2.1 Inductive Reasoning and Conjecture

- **Conjecture** – educated guess based on Known info.
- **Inductive Reasoning** reasoning that uses examples to arrive at a prediction.

Example

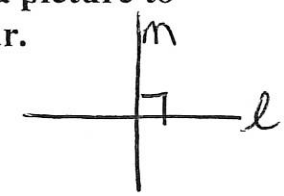
Make a conjecture about the next number based on the pattern: 2, 4, 12, 48, 240

x2 x3 x4 x5

1440

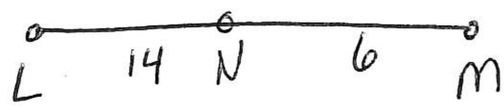
Example

Make a conjecture based on the information given. Draw a picture to represent your conjecture. Lines *l* and *m* are perpendicular.



Example

For points L, M, and N, LM = 20, MN = 6, and LN = 14. Make a conjecture and draw a figure to illustrate your conjecture.



- **Counterexample** – shows false

Example

Determine whether the following is true or false. If false provide a counterexample.

Given: It is the weekend.

Conjecture: It is Saturday.

False, it could be Sunday

Example

Determine whether the following is true or false. If false provide a counterexample.

Given: *a* and *b* are real numbers

Conjecture: ab > b

False

$a = 3$

$b = -7$

$3 \cdot -7 > -7$

$-21 > -7$

$3 \cdot 0 > 0$

$0 > 0$

2.2 Logic

- **Statement** – a sentence that is True or False, not both
- Truth value – **the value the statement holds, T or F**
- Negation – **a statement with opposite meaning and an opposite truth value** ~ ← Symbol

Example

p: Lansing is a city in Michigan T

~p: Lansing is not a city in Michigan F

- **Compound Statement –** two Statements are joined

T **P: Lansing is a city in Michigan.**

T **Q: Lansing is the capital of Michigan.**

P∧Q: Lansing is a city in Mich. and Lansing is the capital of michigan.

True

- **Conjunction – a compound statement formed by using the word AND**

  Symbol: $p \wedge q$

*A conjunction is true only when both statement in it are true.**

Example

Use the following compound statement for each conjunction then find its truth value.

P: One foot is 14 inches. F

Q: September has 30 days. T

R: A plane is defined by three noncollinear points. T

False **a) p and q**

F T

False **b) $r \wedge p$**

T F

False **c) $\sim q \wedge r$**

F

d) $\sim p \wedge r$ One foot is not 14 inches and a plane is defined by 3 noncollinear pts.

T T

True

➢ Disjunction – a compound statement formed by the word or.
Symbol: $\vee$
A disjunction is true when at least one of the statements is true.

TF	T
FT	T
FF	F
TT	T

Example
Use the following compound statements for the disjunction then find its truth value.

P: centimeters are metric units T
Q: 9 is a prime number F

$P \vee Q$ True

➢ Truth Tables – used to help determine values of compound statements.

2.3 Conditional Statements

➢ Conditional Statement – a statement that can be written in if-then form.

If you want to graduate, then you have to pass Geometry.

hypothesis conclusion

Example
Identify the hypothesis and conclusion of each statement.

a) If a polygon has 6 sides, then it is a hexagon.

b) Tamika will advance to the next level of play if she completes the maze in her computer game.

H

Example
Identify the hypothesis and conclusion of each statement. Then write each statement in the if-then form.

a) Distance is positive

If it is a distance, then it is pos.

b) A five-sided polygon is a pentagon

If 5-sided polygon, then it is a pentagon.

p	$\sim p$
T	F
F	T

and

p	q	$p \wedge q$
T	T	T
F	F	F
T	F	F
F	T	F

<u>Conditional</u> $p \rightarrow q$

Converse $q \rightarrow p$

Inverse $\sim p \rightarrow \sim q$

Contrapositive $\sim q \rightarrow \sim p$

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

*

p	q	$\sim p$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

p	q	$\sim q$	$p \vee \sim q$
T	T		
T	F		
F	T		
F	F		

* * or

p	q	r	$\sim q$	$\sim q \wedge r$	$p \vee (\sim q \wedge r)$
T	~~T~~	~~T~~	~~F~~	F	T
T	~~T~~	~~F~~	~~F~~	F	T
T	~~F~~	~~T~~	~~T~~	T	T
F	~~T~~	~~T~~	~~F~~	F	F
T	~~F~~	~~F~~	~~T~~	F	T
F	~~F~~	~~T~~	~~T~~	T	T
F	~~F~~	~~F~~	~~T~~	F	F
F	~~T~~	~~F~~	~~F~~	F	F

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2-4 Study Guide and Intervention

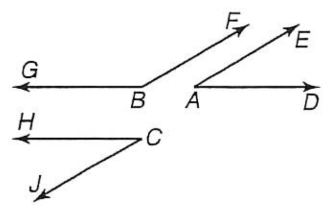
Deductive Reasoning

Law of Detachment **Deductive reasoning** is the process of using facts, rules, definitions, or properties to reach conclusions. One form of deductive reasoning that draws conclusions from a true conditional $p \rightarrow q$ and a true statement p is called the **Law of Detachment**.

Law of Detachment	If $p \rightarrow q$ is true and p is true, then q is true.
Symbols	$[(p \rightarrow q)] \wedge p] \rightarrow q$

Example **The statement *If two angles are supplementary to the same angle, then they are congruent* is a true conditional. Determine whether each conclusion is valid based on the given information. Explain your reasoning.**

a. Given: $\angle A$ and $\angle C$ are supplementary to $\angle B$.
Conclusion: $\angle A$ is congruent to $\angle C$.



The statement *$\angle A$ and $\angle C$ are supplementary to $\angle B$* is the hypothesis of the conditional. Therefore, by the Law of Detachment, the conclusion is true.

b. Given: $\angle A$ is congruent to $\angle C$.
Conclusion: $\angle A$ and $\angle C$ are supplementary to $\angle B$.

The statement *$\angle A$ is congruent to $\angle C$* is not the hypothesis of the conditional, so the Law of Detachment cannot be used. The conclusion is not valid.

Exercises

Determine whether each conclusion is valid based on the true conditional given. If not, write *invalid*. Explain your reasoning.

If two angles are complementary to the same angle, then the angles are congruent.

1. **Given:** $\angle A$ and $\angle C$ are complementary to $\angle B$. ← hypothesis
   Conclusion: $\angle A$ is congruent to $\angle C$. ← conclusion

   valid

2. **Given:** $\angle A \cong \angle C$ ← conclusion
   Conclusion: $\angle A$ and $\angle C$ are complements of $\angle B$. ← hypoth.

   Invalid

3. **Given:** $\angle E$ and $\angle F$ are complementary to $\angle G$. ← hypoth.
   Conclusion: $\angle E$ and $\angle F$ are vertical angles. ?

   Invalid

Lesson 2-4

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2-4 Study Guide and Intervention *(continued)*

Deductive Reasoning

Law of Syllogism Another way to make a valid conclusion is to use the **Law of Syllogism**. It is similar to the Transitive Property.

Law of Syllogism	If $p \rightarrow q$ is true and $q \rightarrow r$ is true, then $p \rightarrow r$ is also true.
Symbols	$[(p \rightarrow q)] \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

~~If you~~
• If its a weekday, then you come to school.

Example **The two conditional statements below are true. Use the Law of Syllogism to find a valid conclusion. State the conclusion.**

• If you come to school, then you go to Geometry.

(1) If a number is a whole number, then the number is an integer.
(2) If a number is an integer, then it is a rational number.

p: A number is a whole number.
q: A number is an integer.
r: A number is a rational number.

If weekday, then go to Geometry.

The two conditional statements are $p \rightarrow q$ and $q \rightarrow r$. Using the Law of Syllogism, a valid conclusion is $p \rightarrow r$. A statement of $p \rightarrow r$ is "if a number is a whole number, then it is a rational number."

Exercises

Determine whether you can use the Law of Syllogism to reach a valid conclusion from each set of statements.

$x = 4$
$4 = y$

1. If a dog eats Superdog Dog Food, he will be happy.
Rover is happy.
No

2. If an angle is supplementary to an obtuse angle, ~~then it is acute.~~
~~If an angle is acute,~~ then its measure is less than 90.
If an angle is supp. to obtuse angle, then measure is less than 90.

3. If the measure of $\angle A$ is less than 90, ~~then $\angle A$ is acute~~.
~~If $\angle A$ is acute~~, then $\angle A \cong \angle B$.
If the measure of ∠A less than 90, then $\angle A \cong \angle B$

4. If an angle is a right angle, then the measure of the angle is 90.
If two lines are perpendicular, then they form a right angle.
If 2 lines are perp., then measure of angle is 90.

5. If you study for the test, then you will receive a high grade.
Your grade on the test is high.
No

2.5 Postulates and Paragraph Proofs

- **Postulate** – statement that is true and describes a basic geometric relationship

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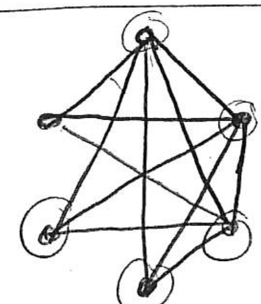
Example

Some snow crystals are shaped like regular hexagons. How many lines must be drawn to interconnect all vertices of a hexagonal snow crystal.

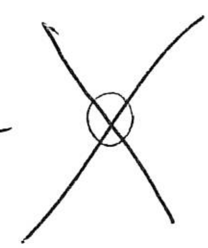


5
4
3
2
1

15

Postulates

- Through any two points there is exactly one line
- Through any three points not on the same line, there is exactly one plane
- A line contains at least 2 points
- If two lines intersect, then their intersection is exactly 1 point
- If two planes intersect, then their intersection is a line



- Theorems a statement proved true

Example

Determine whether each statement is always, sometimes, or never true. Explain.

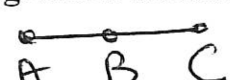
a) If plane T contains line EF and line EF contains point G, then plane T contains point G.

Always →

b) Line GH contains three noncollinear points

Never noncollinear means not on same line.

- Proof – logical argument that shows a statement is true

Given: 

B is midpt.

Prove: $AB \cong BC$

Statement	Reason
1)	Given

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2-6 Study Guide and Intervention

Algebraic Proof

Algebraic Proof The following properties of algebra can be used to justify the steps when solving an algebraic equation.

Reflection→

Property	Statement
Reflexive	For every number a, $a = a$. $3=3$ $5=5$
Symmetric	For all numbers a and b, if $a = b$ then $b = a$. $3=x$ $x=3$
Transitive	For all numbers a, b, and c, if $a = b$ and $b = c$ then $a = c$. $x=5$ $5=y$ $x=y$
+ & − Prop. of Equal.	For all numbers a, b, and c, if $a = b$ then $a + c = b + c$ and $a - c = b - c$.
× & ÷ Prop of Equal.	For all numbers a, b, and c, if $a = b$ then $a \cdot c = b \cdot c$, and if $c \neq 0$ then $\frac{a}{c} = \frac{b}{c}$.
Substitution	For all numbers a and b, if $a = b$ then a may be replaced by b in any equation or expression.
Distributive	For all numbers a, b, and c, $a(b + c) = ab + ac$.

Example Solve $6x + 2(x - 1) = 30$.

Algebraic Steps	Properties
$6x + 2(x - 1) = 30$	Given ←
$6x + 2x - 2 = 30$	Distributive Property
$8x - 2 = 30$	Substitution
$8x - 2 + 2 = 30 + 2$	Addition Property
$8x = 32$	Substitution
$\frac{8x}{8} = \frac{32}{8}$	Division Property
$x = 4$	Substitution

Given: $3x+4=25$
Prove: $x=7$

Statement	Reason
1)	
2)	

Exercises

Complete each proof.

What changed?

1. Given: $\frac{4x + 6}{2} = 9$
Prove: $x = 3$

Statements	Reasons
a. $\frac{4x + 6}{2} = 9$	a. Given
b. $2\left(\frac{4x + 6}{2}\right) = 2(9)$	b. Mult. Prop.
c. $4x + 6 = 18$	c. Substitution
d. $4x + 6 - 6 = 18 - 6$	d. Subtraction
e. $4x =$ 12	e. Substitution
f. $\frac{4x}{4} =$ $\frac{12}{4}$	f. Div. Prop.
g. $x = 3$	g. Substitution

2. Given: $4x + 8 = x + 2$
Prove: $x = -2$

Statements	Reasons
a. $4x + 8 = x + 2$	a. Given
b. $4x + 8 - x = x + 2 - x$	b. Subtraction
c. $3x + 8 = 2$	c. Substitution
d. $3x+8-8=2-8$	d. Subtr. Prop.
e. $3x=-6$	e. Substitution (Simplify)
f. $\frac{3x}{3} = \frac{-6}{3}$	f. Division
g. $x=-2$	g. Substitution

Lesson 2-6

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2-6 Study Guide and Intervention *(continued)*

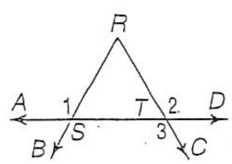
Algebraic Proof

Geometric Proof Geometry deals with numbers as measures, so geometric proofs use properties of numbers. Here are some of the algebraic properties used in proofs.

Property	Segments	Angles
Reflexive	$\overline{AB} \cong \overline{AB}$	$m\angle A = m\angle A$
Symmetric	$\overline{AB} \cong \overline{CD}$ then $\overline{CD} \cong \overline{AB}$	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive	$\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$	If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

$\overline{AB} \cong \overline{EF}$

Example Write a two-column proof.



Given: $m\angle 1 = m\angle 2$, $m\angle 2 = m\angle 3$

Prove: $m\angle 1 = m\angle 3$

Proof:

Statements	Reasons
1. $m\angle 1 = m\angle 2$	1. Given
2. $m\angle 2 = m\angle 3$	2. Given
3. $m\angle 1 = m\angle 3$	3. Transitive Property

Exercises

State the property that justifies each statement.

1. If $m\angle 1 = m\angle 2$, then $m\angle 2 = m\angle 1$. Symmetric
2. If $m\angle 1 = 90$ and $m\angle 2 = m\angle 1$, then $m\angle 2 = 90$. Transitive/Subst.
3. If $AB = RS$ and $RS = WY$, then $AB = WY$. Trans./Sub.
4. If $AB = CD$, then $\frac{1}{2}AB = \frac{1}{2}CD$. Mult.
5. If $m\angle 1 + m\angle 2 = 110$ and $m\angle 2 = m\angle 3$, then $m\angle 1 + m\angle 3 = 110$. Sub.
6. $RS = RS$ Reflexive
7. If $AB = RS$ and $TU = WY$, then $AB + TU = RS + WY$. Addition
8. If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$. Trans./Sub
9. A formula for the area of a triangle is $A = \frac{1}{2}bh$. Prove that bh is equal to 2 times the area of the triangle.

Statement	Reason
1) $A = \frac{1}{2}bh$	Given
2) $2 \cdot A = 2(\frac{1}{2}bh)$	Mult.
3) $2A = bh$	Subst.
$bh = 2A$	Symmetric

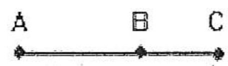
Proving Segment Relationships

Given: 3x+4=16

Prove: x=4

Statements	Reasons
$3x + 4 = 16$	
$3x + 12$	
$x = 4$	

Now we'll add postulates, and use segments in addition to numbers and variables.



-Segment Addition Postulate: AB+BC=AC

-Between any two points there can be only one line.

-The congruence of segments is reflexive, symmetric, and transitive.

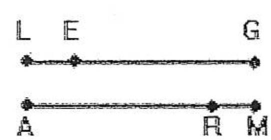
Example:

Given: AB=BC , C is the midpoint of BD

Prove: AB=CD

Statements	Reasons
AB=BC	
BC=CD	
AB=CD	

Practice these proofs.



1) Given: LE = MR and EG = RA , Prove: LG = MA

2) Given: AB = BC and BC = CD , Prove: AB = CD

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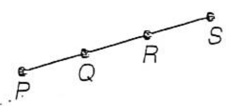
2-7 Study Guide and Intervention

Proving Segment Relationships

Segment Addition Two basic postulates for working with segments and lengths are the Ruler Postulate, which establishes number lines, and the Segment Addition Postulate, which describes what it means for one point to be between two other points.

	The points on any line or line segment can be paired with real numbers so that, given any two points *A* and *B* on a line, *A* corresponds to zero and *B* corresponds to a positive real number.
	B is between *A* and *C* if and only if $AB + BC = AC$.

Example Write a two-column proof.



Given: Q is the midpoint of $\overline{PR}$.
R is the midpoint of $\overline{QS}$.

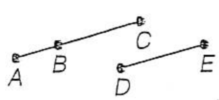
Prove: $PR = QS$

Statements	Reasons
1. Q is the midpoint of $\overline{PR}$.	1. Given
2. $PQ = QR$	2. Definition of midpoint
3. R is the midpoint of $\overline{QS}$.	3. Given
4. $QR = RS$	4. Definition of midpoint
5. $PQ + QR = QR + RS$	5. Addition Property
6. $PQ + QR = PR$, $QR + RS = QS$	6. Segment Addition Postulate
7. $PR = QS$	7. Substitution

Exercises

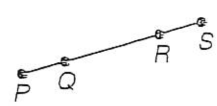
Complete each proof.

1. Given: $BC = DE$
Prove: $AB + DE = AC$



Statements	Reasons
a. $BC = DE$	a. __________
b. __________	b. Seg. Add. Post.
c. $AB + DE = AC$	c. __________

2. Given: Q is between P and R, R is between Q and S, $PR = QS$.
Prove: $PQ = RS$



Statements	Reasons
a. Q is between P and R.	a. Given
b. $PQ + QR = PR$	b. __________
c. R is between Q and S.	c. __________
d. __________	d. Seg. Add. Post.
e. $PR = QS$	e. __________
f. $PQ + QR = QR + RS$	f. __________
g. $PQ + QR - QR = QR + RS - QR$	g. __________
h. __________	h. Substitution

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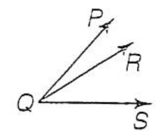
2-8 Study Guide and Intervention

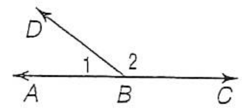
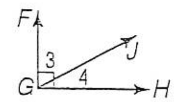
Proving Angle Relationships

Supplementary and Complementary Angles There are two basic postulates for working with angles. The Protractor Postulate assigns numbers to angle measures, and the Angle Addition Postulate relates parts of an angle to the whole angle.

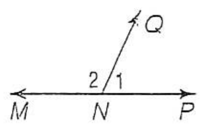
Protractor Postulate	Given $\overrightarrow{AB}$ and a number r between 0 and 180, there is exactly one ray with endpoint A, extending on either side of $\overrightarrow{AB}$, such that the measure of the angle formed is r.
Angle Addition Postulate	R is in the interior of $\angle PQS$ if and only if $m\angle PQR + m\angle RQS = m\angle PQS$.



The two postulates can be used to prove the following two theorems.

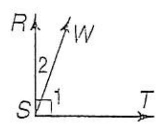
Supplement Theorem	If two angles form a linear pair, then they are supplementary angles. If $\angle 1$ and $\angle 2$ form a linear pair, then $m\angle 1 + m\angle 2 = 180$.	
Complement Theorem	If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles. If $\overrightarrow{GF} \perp \overrightarrow{GH}$, then $m\angle 3 + m\angle 4 = 90$.	

Example 1 If $\angle 1$ and $\angle 2$ form a linear pair and $m\angle 2 = 115$, find $m\angle 1$.



$m\angle 1 + m\angle 2 = 180$	Suppl. Theorem
$m\angle 1 + 115 = 180$	Substitution
$m\angle 1 = 65$	Subtraction Prop.

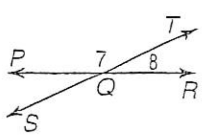
Example 2 If $\angle 1$ and $\angle 2$ form a right angle and $m\angle 2 = 20$, find $m\angle 1$.



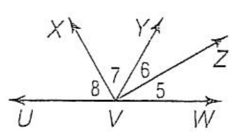
$m\angle 1 + m\angle 2 = 90$	Compl. Theorem
$m\angle 1 + 20 = 90$	Substitution
$m\angle 1 = 70$	Subtraction Prop.

Exercises

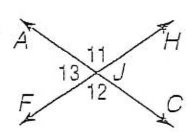
Find the measure of each numbered angle.

1. 

$m\angle 7 = 5x + 5$,
$m\angle 8 = x - 5$

2. 

$m\angle 5 = 5x$, $m\angle 6 = 4x + 6$,
$m\angle 7 = 10x$,
$m\angle 8 = 12x - 12$

3. 

$m\angle 11 = 11x$,
$m\angle 12 = 10x + 10$

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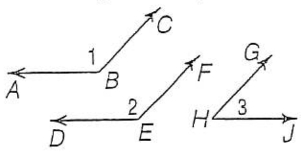
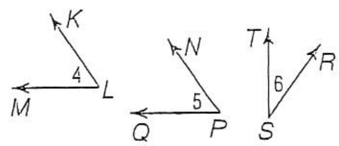
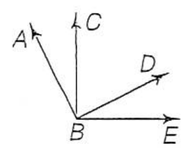
2-8 Study Guide and Intervention *(continued)*

Proving Angle Relationships

Congruent and Right Angles Three properties of angles can be proved as theorer

Congruence of angles is reflexive, symmetric, and transitive.	
Angles supplementary to the same angle or to congruent angles are congruent.  If $\angle 1$ and $\angle 2$ are supplementary to $\angle 3$, then $\angle 1 \cong \angle 2$.	Angles complementary to the same angle or to congruent angles are congruent.  If $\angle 4$ and $\angle 5$ are complementary to $\angle 6$, then $\angle 4 \cong \angle 5$.

Example Write a two-column proof.



Given: $\angle ABC$ and $\angle CBD$ are complementary.
$\angle DBE$ and $\angle CBD$ form a right angle.

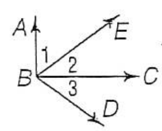
Prove: $\angle ABC \cong \angle DBE$

Statements	Reasons
1. $\angle ABC$ and $\angle CBD$ are complementary. $\angle DBE$ and $\angle CBD$ form a right angle.	1. Given
2. $\angle DBE$ and $\angle CBD$ are complementary.	2. Complement Theorem
3. $\angle ABC \cong \angle DBE$	3. Angles complementary to the same $\angle$ are

Exercises

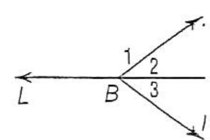
Complete each proof.

1. Given: $\overline{AB} \perp \overline{BC}$; $\angle 1$ and $\angle 3$ are complementary.
Prove: $\angle 2 \cong \angle 3$



Statements	Reasons
a. $\overline{AB} \perp \overline{BC}$	a. ____________
b. ____________	b. Definition of $\perp$
c. $m\angle 1 + m\angle 2 = m\angle ABC$	c. ____________
d. $\angle 1$ and $\angle 2$ form a rt $\angle$.	d. ____________
e. $\angle 1$ and $\angle 2$ are compl.	e. ____________
f. ____________	f. Given
g. $\angle 2 \cong \angle 3$	g. ____________

2. Given: $\angle 1$ and $\angle 2$ form a linear pair. $m\angle 1 + m\angle 3 = 180$
Prove: $\angle 2 \cong \angle 3$



Statements	Reasons
a. $\angle 1$ and $\angle 2$ form a linear pair. $m\angle 1 + m\angle 3 = 180$	a. Given
b. ____________	b. Suppl. Theorem
c. $\angle 1$ is suppl. to $\angle 3$.	c. ____________
d. ____________	d. $\measuredangle$ suppl to same $\angle$ e