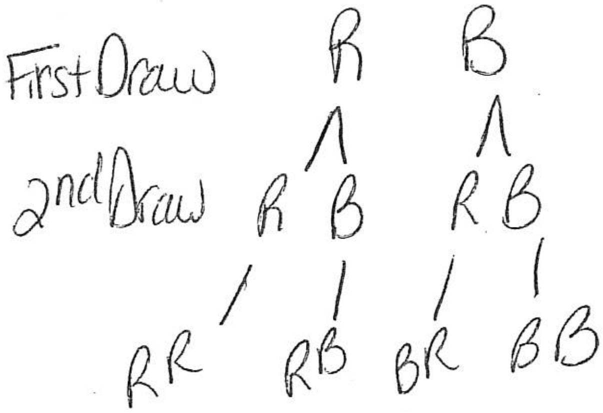
## 13.1 Representing Sample Spaces

- Experiment – situation involving chance
- Outcome – result of a trial
- Event – one or more outcomes of an experiment.
- Sample Space – set of all possible outcomes
- Tree Diagram – way to organize information in multi-stage experiments

Example

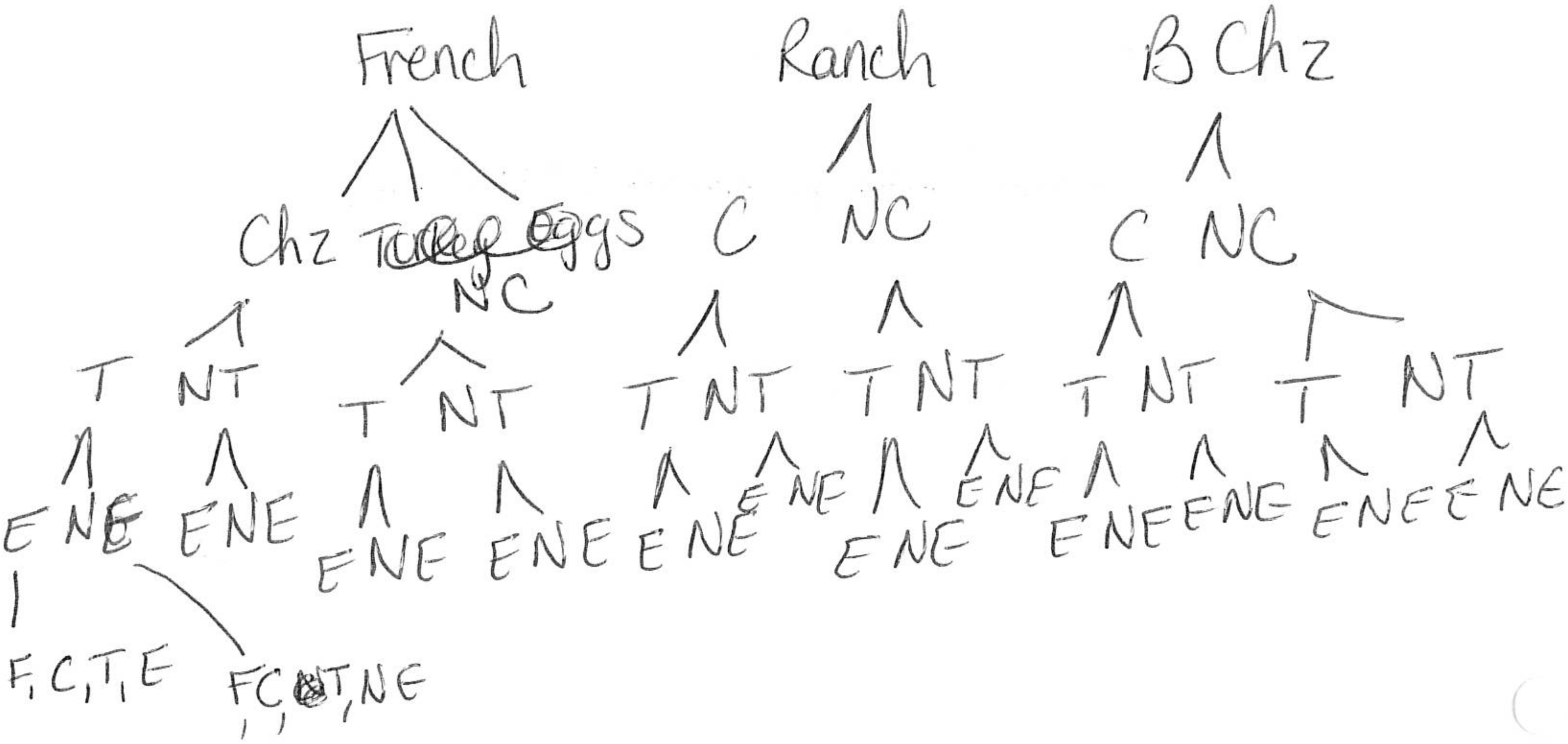
**One red token and one black token are placed in a bag. A token is drawn and the color is recorded. It is then returned to the bag and a second draw is made. Represent the sample space for this experiment by making an organized list a table, and a tree diagram.**

| Outcomes | Red | Blk |
|---|---|---|
| Red | R R | RB |
| Blk | B R | B B |



Example

A chef's salad at a local restaurant comes with a choice of French, ranch, or blue cheese dressings and optional toppings of chees, turkey, and eggs. Draw a tree diagram to represent the sample space for salad orders.




- Fundamental Counting Principle – if one event can occur in m ways and another can occur in n ways, then the total possible outcomes can be found using $m \cdot n$

Examples

New cars are available with a wide selection of options for the consumer. One option is chosen from each category shown. How many different cars could a consumer create in the chosen make and model?

| Car Options | Exterior Color | Interior Color | Seat Material | Engine | Computer Navigation System | Wheels | Doors |
|---|---|---|---|---|---|---|---|
| Number of Choices | 11 | 7 | 5 | 3 | 6 | 4 | 3 |

$11 \cdot 7 \cdot 5 \cdot 3 \cdot 6 \cdot 4 \cdot 3$

83,160

Example

License plates are available so that there are three letters followed by three numbers in certain states. 26 options 10 digits 0-9

a) How many possible license plates are there if you can repeat letters and numbers?

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$

17,576,000

b) How many possible license plates are there if you cannot repeat letters?

$26 \cdot 25 \cdot 24 \cdot 10 \cdot 10 \cdot 10$

15,600,000

c) How many possible license plates are there if you cannot use the letters O and I, but can repeat numerals and letters?

$24 \cdot 24 \cdot 24 \cdot 10 \cdot 10 \cdot 10$

13,824,000

## 13.2 Probability with Permutations and Combinations

- Permutation – arrangement of elements where order matters
  $_5P_2$
- Combination – arrangement where order doesn't matter
  $_5C_2$

Determine if the following are permutations or combinations and justify your thinking.

a) The security code on an alarm system. Perm.

b) Choosing 5 people to be on an advisory committee Combo.

c) Selecting a sophomore class president, vice president, secretary, and treasurer Perm.

d) Selecting 10 freshmen to go on a class trip Combo.

e) The numbers you enter to open your locker Perm.

Examples

**TALENT SHOW** Eli and Mia, along with 30 other people, sign up to audition for a talent show. Contestants are called at random to perform for the judges. What is the probability that Eli will be called to perform first and Mia will be called second?

$_{30}P_2 = 870$

$\frac{1}{870}$

There are 12 puppies for sale at the local pet shop. Four are brown, four are black, three are spotted, and one is white. What is the probability that all the brown puppies will be sold first?

$_{12}P_4$ $\frac{1}{495}$

**TILES** A box of floor tiles contains 5 blue (bl) tiles, 2 gold (gd) tiles, and 2 green (gr) tiles in random order. The desired pattern is bl, gd, bl, gr, bl, gd, bl, gr, and bl. If you selected a permutation of these tiles at random, what is the probability that they would be chosen in the correct sequence?

$\frac{1}{756}$

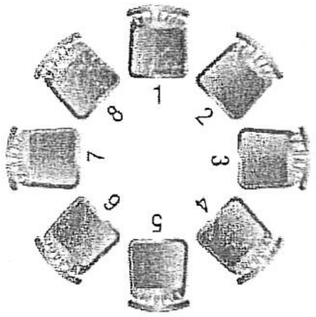
**TILES** A box of floor tiles contains 3 red (rd) tiles, 3 purple (pr) tiles, and 2 orange (or) tiles in random order. The desired patter is rd, rd, pr , pr, or, rd, pr, and or. If you selected a permutation of these tiles at random, what is the probability that they would be chosen in the correct sequence?

- Circular Permutation – objects arranged in a circle or loop

$$\frac{n!}{n} \text{ or } (n-1)!$$

Example

A. SEATING If 8 students sit at random in the circle of chairs shown, what is the probability that the students sit in the arrangement shown? Explain your reasoning.




$$\frac{1}{5040}$$

B. CRAYONS You purchase a box of 8 crayons. If the crayons are packaged in random order, what is the probability that the crayon on the far left is red? Explain your reasoning.

$\frac{1}{8}$ Row

Example

A set of alphabet magnets are placed in a bag. If 5 magnets are drawn from the bag at random, what is the probability that they will be the letters *a*, *e*, *i*, *o*, and *u*?

$$\frac{1}{65,780}$$

## 13.3 Geometric Probability

- Geometric Probability – probability that involves a geometric measure such as length or area

### Examples

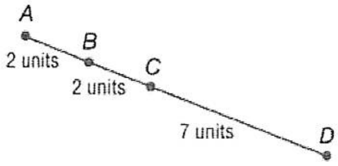
5280ft

1) The Roadrunner runs down a mile-long stretch of highway. He stops once at random. If the Roadrunner stops on the 100-foot section patrolled by the Coyote, the Coyote will chase him. What is the probability that this will happen?


$\frac{100}{5280} = \frac{5}{264}$ (1.9%)

2)

Point $Z$ is chosen at random on $\overline{AD}$. Find the probability that $Z$ is on $\overline{AB}$.




$\frac{2}{11} = 18\%$

3)

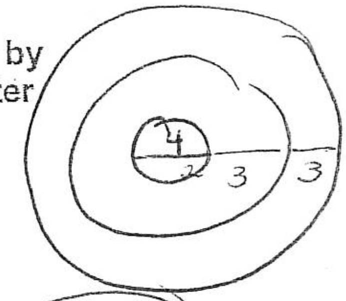
**ORBITS** Halley's Comet orbits the earth every 76 years. What is the probability that Halley's Comet will complete an orbit within the next decade?

$\frac{10}{76} = 13\%$

$\frac{5}{38}$

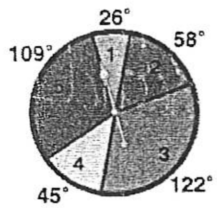
4)

**DARTS** The targets of a dartboard are formed by 3 concentric circles. If the diameter of the center circle is 4 inches and the circles are spread 3 inches apart, what is the probability that a player will throw a dart into the center circle?




$$\frac{\pi 2^2}{\pi 8^2} = \frac{1}{16} \quad 6.25\%$$

5) Use the spinner to the right to answer the following questions.




a. P(Pointer landing on section 3)

$$\frac{122}{360} \quad 34\%$$

b. P(pointer landing on section 1)

$$\frac{26}{360} \quad 7.2\%$$

c. P(pointer landing on section 2 or 4)

$$\frac{58+45}{360} \quad \frac{103}{360} \quad 28.6\%$$

## 13.4 Simulations

- Probability Model – mathematical model used to match a random phenomenon
- Simulation - the use of probability to recreate a situation so that the likelihood of outcomes can be estimated. It consists of four key parts:
  1) Determine each possible outcome + its theoretical probability
  2) State any assumptions
  3) Describe the probability for each situation

4) Define what a trial is and the number of trials to perform.

You can conduct simulations using spinners, coins, dice and random number generators (Calculators)

RandInt(1, 27)

Examples

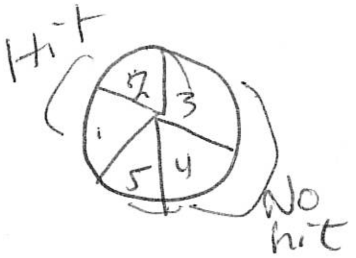
**BASEBALL** **Maria got a hit 40% of the time she was at bat last season. Design a simulation that can be used to estimate the probability that she will get a hit at her next at bat this season.**

RandInt(1, 10)

1-4 Hit

6-10 No hit

or




Now, using the simulation above, conduct the simulation and report the results using the appropriate numerical and graphical summaries.

10 at bats

4, 10, 1, 9, 7, 3, 10, 8, 4, 3

Hits 5

No hit 5


PIZZA A survey of Longmeadow High School students found that 30% preferred cheese pizza, 30% preferred pepperoni, 20% preferred peppers and onions, and 20% preferred sausage. Design a simulation that can be used to estimate the probability that a Longmeadow High School student prefers each of these choices.

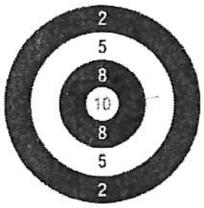
RandInt (1, 10)

Chz 1-3
Pepp. 4-6
Peppers + Onions 7-8
Sausage 9-10

- Random Variable –

- Expected Variable –

## Example




**ARCHERY** Suppose that an arrow is shot at a target. The radius of the center circle is 3 inches, and each successive circle has a radius 5 inches greater than that of the previous circle. The point value for each region is shown.

A. Let the random variable $Y$ represent the point value assigned to a region on the target. Calculate the expected value $E(Y)$ for each shot of the arrow.

B. Design a simulation to estimate the average value or the average of the results of your simulation of shooting this game. How does this value compare with the expected value you found in part a?

Region 2 $\quad \frac{\pi 18^2 - \pi 13^2}{1018} = \frac{155}{324}$

Region 5 $\quad \frac{\pi 13^2 - \pi 8^2}{1018} = \frac{35}{108} = \frac{105}{324}$

Region 8 $\quad \frac{\pi 8^2 - \pi 3^2}{\underset{(\pi 18^2)}{1018}} = \frac{55}{324}$

Region 10 $\quad \frac{\pi 3^2}{\pi 18^2} = \frac{1}{36} = \frac{9}{324}$

$10 \cdot \frac{1}{36} + 8 \cdot \frac{55}{324} + 5 \cdot \frac{35}{108} + 2 \cdot \frac{155}{324}$

4.21

Rand(1, 324)

1-155 Region 2

156-261 Region 5

262-297 Region 8

298-307 Region 10

- Law of Large Numbers – as the number of trials of a random process increases, the average value will approach the expected value.

## 13.5 and 13.6 Probabilities of Events

- Independent Events – Probability of A does not affect Prob of B
- Dependent Events – if prob. that A occurs, impact prob. of B
- Conditional - $P(B|A)$ Probability of B, given that A has already occured

| Type of Event | Example | Probability Rule |
|---|---|---|
| Independent Events | Prob. of Heads and Rolling a 5 $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$ | $P(A \text{ and } B) = P(A) \cdot P(B)$ |
| Dependent Events | Prob. 2 diamonds if don't replace $\frac{13}{52} \cdot \frac{12}{51}$ | $P(A \text{ and } B) = P(A) \cdot P(B\|A)$ B given A |
| Conditional | | $P(B\|A) = \frac{P(A \text{ and } B)}{P(A)}$ |
| Mutually Exclusive | | |
| | | P(A or B) = P(A) + P(B) – P(A and B) |
| Complementary | | |

- Mutually Exclusive –
- Complement -

## Examples

**Determine whether the events are *independent* or *dependent*. Then find the probability.**

**1.** In a game two dice are tossed and both roll a six.

Indep $\frac{1}{36}$

**2.** From a standard deck of 52 cards, a king is drawn without replacement. Then a second king is drawn.

Dep $\frac{1}{221}$

**3.** From a drawer of 8 blue socks and 6 black socks, a blue sock is drawn and not replaced. Then another blue sock is drawn.

Dep $\frac{4}{13}$

**Find each probability.**

**4.** A green marble is selected at random from a bag of 4 yellow, 3 green, and 9 blue marbles and not replaced. What is the probability a second marble selected will be green?

$2/15$

**5.** A die is tossed. If the number rolled is between 2 and 5, inclusive, what is the probability the number rolled is 4?

$1/4$

**6.** A spinner with the 7 colors of the rainbow is spun. Find the probability that the color spun is blue, given the color is one of the three primary colors.

$1/3$

**7. VENDING** Mina wants to buy a drink from a vending machine. In her pocket are 2 nickels, 3 quarters and 5 dimes. What is the probability she first pulls out a quarter and then another quarter?

$1/15$

**8. ESSAYS** Jeremy's English class is drawing randomly for people to critique their essays. Jeremy draws first and his friend, Brandon, draws second. If there are 20 people in their class, what is the probability they will draw each other's names?

$1/380$

## Examples – Day 2

**Determine whether the events are *mutually exclusive* or *not mutually exclusive*. Then find the probability. Round to the nearest tenth of a percent if necessary.**

**1.** drawing a card from a standard deck and choosing a king or an ace

mutually exclusive $\frac{2}{13}$

**2.** rolling a pair of dice and doubles or a sum of 6 is rolled

Not mutually exclusive $\frac{5}{18}$

**3.** drawing a two or a heart from a standard deck of 52 cards

Not mutually exclusive $\frac{4}{13}$

**4.** rolling a pair of dice and a sum of 8 or 12 is rolled

mutually exclusive $\frac{1}{6}$

**Determine the probability of each event.**

**5.** If the chance of being selected for the student bailiff program is 1 in 200, what is the probability of not being chosen?

199/200

**6.** If you have a 40% chance of making a free throw, what is the probability of missing a free throw?

60%

**7.** What is the probability of spinning a spinner numbered 1 to 6 and not landing on 5?

5/6

**8.** Jeanie bought 10 raffle tickets. If 250 were sold, what is the probability that one of Jeanie's tickets will not be selected?

$\frac{240}{250}$