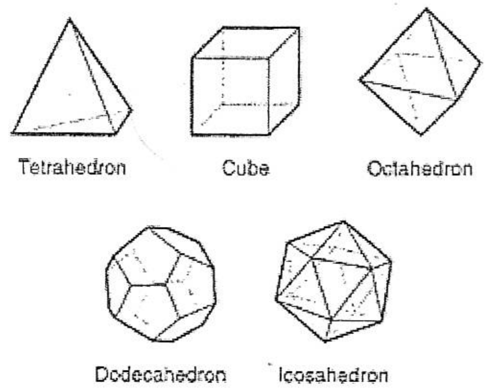
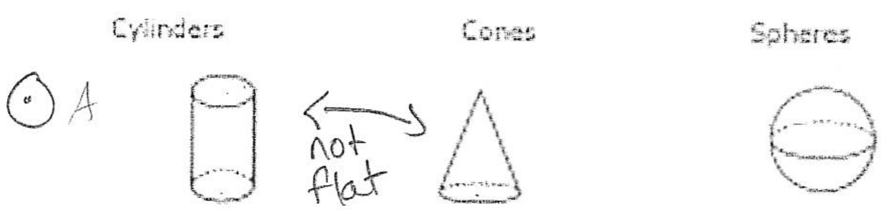
12.1 Representations of Three Dimensional Figures

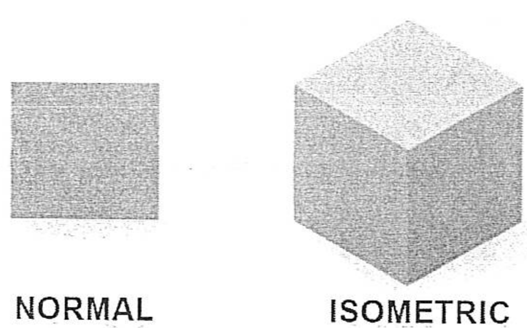
- Polyhedron – Solid figures that enclose a region.
  - Parts
    - Face – flat surfaces
    - Edge – where 2 faces meet
    - Base – flat "usually parallel"
  - Prisms – Solid w/ 2 congruent parallel faces
    - Regular Prisms – regular polygons as bases
  - Pyramid – 1 base, all faces meet at 1 point
  - Regular Polyhedron –



  - Cross Section – Intersection of the solid + the plane
- Not Polyhedrons

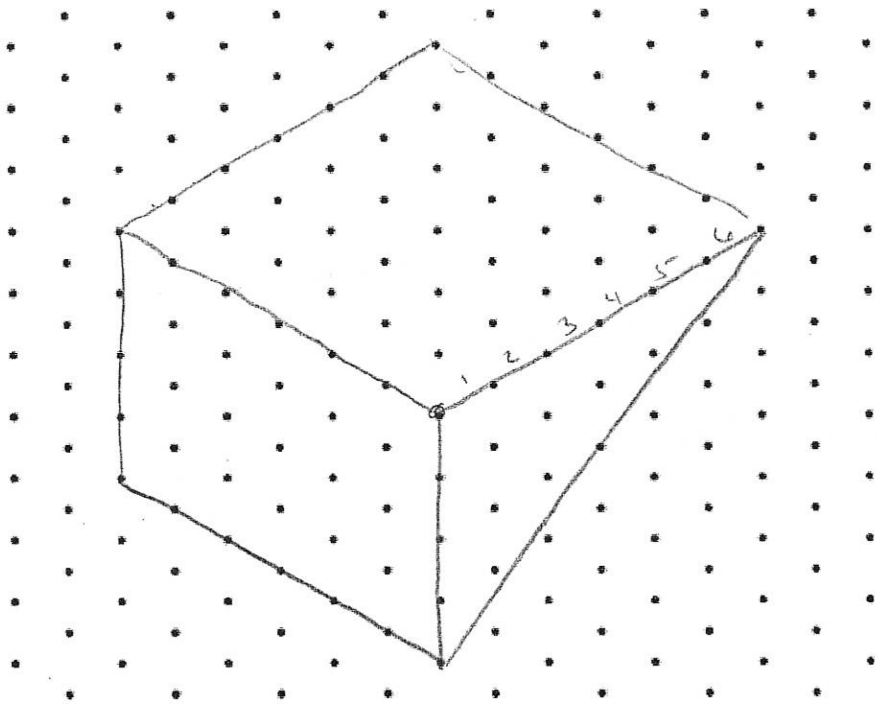


- Isometric View –



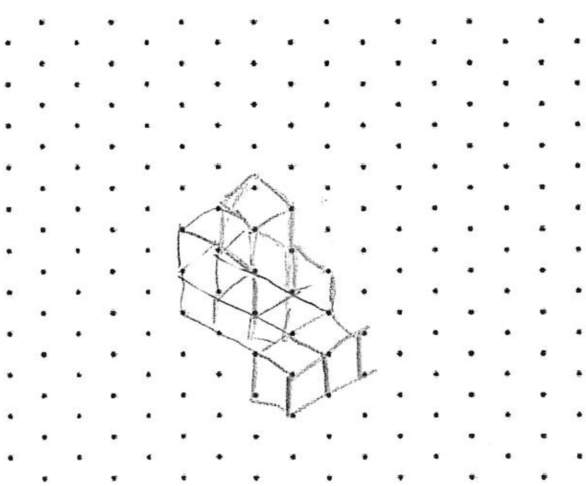
Example

Use isometric dot paper to sketch a triangular prism 6 units high, with bases that are right triangles with legs 6 units and 4 units long.

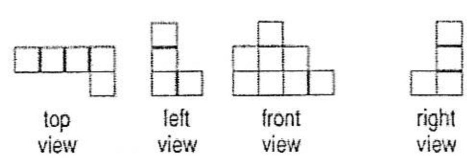


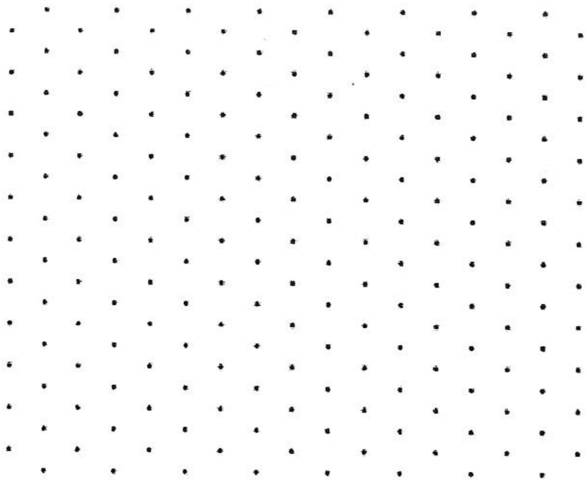
- Orthographic Drawing – shows top, left, rt, & front views

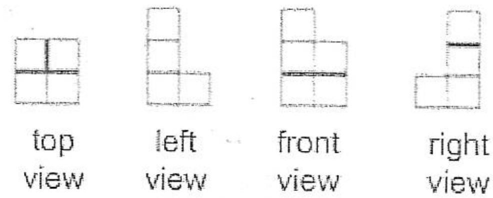
Example



Use isometric dot paper and the orthographic drawing to sketch a solid.

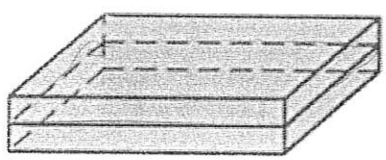






Example

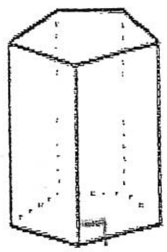
BAKERY A customer ordered a two-layer sheet cake. Determine the shape of each cross section of the cake below.



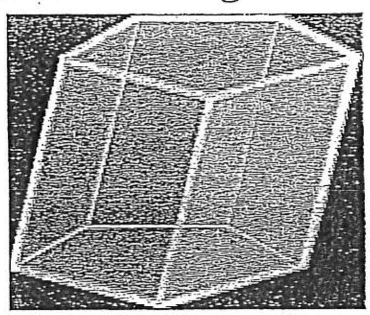
Rectangle

12.2 Surface Areas of Prisms

- Lateral Faces – Faces that are not bases
- Lateral Edges – lateral faces meet.
- Right Prism –



- Oblique Prism – a prism with lateral edges not perpendicular to the bases



- Lateral Area – the sum of the areas of the lateral faces (L)
- Lateral Area of a prism → $P \cdot h$ ← ht of prism

  Perimeter of Base
- Surface Area of a prism

↓

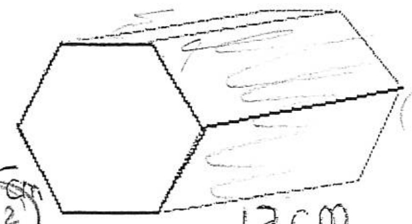
$P \cdot h + 2B$

Area of Base

units^2

Example

Find the lateral area of the regular hexagonal prism.



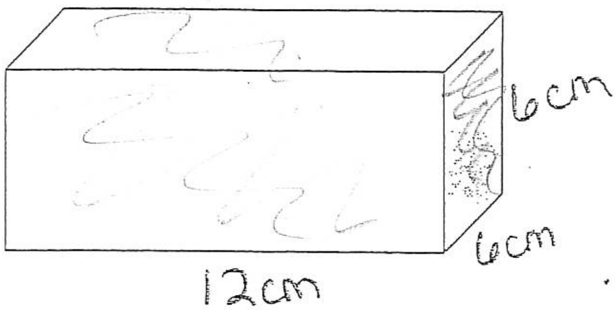
P

30 · 12 = 360 cm^2

Example

Find the surface area of the square prism.

all



36 × 2

72 × 4

= 360 cm^2

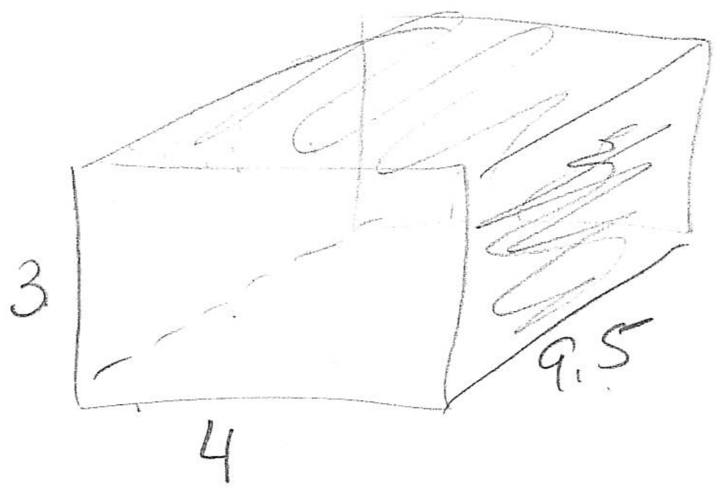
OR

Ph + 2B

24 · 12 + 2 · 36

Example

A solid block of marble will be used for a sculpture. If the block is 3 feet wide, 4 feet long, and 9.5 feet high, find the surface area of the block.



P · h + 2B

27 3

81 + 2 · 4.5 · 9

12 × 2 = 24

9.5 × 3 × 2 = 57

9.5 × 4 × 2 = 76

157 ft^2

12.2 Surface Areas of Cylinders

- Axis – is the segment with endpoints that are centers of the circular bases
- Right Cylinder – perp. base
- Oblique Cylinder

Circumf. $2\pi r$

- Lateral Area of a cylinder $LA = \underbrace{2\pi rh}_{Ph}$ or πdh
- Surface Area of a cylinder $SA = \underbrace{2\pi rh}_{Ph} + \underbrace{2\pi r^2}_{2B}$

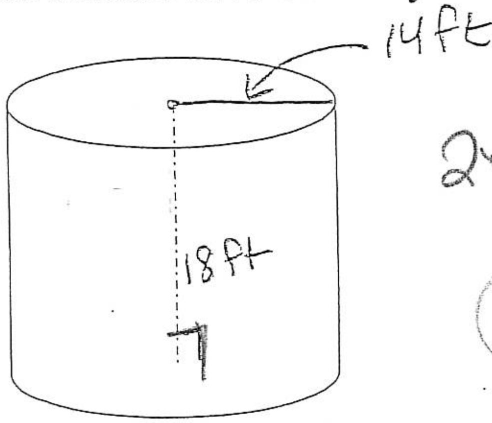
Example

A fruit can is cylindrical with aluminum sides and bases. Each can is 12 centimeters tall and the diameter of the can is 6.3 centimeters. How many square centimeters of aluminum are used to make the sides of the can?

$2\pi \cdot 3.15 \cdot 12 = 237.5 cm^2$

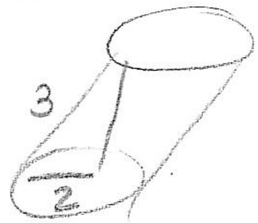
Example

Find the surface area of the cylinder.



$2\pi \cdot 14 \cdot 18 + 2\pi 14^2$

$2814.9 ft^2$



Example

Find the radius of the base of a right cylinder if the surface area is 1658.8 square feet and the height is 10 feet. 12

$2\pi r 10 + 2\pi r^2 = 1658.8$

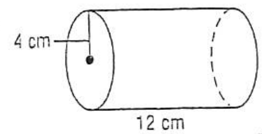
$62.8r + 6.28r^2 = 1658.8$

$6.28r^2 + 62.8r - 1658.8 = 0$

Quad!

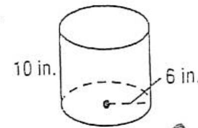
Exercises

Find the lateral area of each cylinder. Round to the nearest tenth.

1. 

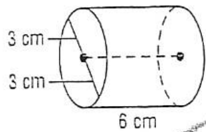
$2\pi 4 \cdot 12$

301.6 cm^2

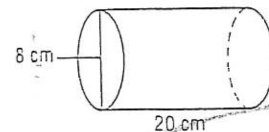
2. 

$2\pi 6 \cdot 10$

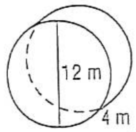
377 in^2

3. 

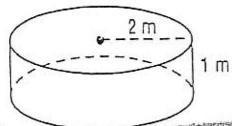
113.1 cm^2

4. 

502.7 cm^2

5. 

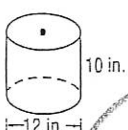
150.8 m^2

6. 

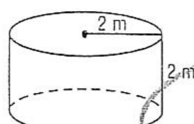
12.6 m^2

Exercises

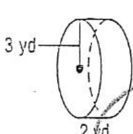
Find the surface area of each cylinder. Round to the nearest tenth.

1. 

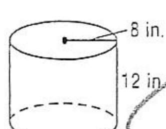
603.2 in^2

2. 

50.3 m^2

3. 

94.2 yd^2

4. 

1005.3 in^2

12.3 Surface Areas of Pyramids

- Regular Pyramid – pyramid w/ regular base

- Lateral Area of a $\frac{1}{2}P\ell$ ← slant
  (P ← perim of base)
- Surface Area of a $\frac{1}{2}P\ell + B$
  (B ← Area of Base)

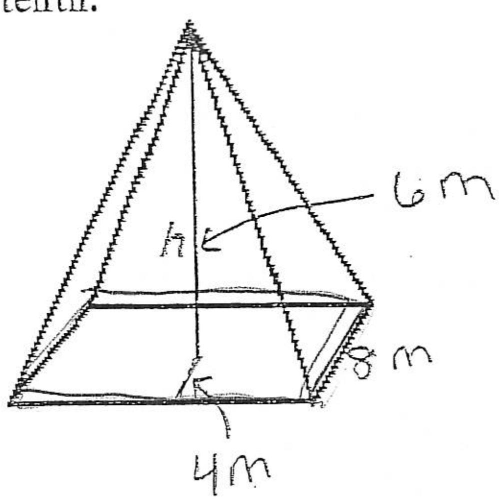
Example

A candle store offers a pyramidal candle that burns for 20 hours. The square base is 6 centimeters on a side and the slant height of the candle is 22 centimeters. Find the lateral area of the candle.

$$\frac{1}{2}\cdot \overset{P}{24} \cdot \overset{\ell}{22} = 264\,cm^2$$

Example

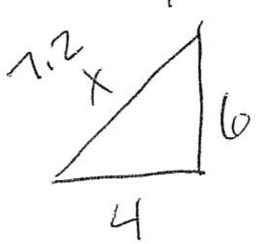
Find the surface area of the regular pyramid to the nearest tenth.



$$\frac{1}{2}P\ell + B \quad (l \cdot w,\ 8 \times 8)$$

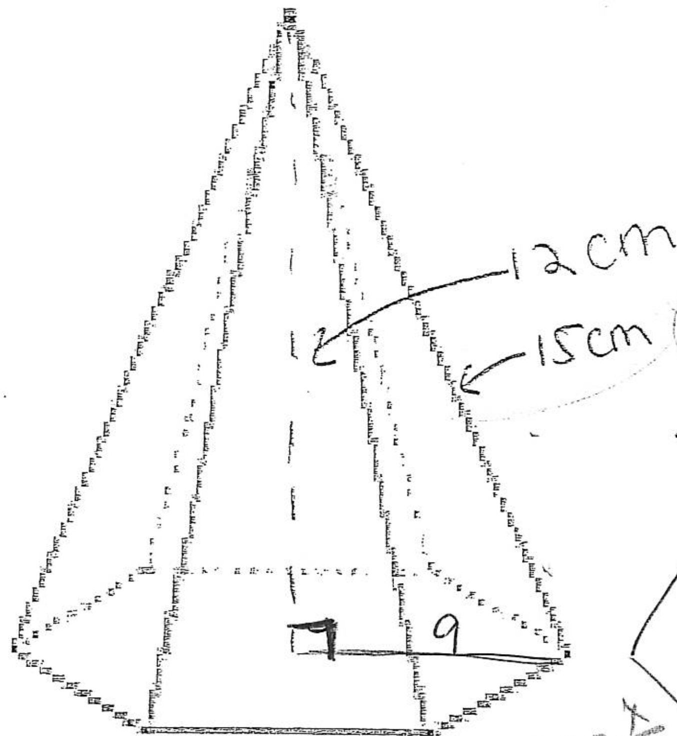
$$\frac{1}{2}\cdot 32\cdot 7.2 + 64$$

$$179.2\,m^2$$



Example

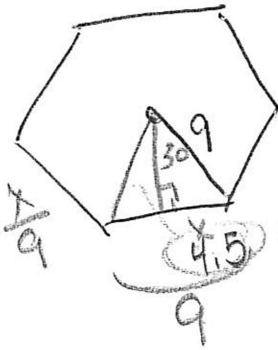
Find the surface area of the regular pyramid. Round to the nearest tenth.



$\frac{1}{2}Pl + B$

$\frac{1}{2} \cdot 54 \cdot 15 + \frac{1}{2}Pa$

$\downarrow 54 \quad \downarrow 7.8$



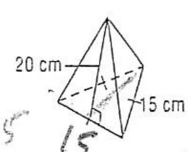
$\sin 30 = \frac{x}{9}$

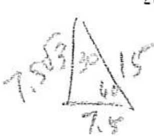
~~439.8 cm²~~

≈615

Find the surface area of each regular pyramid. Round to the nearest tenth if necessary.

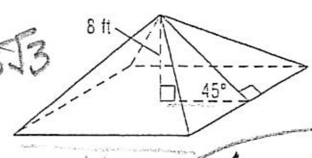
$\frac{1}{2}Pl + B$

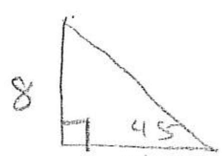
1. 



$\frac{1}{2} \cdot 45 \cdot 20 + \frac{1}{2} \cdot 15 \cdot 7.5\sqrt{3}$

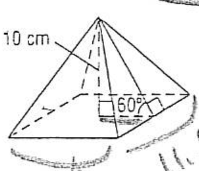
547.5 cm²

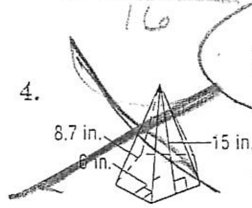
2. 



618 ft²

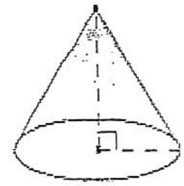
$\frac{1}{2} \cdot 64 \cdot 8\sqrt{2} + 256$

3. 

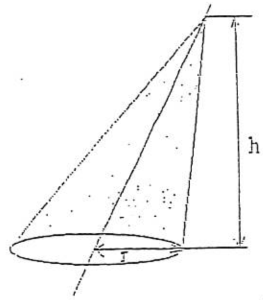
4. 

$\tan 30 = \frac{x}{10}$ 392.16 $\frac{1}{2} \cdot 46.16 \cdot 11.5 + 133$

12.3 Surface Area of Cones



- Rt. Cone – a cone with an axis that is also the altitude
- Oblique



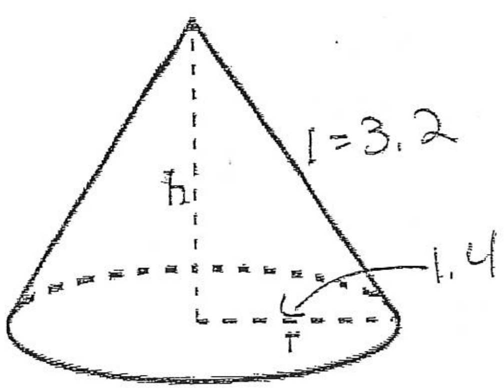
- Lateral Area of a cone $\pi r l$ ← slant
- Surface Area of a $\pi r l + \pi r^2$

Example
A sugar cone has an altitude of 8 inches and a diameter of 2 ½ inches. Find the lateral area of the sugar cone.

$\pi \cdot 1.25 \cdot 8.1$

$31.8 in^2$

Example
Find the surface area of the cone. Round to the nearest tenth.



$\pi r l + \pi r^2$

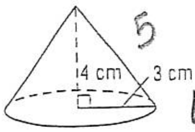
$\pi 1.4 \cdot 3.2 + \pi 1.4^2$

14.

$20.2u^2$

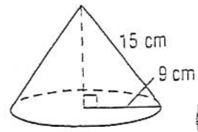
Exercises

Find lateral area of each circular cone. Round to the nearest tenth.

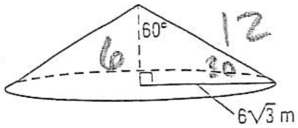
1. 

5

$47.1 cm^2$

2. 

$424.1 cm^2$

3. 

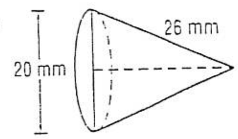
6 12 30

$\sin 60 = \frac{6\sqrt{3}}{x}$ 3

$\pi r l$

$\pi 6\sqrt{3} \cdot 12$

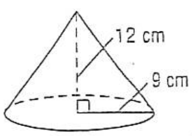
$391.8 \ m^2$

4. 

$816.8 mm^2$

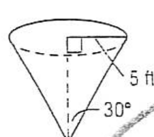
Exercises

Find the surface area of each cone. Round to the nearest tenth.

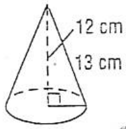
1. 

$\pi r l + \pi r^2$

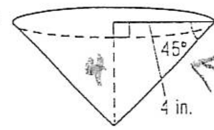
$678.6 cm^2$

2. 

$235.6 f^2$

3. 

$282.7 cm^2$

4. 

$\leftarrow 4\sqrt{2}$

$\pi \cdot 4 \cdot 4\sqrt{2} + \pi 4^2$

$121.4 in^2$

Inside Space

12.4 Volumes of Prisms and Cylinders

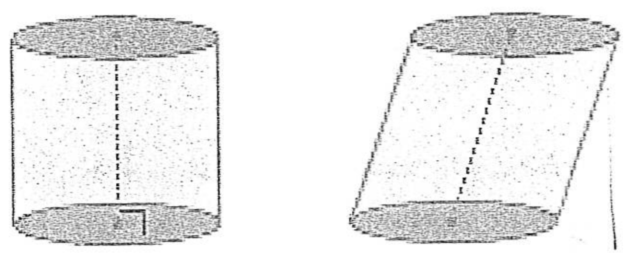
Units3

- **Prisms:** V = Bh ← Area of Base; h ← ht prism
  - Rectangular Prism V = lwh (lw = B)
- **Cylinder:** V = Bh or V = ∏r ² h

Triangular $\frac{1}{2}bh \cdot h$ (bh ← base, h ← prism)

Hex Prism $\frac{1}{2}Pa \cdot h$

Does the formula for right cylinders also work for oblique cylinders? Yes



right cylinder oblique cylinder

Examples

The weight of water is .036 pounds times the volume of water in cubic inches. How many pounds of water would fit into a rectangular child's pool that is 12 inches deep, 3 feet wide, and 4 feet long?

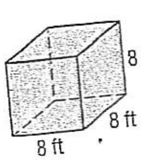
$l \cdot w \cdot h$

$12 \cdot 36 \cdot 48 = 20{,}736\,in^3 \times .036 = 746.5$ pounds

(36 = 3x12, 48 = 4x12)

Find the volume of each.

1.

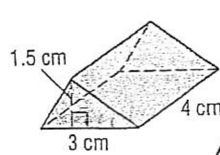


Bh

8x8x8

$512\,ft^3$

2.

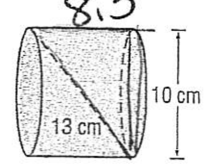


Bh $9\,cm^3$

$\frac{1}{2}bh$

$\frac{1}{2} \cdot 3 \cdot 1.5 \cdot 4$

3.



8.3

Bh

$\pi 5^2 \cdot 8.3$

$651.9\,cm^3$

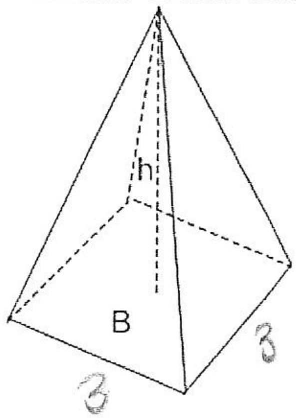
12.5 Volumes of Pyramids

Bh

- **Volume of a Pyramid** – $V = \frac{1}{3}Bh$

Example

Jim has a solid clock that is in the shape of a square pyramid. The clock has a base of 3 inches and a height of 7 inches. Find the volume of the clock.



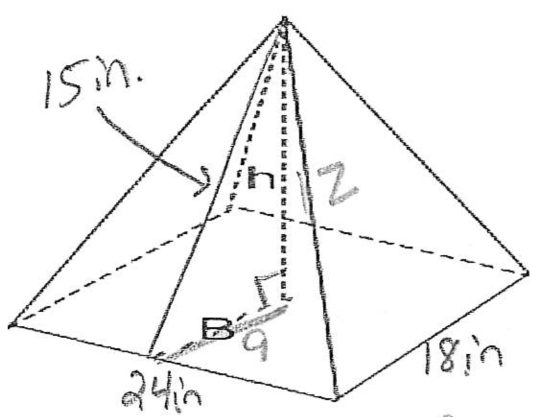
$\frac{1}{3}Bh$

$\frac{1}{3}\cdot 9\cdot 7$

$21 in^3$

Example

Find the volume of the pyramid.



$9^2+x^2=15^2$

$\frac{1}{3}Bh$

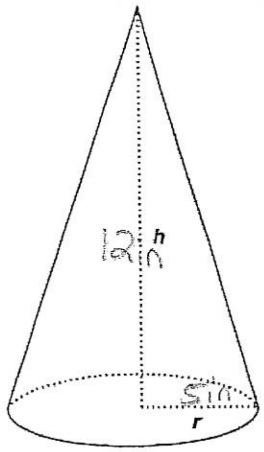
$\frac{1}{3}24\cdot 18\cdot 12$

$1728 in^3$

12.5 Volumes of Cones

- **Volume of a Cone** – V = $\pi r^2 h$ $\frac{1}{3}\pi r^2 h$

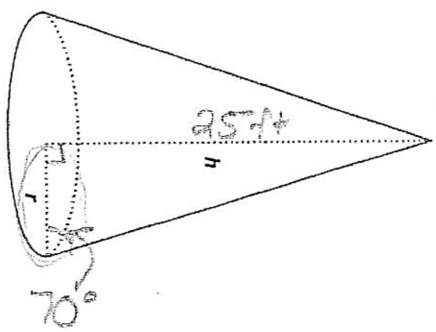
Example
Find the volume of each cone to the nearest tenth.



$\frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi 5^2 \cdot 12$

314.2 in³



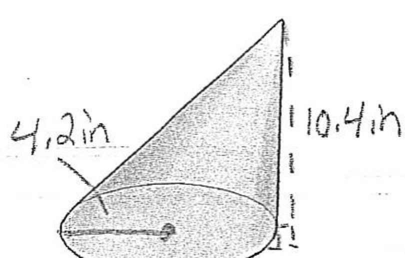
$r \cdot \tan 70 = \frac{25}{r} \cdot r$

$r = 9.1$

$\frac{1}{3}\pi 9.1^2 \cdot 25$

≈ 2168 ft³

Example
Find the volume of the oblique cone to the nearest tenth.



$\frac{1}{3}\pi r^2 h$

$\frac{1}{3}\pi 4.2^2 \cdot 10.4$

192.1 in³

12.6 Surface Areas of Spheres

Can you have the following in a sphere?

- Radius
- Chord
- Diameter
- Tangent Line

Yes

- Great Circle – when a plane intersects a sphere so that it contains the center of the sphere

- Hemisphere – $\frac{1}{2}$ sphere

Work in teams of two to complete the activity on pg. 672
Each sphere has been cut along a great circle.

- Trace the great circle onto a piece of paper.
- Cut out the circle.
- Fold the circle into eight sectors. Then unfold and cut them apart.
- Tape the pieces together like the picture in the book.
- Tape the patter to the sphere

Approximately what fraction of the surface of the sphere is covered by the pattern? $\frac{1}{4}$

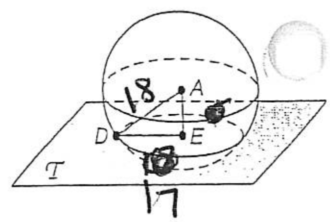
What is the area of the pattern in terms of r, the radius of the sphere? πr^2

Make a conjecture about the formula for the surface area of a sphere. $4 | \pi r^2$

Surface Area of a Sphere $4\pi r^2$

Surface Area of Hemisphere $3\pi r^2$

In the figure, A is the center of the sphere, and plane $\mathcal{T}$ intersects the sphere in circle E. Round to the nearest tenth if necessary.



1. If $AE = 5$ and $DE = 12$, find AD.

13

2. If $AE = 7$ and $DE = 15$, find AD.

16.5

3. If the radius of the sphere is 18 units and the radius of $\odot E$ is 17 units, find AE.

$\sqrt{18^2-17^2} = 5.9$

4. If the radius of the sphere is 10 units and the radius of $\odot E$ is 9 units, find AE.

$\sqrt{10^2-9^2}$ 4.4

5. If M is a point on $\odot E$ and $AD = 23$, find AM.

23

still radius

6. a hemisphere with a radius of the great circle 8 yards $3\pi r^2$ 8

603.2 yd^2

7. a hemisphere with a radius of the great circle 2.5 millimeters

58.9 mm^2

12.6 Volumes of Spheres

- **Volume of a Sphere** – $V = \frac{4}{3}\Pi r^3$.

• Volume Hemisphere
$\frac{2}{3}\pi r^3$

Example

a) Find the volume of a sphere with a radius of 15 centimeters.

$\frac{4}{3}\pi 15^3$

$14{,}137 cm^3$

b) Find the volume of a sphere with a great circle that has a circumference of 25 centimeters.

$2\pi r = 25$

$r = 3.97$

$\frac{4}{3}\pi 3.97^3$

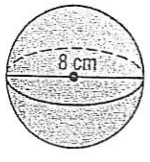
$\approx 262.1 cm^3$

c) Find the volume of a hemisphere with a diameter of 6 feet.

$\frac{2}{3}\pi 3^3$

$= 56.5 ft^3$

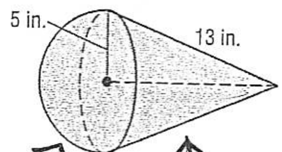
Find the volume of each solid. Round to the nearest tenth.



$\frac{4}{3}\pi 4^3$

$268.1 cm^3$

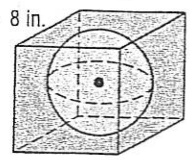
5.



$\frac{2}{3}\pi 5^3 + \frac{1}{3}\cdot\pi 5^2 \cdot 12$

$575.9 in^3$

6.



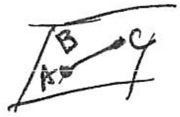
difference between volume of cube and volume of sphere

cube $8^3 = 512$

sphere $\frac{4}{3}\pi 4^3 = 268.1$

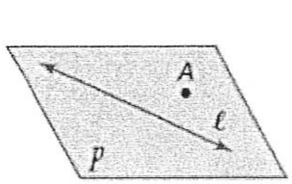
$\approx 243.9 in^3$

12.7 Spherical Geometry



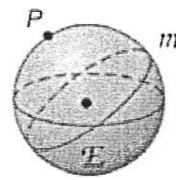
- Euclidean Geometry – uses a plane that extends forever
- Spherical Geometry – geometry on a sphere

Plane Euclidean Geometry



Plane $\mathcal{P}$ contains line ℓ and point A not on line ℓ.

Spherical Geometry

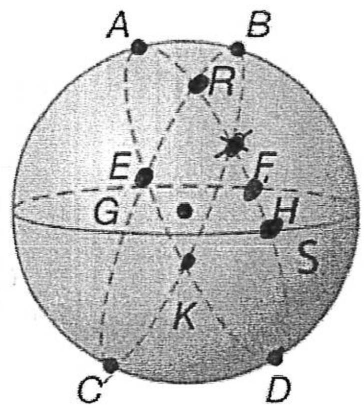


Sphere $\mathcal{E}$ contains great circle m and point P not on m. Great circle m is a line on sphere $\mathcal{E}$.

Great circle

Example

Name each of the following on sphere s.



a) Two lines containing point R

Great circle $\overleftrightarrow{BC}$ $\overleftrightarrow{AD}$

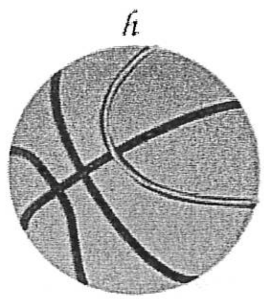
b) A segment containing point C

Part of line $\overline{CR}$ $\overline{CK}$

c) A triangle

$\triangle REF$ $\triangle CRD$

SPORTS Determine whether the line *h* on the basketball shown is a line in spherical geometry. Explain.



No, not great circle

Example

Tell whether the following postulate or property of plane Euclidean geometry has a corresponding statement in spherical geometry. If so, write the corresponding statement. If not, explain your reasoning.

a) If two lines are parallel, they never intersect. True (Note no parallel lines in spherical Geo)

b) Any two distinct lines are parallel or intersect once.

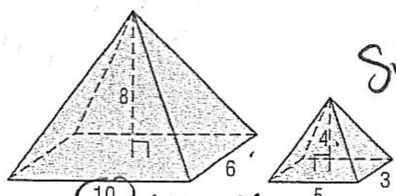
False, intersect 2 times

12.8 Congruent and Similar Solids (All cubes, All spheres)

- Similar Solids – Same shape & consistent scale factor
  $\sim$
- Congruent Solids – Same size & shape
  $\cong$

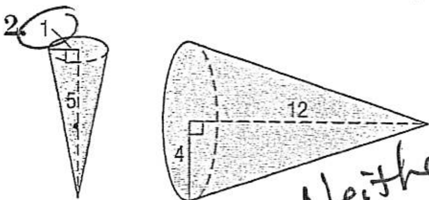
Examples

Determine whether each pair of solids are *similar*, *congruent*, or *neither*.

1. 

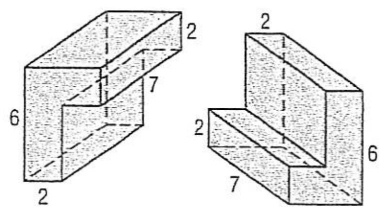
Similar

$\frac{10}{5} = \frac{8}{4} = \frac{6}{3}$

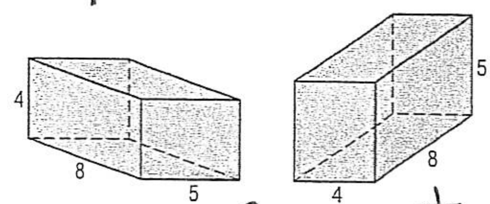
2. 

Neither

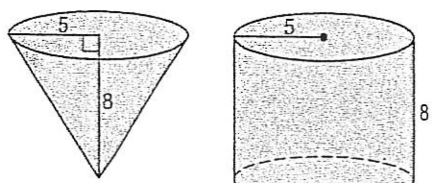
$\frac{1}{4} \neq \frac{5}{12}$

3. 

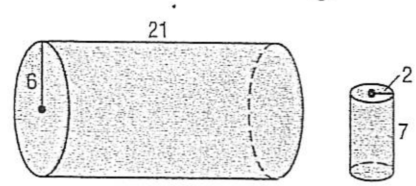
Congruent

4. 

Congruent

5. 

Neither

6. 

Similar $\frac{6}{2} = \frac{21}{7}$

- **Theorem** – If two solids are similar with a scale factor of a:b, then the surface areas have a ratio of $a^2 : b^2$, and the volumes have a ratio of $a^3 : b^3$

Ex

Scale factor	Area Scale factor	Volume Scale Factor
$\frac{1}{2}$	$\frac{1^2}{2^2} = \frac{1}{4}$	$\frac{1^3}{2^3} = \frac{1}{8}$
$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$

Example

Softballs and baseballs are both used to play a game with a bat. A softball has a diameter of 3.8 inches while a baseball has a diameter of about 2.9 inches.

a) Find the scale factor of the two balls. $\frac{3.8}{2.9} = \boxed{\frac{38}{29}}$

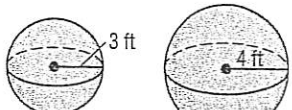
b) Find the ratio of the surface area of the two balls. $\frac{38^2}{29^2}$ $\boxed{\frac{1444}{841}}$

c) Find the ratio of the volumes of the two balls.

$\frac{38^3}{29^3}$ $\boxed{\frac{54{,}872}{24{,}389}}$

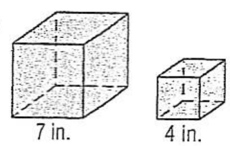
Find the scale factor for each pair of similar figures. Then find the ratio of their surface areas and the ratio of their volumes.

1.



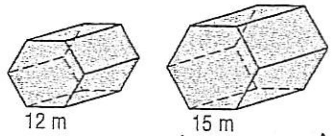
$\frac{3}{4}$ $\frac{9}{16}$ $\frac{27}{64}$

2.



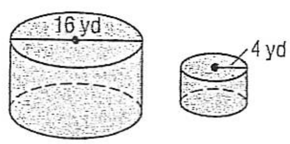
$\frac{7}{4}$ $\frac{49}{16}$ $\frac{343}{64}$

3.



$\frac{4}{5}$ $\frac{16}{25}$ $\frac{64}{125}$

4.



$\frac{2}{1}$ $\frac{4}{1}$ $\frac{8}{1}$

Ex

A small can has radius of 4cm and ht of 3.8cm. A larger can has radius of 5.2cm. Assume similar.

a) Cylinder. Scale factor? $\frac{4}{5.2}$ $\boxed{\frac{10}{13}}$

b) Volume of large can? $\frac{10^3}{13^3}$ $\frac{1000}{2197} = \frac{191}{X}$ smaller $\pi r^2 h$ / Big $419.6\ cm^3$

$h = 4.9$