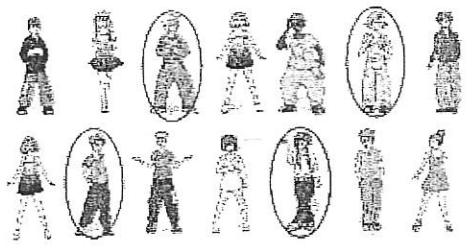
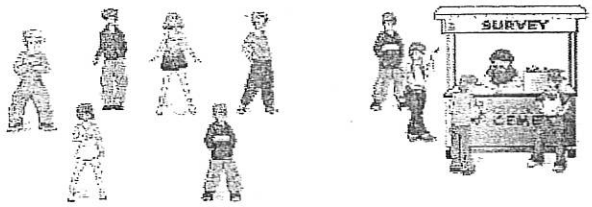
12.1 Samples and Studies

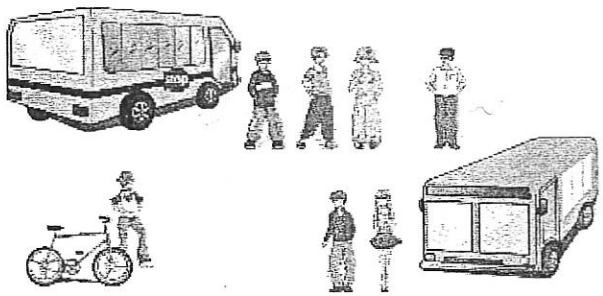
- Population – all members of a group of interest (BHS Students)
- Sample – part of population (4th hr)
  - Simple Sample - each member of pop. has equal chance of selection



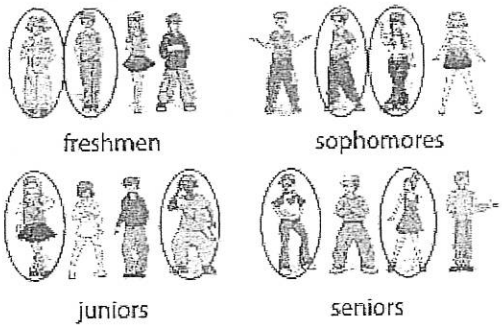
In a **systematic sample**, members are selected according to a specified interval from a random starting point, such as selecting every third student.



In a **self-selected sample**, members volunteer to be included in the sample.



In a **convenience sample**, members that are readily available or easy to reach are selected, such as the students on a particular bus.



In a **stratified sample**, the population is first divided into similar, nonoverlapping groups. Members are then randomly selected from each group.

Examples – No Calculator

1) Every fifth person walking out of a movie theater is asked to name their favorite type of movie.
   a. Identify the sample, and suggest a population from which it was selected.

every 5th person

all moviegoers

b. Classify the sample as simple, systematic, self-selected, convenience, or stratified. Systematic - interval based

2) At a popular restaurant, the manger checks the quality of the burgers every 20 minutes, starting at a randomly selected time.

a. Identify the sample, and suggest a population from which it was selected. Burgers every 20 min ; all burgers at rest.

b. Classify the sample as simple, systematic, self-selected, convenience, or stratified. systematic, interval of time

- Bias - an error that results in a misrepresentation of the population.

Examples – No Calculator

Identify each sample as biased or unbiased. Explain your reasoning.

The student senate surveys the students in one classroom to decide the theme for the spring dance. Biased, only 1 class

The Parent Association surveys the parents of every fifth student to decide whether to hold a fund-raiser. unbiased, random

You can collect the information in a variety of ways!

- Survey
- Observational Study – watch & record
- Experiment – control group vs. experimental group

Examples – No Calculator

Determine whether each situation describes a survey, an observational study, or an experiment. Explain your reasoning.

a) A retailer wants to evaluate their performance in customer service. They contact 1000 random customers asking if they would complete an evaluation form. Survey, responses of sample

b) Researchers analyze the reactions of rats to a vitamin.

Experiment, Same w/ & Same w/o Vitamine

c) A company shows five different commercials that advertise the same product to a group of students. The company records the students' reactions to each.

Obs. Study

d) The city council wants to build a community recreational center. They call 500 random citizens asking if they would consider paying a fee to join if the center is built.

Survey

When designing a fair survey it must not:

Be confusing, encourage a certain response, cause a strong reaction, or address more than one issue at a time.

Examples – No Calculator

Identify each survey question as biased or unbiased. If biased, explain your reasoning.

a) How often do you exercise? unbiased

b) Do you like basketball? If so, do you prefer watching high school, college, or professional? Biased, more than one issue

c) Is your favorite type of ice cream plain vanilla or delicious chocolate? Biased, encourages chocolate by wd usage

Example

A baseball bat manufacturer wants to test a new grip on their bats They selet 75 high school baseball players to try out the bat with the new grips and 75 other players to try out the old grips. Identify the experiment as biased or unbiased. If biased, explain your reasoning.

Biased, not randomly selected

12.2 Statistics and Parameters

- Statistic – describes a sample
- Parameter – describes a whole population

Example

Identify the sample and the population for each situation. Then describe the sampel statistic and the population parameter.

a) A movie rental business selects a random sample of 50 orders in one day. The median number of rentals per order is calculated.

Sample: 50 orders
population: all movie orders 1 day
Statistic: median per 50
parameter: median all orders

b) A stratified random sample of 2 trees of each species is selected from all trees at a nursery. The mean height of trees in the sample is calculated.

Sample: 2 trees each species
pop: all trees at nursery
Stat: mean ht of sample
pop: mean ht of all trees

- Standard Deviation – how spread out data is
  σ
  1-Var Stats
- Variance - Square of Stand dev. (σ^2)

Example

Leo tracked his homework scores for the past week: {100, 0, 100, 50, 0}. Find and interpret the standard deviation of the set.

44.7
Spread out

Kyle can throw a baseball left-handed or right-handed. Below are the speeds in miles per hour of 16 throws from each hand. Compare the means and standard deviations.

Left-Handed			
68	71	70	69
67	67	73	71
74	68	68	71
72	70	66	70

Right-Handed			
71	78	77	70
81	72	74	80
70	69	79	83
81	68	83	82

Mean 69.7 76.1

Stander. 2.2 5.3

Rt hander higher avg speed but greater variability.

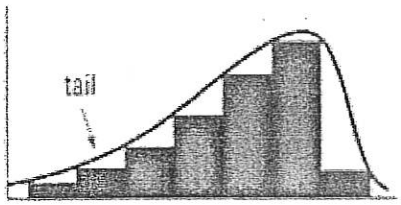
BOWLING Gerald and Erica compared their bowling scores. Compare the means and standard deviations.

Gerald			
166	201	99	210
123	155	121	203
131	138	211	200
142	141	129	192

Erica			
148	153	151	160
149	155	159	172
161	163	158	153
158	163	180	168

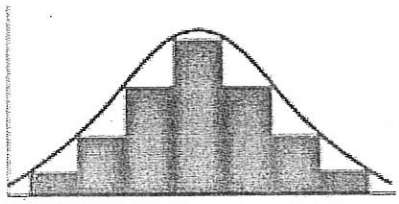
12.3 Distributions of Data

Negatively Skewed Distribution



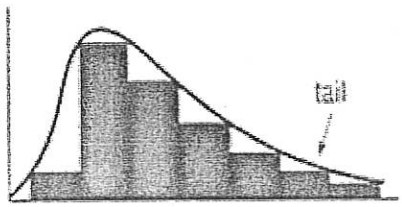
The majority of the data are on the right.

Symmetric Distribution



The data are evenly distributed.

Positively Skewed Distribution



The majority of the data are on the left.

"SAT" "ACT"

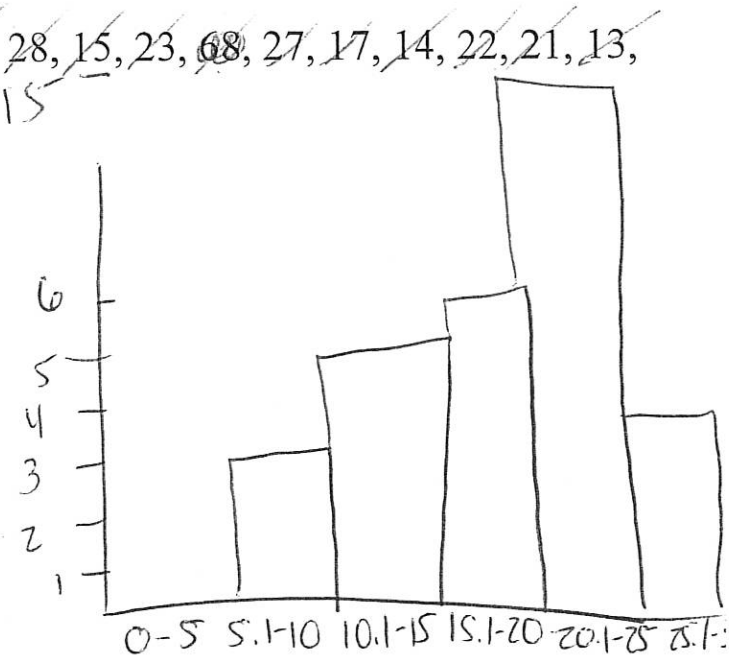
Example

Construct a histogram for the data, and use it to describe the shape of the distribution.

9, 18, 22, 12, 24, 25, 19, 25, 25, 28, 12, 22, 19, 28, 15, 23, 68, 27, 17, 14, 22, 21, 13, 24, 21, 9, 25, 16, 24, 16, 25, 27, 21, 10

Interval	Freq	
0-5		0
5.1-10	III	3
10.1-15	IIII	5
15.1-20	IIII I	6
20.1-25	IIII IIII IIII	15
25.1-30	IIII	4
30.1-35		
	1	1

68

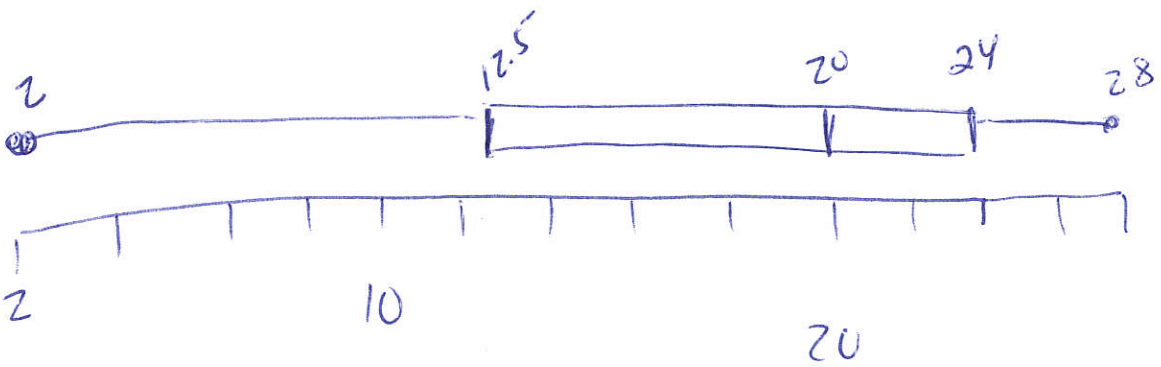


Bars must touch

Neg Skewed

Use a graphing calculator to construct a box-and-whisker plot for the data, and use it to determine the shape of the distribution.

9, 18, 22, 12, 24, 25, 19, 25, 2
5, 28, 12, 22, 19, 28, 15, 23, 6
8, 27, 17, 14, 22, 21, 13, 24, 21
9, 25, 16, 24, 16, 25, 27, 21, 10



Skewed rt.

Sometimes it is more appropriate to use the median over the mean to represent the center of the data!

Use the mean when there are no outliers.

Use the median when there are outliers.

Example

Use the data below from scores on Mr. Smith's recent History test to determine whether it is more appropriate to use the mean or the median and actually find the value. 35, 67, 70, 75, 75, 80, 85, 86, 90, 93, 97

median 80

BOWLING The averages for the bowlers on five teams are shown below. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

Bowling Average				
142	180	161	131	201
179	152	177	196	148
198	123	203	170	187
159	193	176	137	183

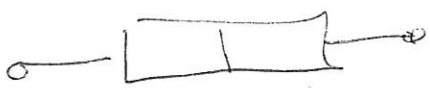
1 Var Stats

Skewed

SHOES The shoe sizes for a group of men are shown below. Describe the center and spread of the data using either the mean and standard deviation or the five-number summary. Justify your choice by constructing a box-and-whisker plot for the data.

low Standdev.

Shoe Size				
10	11	9	13	9
14	12	10	11	12
14	12	10	11	15
9	13	13	12	10



12.4 Comparing Sets of Data

Recall:

- Mean - Add all #s ÷ # of terms (no outliers)
- Median – middle # when ordered (outliers)
- Mode - most often (categories)

Examples

Find the mean, median, mode, range, and standard deviation of the data set obtained after adding 12 to each value.

Either add 1st or at end.

73, 78, 61, 54, 88, 90, 63, 78, 80, 61, 86, 78

86.2
90
90
36
11.3

Find the mean, median, mode, range, and standard deviation of the data set obtained after multiplying each value by 2.5.

4, 2, 3, 1, 4, 6, 2, 3, 7, 5, 1, 4

mean 8.75

med. 8.75
mode 10
range 15
stand dev. 4.5

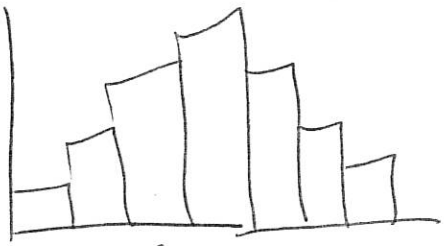
Examples

Brittany and Justin are playing a computer game. Their high scores for each game are shown below. Who is a better player? Justify your choice mathematically.

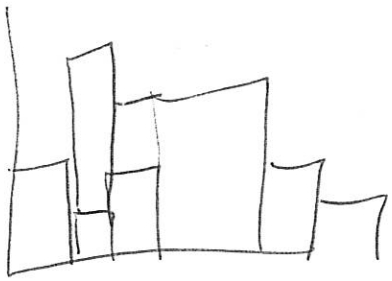
Brittany: 29, 43, 54, 58, 39, 44, 39, 53, 32, 48, 39, 49, 38, 31, 41, 44, 44, 45, 48, 31

Justin: 48, 26, 28, 53, 39, 28, 30, 58, 45, 37, 30, 31, 40, 32, 30, 44, 33, 35, 43, 35

Brittany, Justin skewed pos.



Brittany

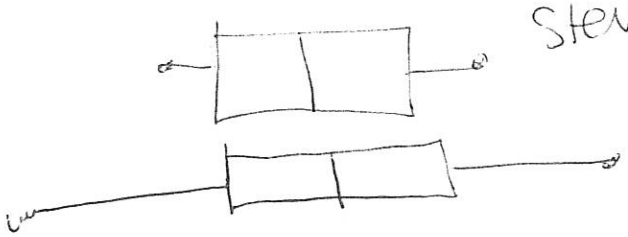


Justin

Steve and Kirt went fishing for the weekend. They weights of the fish they each caught are shown below. Who ultimately had a better day? Justify your answer mathematically.

Steve: 1.6, 2.1, 2.6, 1.3, 2.7, 3.2, 1.4, 2.3, 3.5, 1.9, 2.2, 2.7, 3.5, 1.4, 3.7, 3.4, 1.8, 2.5, 3

Kurt: 1.1, 3.2, 2.3, 3.7, 1.7, 2.7, 2.1, 4, 1, 2.9, 1.2, 3.3, 2.3, 4.5, 2.4, 3.9



Steve ← lower stand dev.

Who is the Greatest Baller?

12.5 Simulations

- Theoretical Probability – favorable / total (what should happen)
- Experimental Probability (AKA Relative Frequency) – event occurs / trials (did happen)
- Simulation – experiment or trial

Examples

Students were asked how they travel to school each morning. Find the experimental probability of randomly selecting a student who does not ride in a car or bus.

Mode	Frequency
Bike	3
Bus	21
Car	17
Walk	6

47

9/47

Examples

Mandy is a pitcher on her high school softball team. Last season 70% of her pitches were strikes. Design a simulation that can be used to estimate the probability that Mandy's next pitch is a strike.

RandInt(1, 10)

1-7 Strike

8-10 Ball

Conduct the above experiment and report the results.

Example

SCHOOL BUS Larry's bus is late 60% of the time. Design a simulation that can be used to estimate the probability that his bus is late.

RandInt (1,10)
1-6 Late
7-10 on time

Conduct the experiment above and report the results.

12.6 Permutations and Combinations

- Permutation – objects are arranged so order matters
  nPr
- Combination – objects arranged so order doesn't matter
  nCr
- Factorial – $5!_0 = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Examples – No Calculator

Identify each situation as a permutation or combination.

a) During a fire drill, a teacher checks the students in her row to see if everyone is present. Combo

b) In preparing for a competition, a tennis coach lists his players in order of ability. Perm.

c) A teacher uses a random number generator to create a seating chart for 20 students. Perm, diff order = diff chart

d) Ten athletes enter a race. The top three finishers move on to the next round.

Combo

e) 15 young ladies enter a beauty pageant. A queen, first runner-up, and second runner-up are crowned.

perm.

Examples

a) Shaquille has a 4-digit pass code to access his e-mail account. The code is made up of the even digits 2, 4, 6, and 8. Each digit can be used only once. How many different pass codes could Shaquille have?

4·3·2·1

(24)

b) A word processing program requires a used to enter a 5-digit registration code made up of the digits 1, 2, 3, 4, 5, 6, and 7. No digit can be used more than once. How many different registration codes are possible?

2520

7 6 5 4 3

c) A group of 4 seniors, 5 juniors, and 7 sophomores have volunteered to be on a fundraising committee. Mr. Davidson needs to choose 12 students out of the group. How many ways can the 12 students be chosen?

4+5+7 $_{16}C_{12} = 1820$

d) 16 people are signed up for a tennis tournament. If people are put into groups of 4 and the draw is determined randomly, what is the probability that Heather, Erin, Michele, and Patrick are put into the same group?

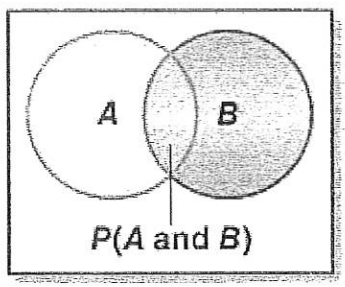
$_{16}C_4$ ✓ order doesn't matter

$\frac{1}{1820}$

12.7 Probability of Compound Events

- Independent Events – outcome of 1 event does not affect the outcome of the other

Ex
A Plane arriving on time
B Luggage lost



$P(A \text{ and } B) = P(A) \cdot P(B)$

Examples

Rae is flying from Birmingham to Chicago on a flight with a 90% on time record. On the same day, the chances of rain in Denver are predicted to be 50%. What is the probability that Rae's flight will be on time and that it will rain in Denver?

$.90 \times .50$

45%

4/21 9/21 4/21 2/21 21 marbles

A bag contains 6 black marbles, 9 blue marbles, 4 yellow marbles, and 2 green marbles. A marble is selected, replaced, and a second marble is selected. Find the probability of selecting:

a) P(blue, green) $\frac{9}{21} \cdot \frac{2}{21}$ $\frac{18}{441}$ 4.1%

b) P(not black, blue)

$\frac{15}{21} \cdot \frac{9}{21} = \frac{135}{441} = 30.6\%$

- Dependent Event – when the outcome of 1 even effects the other
  Ex/ not replacing marble.

P(A and B) = P(A) •P(B following A)

Example

At a school carnival, winners in the ring-toss game (15) are randomly given a prize from a bag that contains 4 sunglasses, 6 hairbrushes, and 5 key chains. Three prizes are randomly chosen from the bag and not replaced. Find each probability.

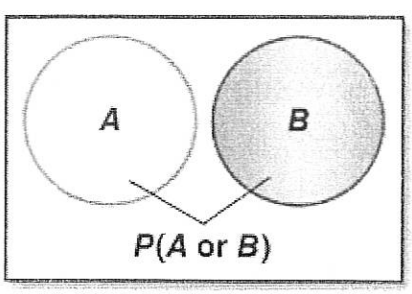
a) P(sunglasses, hairbrush, keychain)

$\frac{4}{15} \cdot \frac{6}{14} \cdot \frac{5}{13} = \frac{120}{2730} = \frac{4}{91}$

b) P(hairbrush, hairbrush, not a hairbrush)

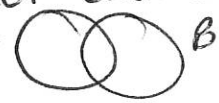
$\frac{6}{15} \cdot \frac{5}{14} \cdot \frac{9}{13} = \frac{270}{2730} \quad \frac{9}{91}$ ($9 \leftarrow 13-4$)

- Mutually Exclusive Events - events that cannot occur at same time
  Ex/ Drawing odd or even #



P(A or B)= P(A) + P(B)

P(A or B) = P(A) + P(B) − P(A and B) not exclusive



Examples

A card is being drawn from a standard deck. Find each probability. ← 52 cards

a) P(7 or 8) $\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$ (15%)

b) P(neither club or heart)

$\frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2}$

A die is being rolled. Find each probability. Die has 6 sides

a) P(less than 3)

$\frac{2}{6} = \frac{1}{3}$ 33%

b) P(even) $\frac{3}{6} = \frac{1}{2}$ (50%)

c) P(3 or 5)

$\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ (33%)

12.8 Probability Distributions

- Random Variable – Variable that is an outcome of a random event
- Discrete Random Variable – Variable with a countable # of possibilities

Example

A. The owner of a pet store asked customers how many pets they owned. The results of this survey are shown in the table. Find the probability that a randomly chosen customer has 2 pets.

Number of Pets	Number of Customers
0	3
1	37
2	33
3	18
4	9

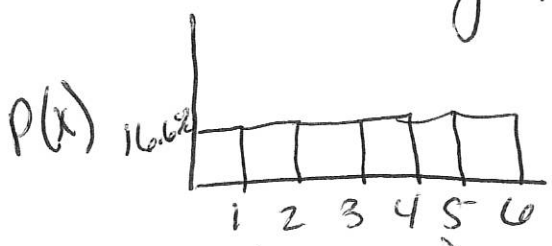
100

$\frac{33}{100} = 33\%$

B. Find the probability that a randomly chosen customer has at least 3 pets.

$18+9=\frac{27}{100}$ (27%)

- Probability Distribution – Probability of every possible value of the random variable
- Probability Graph – bar graph that displays a distribution



X = Number on Dice

1/6 = 16.6%

Example

A. POPULATION The table shows the probability distribution of the number of students in each grade at Sunnybrook High School. Show that the distribution is valid.

X = Grade	$P(X)$
9	0.29
10	0.26
11	0.25
12	0.2

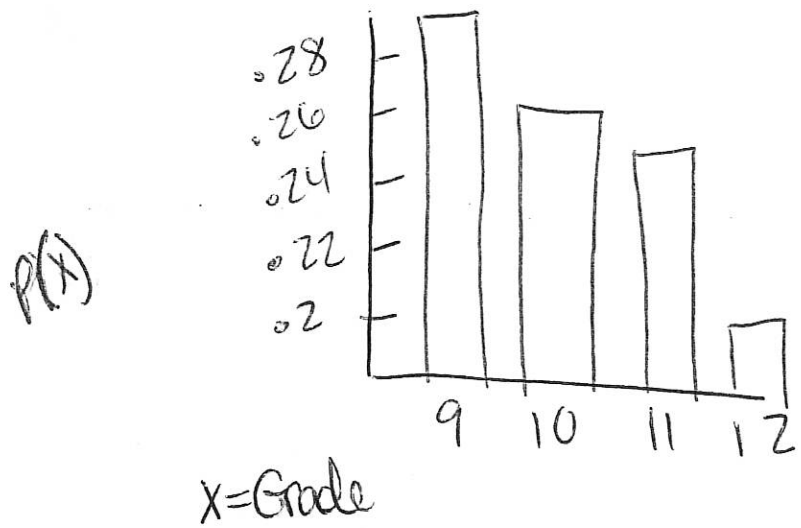
.29+.26+.25+.2=1

B. If a student is chosen at random, what is the probability that he or she is in grade 11 or 12?

.45

.25+.2

C. Make a probability graph of the data.



- Expected Value -

The expected value of a discrete random variable is the weighted average of the values of the variable. It is calculated by finding the sum of the products of every possible value of X and its associated probability $P(X)$.

$E(X) = [X_1 \cdot P(X_1)] + [X_2 \cdot P(X_2)] + \ldots + [X_n \cdot P(X_n)]$, where n is the total number of values of X

Example

Nikki paid $5 for an entry into a contest with the following prize values.

A. Create a probability distribution.

Prize Value	Probability
$500	1 in 5000
$5000	1 in 50,000
$20,000	1 in 500,000
$50,000	1 in 2,000,000

X	P(X)
500	.0002
5000	.00002
20,000	.000002
50,000	.0000005
0	.9979775 ← 1 − #

Sum .0020225

chance of winning →

B. Calculate the expected value.

X · P(X) Sum $500(.0002)+5000(.0002)+20{,}000(.000002)+\ldots = .265$

C. Interpret your results.

- One entry is expected to win ≈ 27 cents
- Maker of contest spends about 27 cents per entry, so they make 4.73 per entry.