

## 2.1 Writing Expressions

When writing expressions look for key words to signal operations and variables.

Key Terms for operations/variables

Addition	Sum, in addition, plus
Subtraction	take away, less than, difference decrease (3 less than x) $x - 3$
Multiplication	mult, product, times, by (Area)
Division	Divide, Quotient $\frac{2}{3}$
Variable	a, b, c, ... "a number", "x"

Verb "is" = Quantity ( )

Example – No Calculator

Translate each sentence into an equation.

a) A number b divided by three is six less than c.

$$\frac{b}{3} = c - 6$$

b) Fifteen more than z times 6 is 11 less than y times 2.

$$15 + z \cdot 6 = 2y - 11$$

c) Three times x less than 10 equals 4.

$$10 - 3x = 4$$

d) The <sup>+</sup>sum of y and four times t is equal to 12.

$$y + 4t = 12$$

### Example

A jelly bean manufacturer produces 1,250,000 jelly beans per hour. How many hours does it take to produce 10,000,000 jelly beans?

8 hrs

$$\begin{array}{r} 1,250,000 \times = 10,000,000 \\ \hline 1,250,000 \end{array}$$

Total

There are more than 87,000 flights per day in the United States. How many days will it take for 261,000 flights to have occurred?

$$\begin{array}{r} 87,000 \times = 261,000 \\ \hline 87,000 \end{array}$$

3 days

- Formula – a rule for a relationship between 2 quantities (= sign)

### Example – No Calculator

Translate the sentence into a formula.

a) The perimeter of a square equals four times the length of a side.

$$P = 4s$$

b) Four times  $\pi$  times the radius squared is the surface area of a sphere.

$$4(\pi r^2) = S$$

### Example – No Calculator

Translate each equation into a sentence.

a)  $12 - 2x = -5$

2x less than 12 is equivalent to -5

b)  $a^2 + 3b = \frac{c}{6}$

a squared plus 3 time b equals c divided by 6

### Example

Write a problem based on the given information:

F = cost of fries

$F + 1.50 =$  cost of burger

$\rightarrow 4(\underbrace{F + 1.50}_{\text{Burger}}) - f = 8.25.$

The cost of a burger is \$1.50 more than the cost of fries. Four times the cost of a burger minus fries equals \$8.25. How much do fries cost?

### Example

Write a problem based on the given information.

P = the number of players in the game;  $5p + 7 =$  the number of cards in a deck

Five times p (# of players in game) plus seven is the number of cards in a deck. How many players are in the game?

## 2.2 Solving One-Step Equations

- Equivalent Equations – equations w/ same solution  
 $2x + 1 = 11$        $3x = 15$

Key Ideas:

- 1) Whatever you do to one side of an equation you must do to the other!
- 2) You solve equations using opposite operations

### Examples

Solve the following

$$h - 12 = -27$$

$$+12 \quad +12$$

$$h = -15$$

$$c + 102 = 36$$

$$-102 \quad -102$$

$$c = -66$$

$$-1\frac{3}{8}k = \frac{2}{3}$$

$$\frac{-11}{8} \quad \frac{-11}{8}$$

$$k = -\frac{16}{33}$$

$$-75 = -15b$$

$$\div -15 \quad \div -15$$

$$b = 5$$

P  
E  
m D  
A S ↑

$$-\frac{1}{4} = \frac{2}{3}x$$

$$\frac{2}{3} \quad \frac{2}{3}$$

$$x = -\frac{3}{8}$$

### Examples

Ricardo is driving 780 miles to Memphis. He drove about  $\frac{3}{5}$  of the distance on the first day. About how many miles did Ricardo drive?

$$= 780 \cdot \frac{3}{5}$$

$$468$$

Allison is making a stained glass window. Her pattern requires that one fifth of the glass should be blue. She has 288 square inches of blue glass. If she intends to use all of her blue glass, how much glass will she need for the entire project?

$$\frac{1}{5}x = 288$$

$$1440$$

### 2.3 Solving Multi-Step Equations

To solve multi-step equations you must use the order of operations in reverse order!

P  
E  
M  
D  
A  
S ↑

$$3x + 2 = 10$$

$$\frac{3x}{3} = \frac{8}{3}$$

$$x = 2.\overline{6}$$

Example

Solve each equation.

$$2x + 11 = 3$$

$$\begin{array}{r} -11 \quad -11 \\ \hline \end{array}$$

$$\begin{array}{r} 2x = -8 \\ \hline 2 \quad 2 \end{array}$$

$$\boxed{x = -4}$$

$$12 = -7x - 9$$

$$\begin{array}{r} +9 \quad +9 \\ \hline \end{array}$$

$$\begin{array}{r} 21 = -7x \\ \hline -7 \quad -7 \end{array}$$

$$\boxed{x = -3}$$

$$\begin{array}{r} \frac{3}{2}x - 8 = 11 \\ +8 \quad +8 \end{array}$$

$$\frac{3}{2}x = 19$$

$$12.6$$

$$\boxed{12 \frac{2}{3}}$$

$$12 \cdot \frac{k+9}{12} = -2 \cdot 12$$

$$\begin{array}{r} k+9 = -24 \\ -9 \quad -9 \end{array}$$

$$\boxed{k = -33}$$

$$3 \cdot \frac{c+1}{-3} = -21 \cdot -3$$

$$c+1 = 63$$

$$\boxed{c = 62}$$

$$7 \cdot \frac{x-5}{7} = 8 \cdot 7$$

$$x-5=56$$

$$x=61$$

## Day 2 – Real World Applications

### Example

Susan had a \$10 coupon for the purchase of any item. She bought a coat that was on sale for  $\frac{1}{2}$  its original price. After using the coupon, Susan paid \$125 for the coat before taxes. What was the original price of the coat? Write an equation for the problem. Then solve the equation.

$$\frac{1}{2}x - 10 = 125$$

$$\frac{1}{2}x = 135$$

$$\$270$$

Len read  $\frac{3}{4}$  of a novel over the weekend. Monday, he read 22 more pages. If he has read 220 pages, how many pages does the book have? Write an equation and solve it.

$$\frac{3}{4}x + 22 = 220$$

$$\frac{3}{4}x = 198$$

$$264$$

- Consecutive Integers –

Integers in counting order  
 $x, x+1, x+2$

- Consecutive Even Integers –

Integers Div. by 2 in counting order  
 $x, x+2, x+4$

- Consecutive Odd Integers –

$x, x+2, x+4$

- Number theory –

Study of #s & relationship between them.

### Examples

Write an equation for the following problems. Then solve the equation and answer the problem.

- a) Find three consecutive odd integers with a sum of 57

$$x + x+2 + x+4 = 57$$

$$3x + 6 = 57$$

$$3x = 51$$

$$x = 17$$

17, 19, 21

- b) Find three consecutive integers with a sum of 21

$$x + x+1 + x+2 = 21$$

$$3x + 3 = 21$$

$$3x = 18$$

$$x = 6$$

6, 7, 8



c) Find three consecutive odd integers with a sum of -51

$$x + x + 2 + x + 4 = -51$$

$$3x + 6 = -51$$

$$3x = -57$$

$$x = -19$$

$$-19$$

$$-17$$

$$-15$$

## 2.4 Solving Equations with the Variable on Each Side

### Examples

Solve  $8 + 5c = 7c - 2$

$$-5c \quad -5c$$

$$8 = 2c - 2$$

$$10 = 2c$$

$$c = 5$$

Solve  $5x + 2 = 6 - 7x$

$$-5x \quad -5x$$

$$2 = 6 - 12x$$

$$-4 = -12x$$

$$x = \frac{1}{3}$$

Solve  $1.3x = 3.3x + 2.8$

$$-3.3x \quad -3.3x$$

$$-2x = 2.8$$

$$x = 1.4$$

Solve  $\frac{1}{3}(18 + 12x) = 6(2x - 7)$

$$6 + 4x = 12x - 42$$

$$6 = 8x - 42$$

$$48 = 8x$$

$$x = 6$$

Solve  $7(x - 1) = -2(3 + x)$

$$7x - 7 = -6 - 2x$$

$$7x + 1 = -2x$$

$$-1 = -9x$$

$$x = \frac{1}{9}$$

### Special Situations

$x = 6$	1 Solution
$3 = 5$	No Solution
$5 = 5$	Inf. Sol.

Solve  $8(5x - 2) = 10(32 + 4x)$

$$40x - 16 = 320 + 40x$$

$$40x = 336 + 40x$$

$$0 = 336$$

No Sol.

Solve  $4(x + 20) = \frac{1}{5}(20x + 400)$

$$4x + 80 = 4x + 80$$

All #s

Solve  $7x + 5(x - 1) = -5 + 12x$

$$7x + 5x - 5 = -5 + 12x$$

$$12x - 5 = -5 + 12x$$

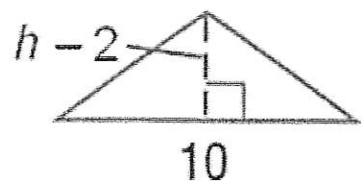
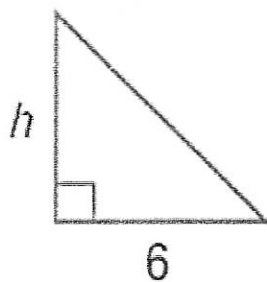
$$12x = 12x$$

All #s

### Example

Find the value of  $h$   
so that the figures  
have the same area.

Triangle =  $\frac{1}{2}bh$



$$\frac{1}{2} \cdot 6 \cdot h = \frac{1}{2} \cdot 10 \cdot (h-2)$$

$$3 \cdot h = 5(h-2)$$

$$3h = 5h - 10$$

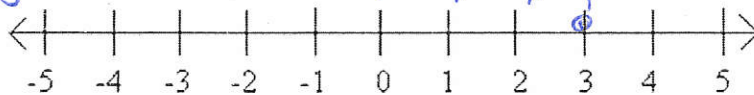
$$-2h = -10$$

$$h = 5$$

## 2.5 Solving Equations with Absolute Value

What does absolute value represent?

Distance from Zero!  
Always Pos



### Examples – Non Calculator

Evaluate  $|x - 7| + 15$  if  $x = 5$ .

Handwritten calculation:  $5$  is substituted for  $x$ , resulting in  $2 + 15 = 17$ .

Evaluate  $23 - |3 - 4x|$  if  $x = 2$ .

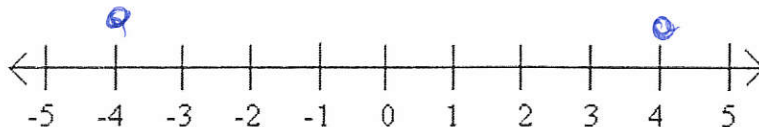
Handwritten calculation:  $23 - |3 - 8| = 23 - 5 = 18$ .

Evaluate  $|2x - z| + 6$  if  $x = 3$  and  $z = 4$ .

Handwritten calculation:  $|6 - 4| + 6 = 2 + 6 = 8$ .

- When you solve absolute value equations think about how many solutions would make sense.

Handwritten notes: 2 Sol,  $|x| = 4$



Solve  $|x| = 8$

Handwritten solution: 8 or -8

$|x| = 6$

Handwritten solution: 6 or -6

$|x| = -3$

Handwritten solution: No Sol.

## Examples

Solve the following and graph the solution set.

$$|2x - 1| = 7$$

$$\underbrace{2x - 1}_{7} = 7$$

$$2x - 1 = 7$$

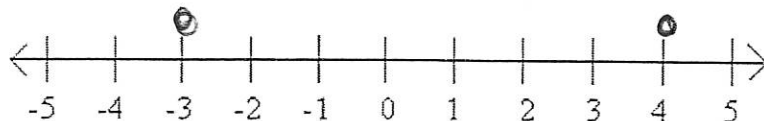
$$2x = 8$$

$$x = 4$$

$$2x - 1 = -7$$

$$2x = -6$$

$$x = -3$$



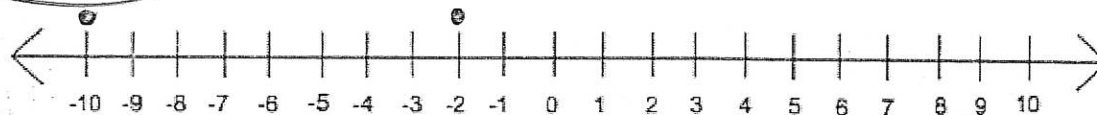
$$|x + 6| = 4$$

$$x + 6 = 4$$

$$x = -2$$

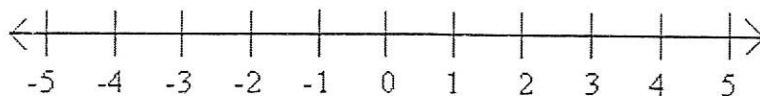
$$x + 6 = -4$$

$$x = -10$$



$$|3x + 1| = -4$$

⊗ ~~⊗~~ Can't be neg.



## Example

The average temperature in January in a northern Canadian city is 1 degree F. The actual January temperature for that city may be about 5 degrees warmer or colder. Write and solve an equation to find the maximum and minimum temperatures.

$$|t - 1| = 5$$

$$-4 \text{ to } 6$$

$$t - 1 = 5$$

$$t = 6$$

$$t - 1 = -5$$

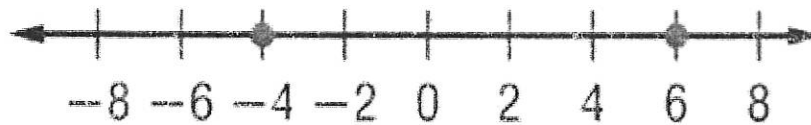
$$t = -4$$

For a company to invest in a product, they must believe they will receive a 12% return on investment (ROI) plus or minus 3%. Write an equation to find the least a greatest ROI they believe they will receive.

$X \pm$

Example

Write an equation involving absolute value for the graph.



## 2.6 Ratios and Proportions

- Ratio - comparison of 2 #s by division

### Example

Determine whether  $\frac{7}{8}$  and  $\frac{49}{56}$  are equivalent ratios and justify your answer.

.875 .875 Same

- Proportion - equation stating 2 ratios equal

$$\frac{a}{c} = \frac{d}{b} \rightarrow a \times b = c \times d$$

extremes

### Examples

Use cross products to determine whether each pair of ratios form a proportion.

a)  $\frac{0.25}{0.6}, \frac{1.25}{2}$

.5 .75 No

b)  $\frac{2}{2.5}, \frac{16}{20}$

40 40 Yes

## Examples

Solve each proportion.

a)  $\frac{n}{12} = \frac{3}{8}$

$$\frac{8n}{8} = \frac{36}{8}$$
$$n = 4.5$$

b)  $\frac{x}{10} = \frac{3}{5}$

$$\frac{5x}{5} = \frac{30}{5}$$
$$x = 6$$

c)  $\frac{(x+4)}{12} = \frac{3}{4}$

$$4(x+4) = 36$$
$$4x + 16 = 36$$
$$4x = 20$$
$$x = 5$$

d)  $\frac{x-2}{14} = \frac{2}{7}$

$$7(x-2) = 28$$
$$7x - 14 = 28$$
$$7x = 42$$
$$x = 6$$

## Examples

The gear on a bicycle is 8:5. This means that for every 8 turns of the pedals, the wheel turns 5 times. Suppose the bicycle wheel turns about 2435 times during a trip. How many times would you have to crank the pedals during the trip?

pedal  $\frac{8}{5} = \frac{x}{2435}$   
wheel

$$5x = 19480$$

$$x = 3896$$



In a road atlas, the scale for the map of Connecticut is 5 inches = 41 miles. What is the distance in miles represented by 2.5 inches on the map?

$$\begin{array}{l} \text{in} \\ \text{miles} \end{array} \quad \frac{5}{41} = \frac{2.5}{x}$$

$$5x = 102.5$$

$$x = 20.5$$

## 2.7 Percent of Change

The price per share of a stock decreased from \$90 per share to \$36 per share. By what percent did the price of the stock decrease?

$$\frac{90-36}{90} = \frac{54}{90} \quad (60\%)$$

- Percent of Change - ratio of change in an amount to original amt
- Percent of Increase - new # greater than original
- Percent of Decrease - old # greater than new.

### Examples

The price a used book store pays to buy a book is \$5. The store sells the book for 28% above the price that it pays for the book. What is the selling price of the book?

$$5 + 5 \times .28 = X$$

$$\text{\$}6.40$$

A meal for two at a restaurant costs \$32.75. If the sales tax is 5%, what is the total price of the meal?

$$32.75 \times .05 + 32.75$$

$$\text{\$}34.39$$

A dog toy is on sale for 20% off the original price. If the original price of the toy is \$3.80, what is the discounted price?

$$3.80(.20) = .76 \text{ off}$$

$$3.80 - .76 = \text{\$}3.04$$

Find the price for each of the following:

- a) Shoes \$60, discount 40%

$$60 \times (.60) = 36$$

100-40 = 60% paying 60%

- b) Belt \$14.99, discount 20%

$$14.99 (.80) = 11.99$$

- c) Sweater \$12.99, discount 10%, tax 6%

$$12.99 (.90) = 11.69$$

$$11.69 \times .06 = .70$$

$$.70 + 11.69 = 12.39$$

- Percentiles - tells what percent of total scores were below a given score.

## 2.8 Literal Equations and Dimensional Analysis

### Example

The formula  $P = 100 + \frac{a}{2}$  relates normal blood pressure  $P$  in millimeters and age  $a$  in years. Solve the formula for  $a$ .

$$P = 100 + \frac{a}{2}$$
$$(P - 100 = \frac{a}{2}) \times 2$$

### Example

$$2P - 200 = a$$

A car's fuel economy  $E$  (miles per gallon) is given by the formula  $E = \frac{m}{g}$  where  $m$  is the number of miles driven and  $g$  is the number of gallons of fuel used.

a) Solve for  $m$ .

$$g \cdot E = \frac{m}{g} \cdot g$$
$$Eg = m$$

b) If Angie's car has an average fuel consumption of 30 miles per gallon and she used 9.5 gallons, how far did she drive?

$$30 = \frac{m}{9.5}$$

285 miles

### Examples – No Calculator

Solve for the indicated variable.

a)  $5x + 12y = 9$  for  $x$

$$-12y \quad -12y$$

$$\frac{5x = 9 - 12y}{5}$$

$$x = \frac{9}{5} - \frac{12}{5}y$$

b)  $2\pi r = C$  for  $r$

$$\frac{2\pi r}{2\pi} = \frac{C}{2\pi}$$

$$r = \frac{C}{2\pi}$$

c)  $r = \frac{2}{3}t + v$  for  $t$

$$\frac{r-v}{\frac{2}{3}} = \frac{\frac{2}{3}t}{\frac{2}{3}}$$

$$\frac{r-v}{\frac{2}{3}} = t$$

### Conversions

1 centimeter (cm) = 10 millimeters (mm)

1 inch = 2.54 centimeters (cm)

1 foot = 0.3048 meters (m)

1 foot = 12 inches

1 yard = 3 feet

1 meter (m) = 100 centimeters (cm)

1 kilometer (km) = 1000 meters (m)

1 kilometer (km)  $\cong$  0.62137119 miles

1 mile = 5280 ft

1 mile = 1.609344 kilometers (km)

### Weight

1 pound (lb) = 16 ounces

1 kilogram (kg) = 1000 grams

1. 55 feet per second is how many miles per hour?

$$\frac{55 \cancel{\text{ft}}}{1 \cancel{\text{sec}}} \cdot \frac{1 \text{ mi}}{5280 \cancel{\text{ft}}} \cdot \frac{3600 \text{ sec}}{1 \text{ hr}} = \frac{198000 \text{ miles}}{5280 \text{ hr}}$$

37.5 m/hr

2. 40 miles per hour is how many feet per second?

$$\frac{40 \cancel{\text{mi}}}{1 \cancel{\text{hr}}} \cdot \frac{5280 \text{ ft}}{1 \cancel{\text{mi}}} \cdot \frac{1 \cancel{\text{hr}}}{3600 \text{ sec}} = \frac{211200 \text{ ft}}{3600 \text{ sec}}$$

58.7 ft/sec

3. 240 milliliters in 12 hours is how many microdrops per minute?

60 microdrops = 1 milliliter

$$\frac{240 \cancel{\text{ml}}}{12 \cancel{\text{hr}}} \cdot \frac{60 \text{ md}}{1 \cancel{\text{ml}}} \cdot \frac{1 \cancel{\text{hr}}}{60 \text{ min}} = \frac{14400 \text{ md}}{720 \text{ min}}$$

20 md/min

Are the following conversions done correctly? If not explain what was done wrong.

4. Walter Payton has been clocked at 4 seconds in the 40 (40yd). How many miles per hour is this.

$$\frac{40 \text{ yds}}{4 \text{ sec}} * \frac{3 \text{ ft}}{1 \text{ yd}} * \frac{1 \text{ mile}}{5280 \text{ ft}} * \frac{60 \text{ sec}}{1 \text{ min}} * \frac{60 \text{ min}}{1 \text{ hr}} = \frac{40 * 3 * 1 * 60 * 60}{1 * 5280 * 1 * 1} = \frac{432000}{5280} = 81 \text{ mph}$$

No

5. You are driving your M/C in Germany; the speed limit is 90 k/h. Your speedometer only has miles per hour. How fast can you go?

$$\frac{90 \text{ k}}{1 \text{ hr}} * \frac{1 \text{ k}}{0.6214 \text{ mi}} = \frac{90}{0.6214} = 144.8 \text{ m/h}$$



## 2.9 Weighted Averages

- Weighted Average – multiply each data value by its weight and find mean of new set.

Regular Average	Weighted Average
<p>Marco scored an 85, 90, 72, 84, and 68 on 5 tests. What was his average on the tests?</p> $\frac{85+90+72+84+68}{5}$ $= 79.8$	<p>Marco averaged a 86% first marking period, a 79% second marking period, and a 75% on his marking period exam. Each marking period counts as 40% of his semester grade and the exam counts as 20%. What is Marco's overall semester grade?</p> $86 \cdot 0.4 + 79 \cdot 0.4 + 75 \cdot 0.2$ $81$



### Example

Mandisha feeds her cat gourmet cat food that costs \$1.75 per pound. She combines it with cheaper food that costs \$0.50 per pound. How many pounds of cheaper food should Mandisha buy to go with 5 pounds of gourmet food, if she wants the average price to be \$1.00 per pound?

	Units	Price Per Unit	Total Price
Gourmet Cat Food	5	1.75	8.75
Cheap Cat Food	X	.50	.50X
Mixed Food	5+X	1.00	5+X

$$8.75 + .50x = 5 + x$$

$$3.75 = .50x$$

$$7.5 \text{ lbs}$$

### Example

A car's radiator should contain a solution of 50% antifreeze. George has 2 gallons of <sup>sol.</sup> 35% antifreeze. How many gallons of 100% antifreeze should he add to his solution to produce a solution of 50% antifreeze?

	Amount of Solution	Amount of Antifreeze
35% solution	2	$2(.35)$ 0.7
100% solution	X	1X
50% solution	$2+X$	$1+.5X$

$$0.7 + 1x = 1 + .5x$$

$$1x = .3 + .5x$$

$$.5x = .3$$

$$.6 \text{ gal of } 100\%$$

### Example

A railroad switching operator has discovered that two trains are heading toward each other on the same track. Currently, the trains are 53 miles apart. One train is traveling at 75 miles per hour and the other 40 miles per hour. The faster train will require 5 miles to stop safely and the slower train will require 3 miles to stop safely. About how many minutes does the operator have to warn the train engineers to stop their trains?

	R	T	$D=rt$
Train A	75		
Train B	40		