$

$7y+7 = 3y-9$

$4y = -16$

$y = -4$

b) $\frac{5}{x+1} - \frac{1}{x} = \frac{2}{x^2+x}$

combine 1st

$\frac{5x}{x^2+1x} - \frac{x+1}{x^2+1x}$

$\frac{5x-x-1}{x^2+1x}$

$\frac{4x-1}{x^2+1x} = \frac{2}{x^2+x}$ same

$4x-1 = 2$

$4x = 3$

$x = 3/4$

c) $\frac{3x}{x-1} + \frac{6x-9}{x-1} = 6$

$\frac{9x-9}{x-1} = \frac{6}{1}$

$9x-9 = 6x-6$

$3x = 3$

$x = 1$ No Sol.
b/c can't divide by 1

Examples-Real World

On Saturdays, Lee helps her father install satellite TV systems. The jobs normally take Lee's father about 2.5 hours. But when Lee helps the jobs only take them 1.5 hours. If Lee were installing a satellite system herself, how long would the job take?

Father Rate $\frac{1 \text{ Jobs}}{2.5 \text{ hrs}} = .4 \text{ Job/hr}$ $\div 4/5$

Lee + Father $\frac{1 \text{ Jobs}}{1.5 \text{ hrs}} = \frac{2}{3} \text{ jobs/hr}$

3 3/4 hrs

A bus leaves a station and travels an average of 50 miles per hour towards a city. Another bus leaves the same station 20 minutes later and travels to the same city traveling 60 miles per hour. How long will it take the second bus to pass the first bus?

20/60

$$50x = 60\left(x - \frac{1}{3}\right)$$

$$50x = 60x - 20$$

$$-10x = -20$$

$$x = 2$$

$$2 - \frac{1}{3} = 1.\overline{6}$$

1 2/3 hours