Key

3.1 Properties of Parallel Lines

- **Parallel Lines –** coplanar lines that do not intersect

- **Transversal –** a line that intersects 2 or more lines in a plane.

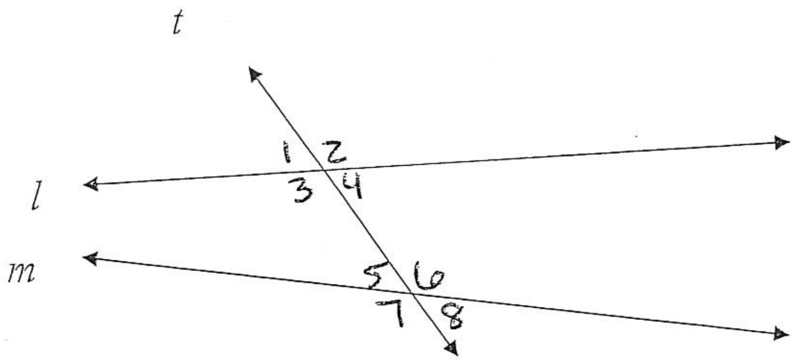
Identifying Angles

Interior – between lines

Exterior – not between the lines

Alternate – opposite sides of transversal

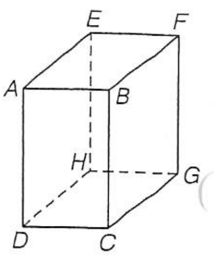
Corresponding – same position with respect to intersection.



- Alternate Interior Angles
- Same-Side Interior Angles
- Corresponding Angles
- Alternate Exterior Angles
- Same-Side Exterior Angles

- **Skew lines -** lines that do not intersect & are not coplanar

For Exercises 1–4, refer to the figure at the right.



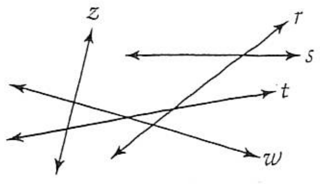
1. Name all planes that are parallel to plane DEH. plane FBG

2. Name all segments that are parallel to $\overline{AB}$. $\overline{EF}$, $\overline{CD}$, $\overline{HG}$

3. Name all segments that intersect $\overline{GH}$. $\overline{CG}$, $\overline{DH}$, $\overline{EH}$, $\overline{FG}$

4. Name all segments that are skew to $\overline{CD}$. $\overline{AE}$, $\overline{BF}$, $\overline{EH}$, $\overline{FG}$

Identify the sets of lines to which each given line is a transversal.

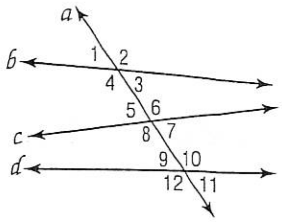


5. r S+t S+w t+w

6. s

7. w

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.



8. $\angle 2$ and $\angle 8$
alt ext.

9. $\angle 3$ and $\angle 6$
consec. int.

10. $\angle 1$ and $\angle 9$
corresponding

11. $\angle 3$ and $\angle 9$
alt. int.

12. $\angle 6$ and $\angle 12$
alt ext.

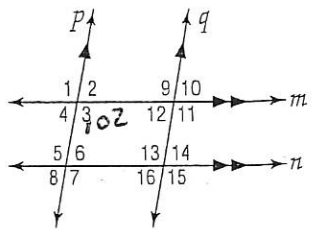
13. $\angle 7$ and $\angle 11$
corr.

3.2: Angles and Parallel Lines

IF A TRANSVERSAL INTERSECTS TWO PARALLEL LINES, …

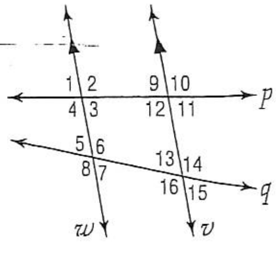
- *Corresponding Angles Postulate* –… then corresponding angles are Congruent
- *Alternate Interior Angles Theorem* – … then alternate interior angles are Congruent
- *Same-Side Interior Angles Theorem* – … *then same-side interior angles are* Supplementary
- *Alternate Exterior Angles Theorem* –… *then alternate exterior angles are* Congruent
- *Same-Side Exterior Angles Theorem* - … then same-side exterior angles are Supplementary

In the figure, $m\angle 3 = 102$. Find the measure of each angle.



1. $\angle 5$ 102
2. $\angle 6$ 78
3. $\angle 11$ 102
4. $\angle 7$ 102
5. $\angle 15$ 102
6. $\angle 14$ 78

In the figure, $m\angle 9 = 80$ and $m\angle 5 = 68$. Find the measure of each angle.



7. $\angle 12$ 100
8. $\angle 1$ 80
9. $\angle 4$ 100
10. $\angle 3$ 80
11. $\angle 7$ 68
12. $\angle 16$ 112

NAME ______________________ DATE __________ PERIOD _____

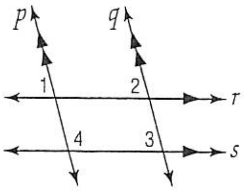
3-2 Study Guide and Intervention *(continued)*

Angles and Parallel Lines

Algebra and Angle Measures

Algebra can be used to find unknown values in angles formed by a transversal and parallel lines.

Example **If $m\angle 1 = 3x + 15$, $m\angle 2 = 4x - 5$, $m\angle 3 = 5y$, and $m\angle 4 = 6z + 3$, find x and y.**



$p \parallel q$, so $m\angle 1 = m\angle 2$ because they are corresponding angles.

$$3x + 15 = 4x - 5$$
$$3x + 15 - 3x = 4x - 5 - 3x$$
$$15 = x - 5$$
$$15 + 5 = x - 5 + 5$$
$$20 = x$$

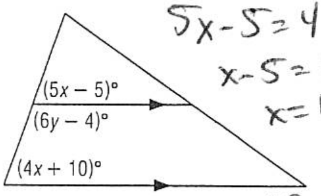
$r \parallel s$, so $m\angle 2 = m\angle 3$ because they are corresponding angles.

$$m\angle 2 = m\angle 3$$
$$75 = 5y$$
$$\frac{75}{5} = \frac{5y}{5}$$
$$15 = y$$

Exercises

Find x and y in each figure.

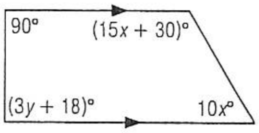
1.



5x-5=4x+10
x-5=10
x=15

x=15 y=19

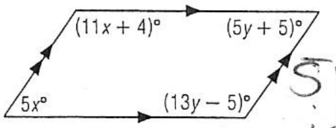
2.



10x+15x+30=180
25x+30=180

x=6 y=24

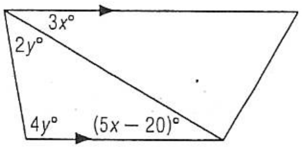
3.



5y+5+13y-5=180
18y=180

y=10 x=11

4.

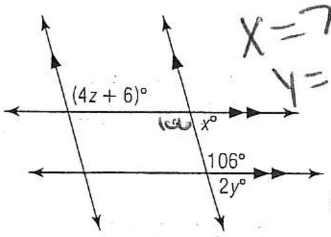


5x-20=3x
-20=-2x
x=10

x=10 y=25

Find x, y, and z in each figure.

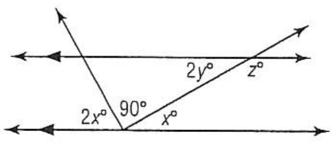
5.



x=74
y=37
z=25

4z+6=106
4z=100
z=25

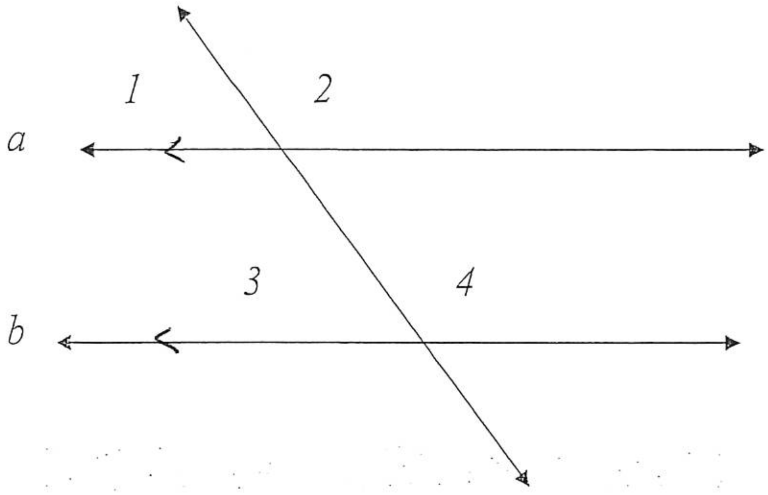
6.



2x+90+x=180
3x=90
x=30

x=30
z=150
y=15

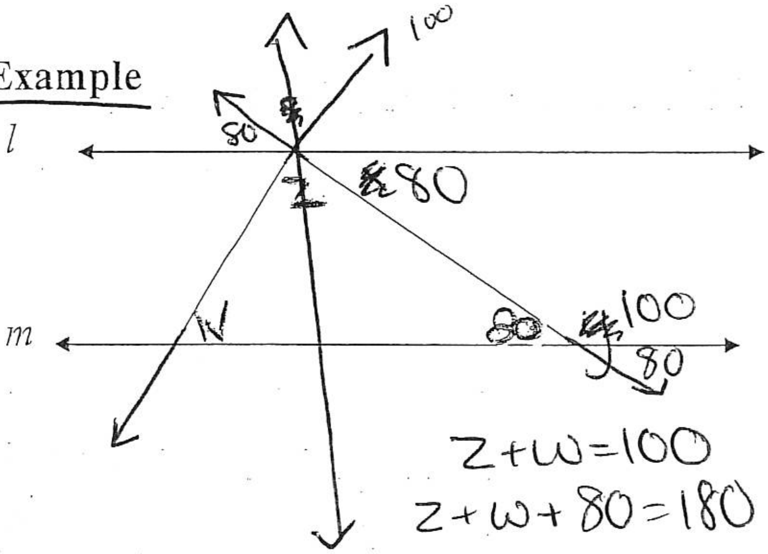
Example



Given a $\|$ b. Prove that $\angle 1$ and $\angle 4$ are supplementary.

Statements	Reasons
1. a $\|$ b	Given
2. $m\angle 1 + m\angle 2 = 180$	Linear Pair
3. $\angle 2 \cong \angle 4$	Corresponding Angles
4. $m\angle 1 + m\angle 4 = 180$	Sub.
5. $\angle 1$ & $\angle 4$ Supp.	def. of linear pair

Example



$z + w = 100$

$z + w + 80 = 180$

Solve for x, y, z, w.

$y = 100$

$x = 80$

$z =$

K W L

3.3 Slopes of Lines

- Slope = Vertical Rise / Horizontal Run $= \frac{y_2 - y_1}{x_2 - x_1}$ pos neg. 0 undefined or no

- Rate of Change – how a quantity changes over time

- Two nonvertical lines have the same slope if and only if they are parallel $y = -\frac{1}{2}x + 1$ $y = -\frac{1}{2}x + 3$

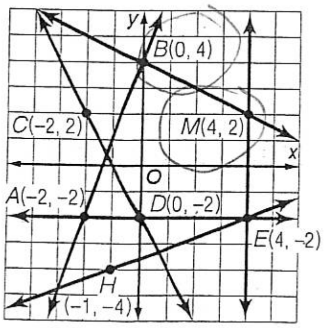
- Two nonvertical lines are perpendicular if and only if the product of their slopes is -1 $y = 3x - 2$ $-\frac{1}{3}$ $\frac{2}{3}$ $-\frac{3}{2}$

Determine the slope of the line that contains the given points.

1. $J(0, 0)$, $K(-2, 8)$ $-\frac{8}{2} = -4$
2. $R(-2, -3)$, $S(3, -5)$ x_1 y_1 x_2 y_2 $-\frac{2}{5}$
3. $L(1, -2)$, $N(-6, 3)$ $-\frac{5}{7}$
4. $P(-1, 2)$, $Q(-9, 6)$ $-\frac{1}{2}$

Find the slope of each line.

7. $\overleftrightarrow{AB}$ $(-2, -2)$ $(0, 4)$ $\frac{-6}{-2}$ 3
8. $\overleftrightarrow{CD}$ $(-2, 2)$ $(0, -2)$ $\frac{4}{-2} = -2$
9. $\overleftrightarrow{EM}$ undefined
10. $\overleftrightarrow{AE}$ 0
11. $\overleftrightarrow{EH}$ $\frac{2}{5}$
12. $\overleftrightarrow{BM}$ $\frac{2}{-4} = -\frac{1}{2}$ $(0, 4)$ $(4, 2)$ $-\frac{2}{4}$



Example

For one manufacturer of camping equipment, between 1990 and 2000 annual sales increased by 7.4 million dollars per year. In 2000, the total sales were 85.9 million. If sales increase at the same rate, what will be the total sales in 2010? 7.4

$(2000, 85.9)$

$(2010, y)$

$\frac{85.9 - y}{2000 - 2010} = 7.4$

$\frac{85.9 - y}{-10} = 7.4 \cdot -10$

$85.9 - y = -74$

$-y = -159.9$

$y = 159.9$ mill

3.4 Equations of Lines

- Slope-Intercept Form $y = mx + b$

slope y-intercept

Example

Write an equation in slope-intercept form of the line with slope 6 and y-intercept of -3.

$y = 6x - 3$

- Point-Slope Form $y - y_1 = m(x - x_1)$

slope

replaced w/ #s

Example

Write an equation in point-slope form of the line whose slope is -3/5 that contains (-10, 8).

$y - 8 = -3/5(x + 10)$

Example

Write an equation in slope-intercept form for a line containing (4, 9) and (-2, 0).

Slope $\frac{y-y}{x-x}$

$\frac{9-0}{4--2}$

$= \frac{9}{6}$

$= 3/2$

$y - 0 = 3/2(x + 2)$

$\boxed{y = 3/2x + 3}$

or

$y - 9 = 3/2(x - 4)$

$y - 9 = 3/2x - 12/2$

$y - 9 = 3/2x - 6$

$\boxed{y = 3/2x + 3}$

Example

Write an equation in slope-intercept form for a line containing (1, 7) that is perpendicular to the line y = -1/2x + 1

Slope = $-\frac{1}{2}$

perp 2

$y - 7 = 2(x - 1)$

$y - 7 = 2x - 2$

$\boxed{y = 2x + 5}$

Example

An apartment complex charges \$525 per month plus a \$750 security deposit.

a) Write an equation to represent the total annual cost *A* for *r* months of rent.

$A = 750 + 525r$

b) Compare this rental cost to a complex which charges a \$200 security deposit but \$600 per month for rent. If a person expects to stay in an apartment for one year, which complex offers the better rate?

$B = 200 + 600r$

$A = 7050$

$B = 7400$

Complex A is better

Try it:

Write equations in point-slope form and slope-intercept form of the line having the given slope and containing the given point.

5. m: 2, (5, 2)

$\boxed{y - 2 = 2(x - 5)}$

$y - 2 = 2x - 10$

$\boxed{y = 2x - 8}$

6. m: −3, (2, −4)

$\boxed{y + 4 = -3(x - 2)}$

$y + 4 = -3x + 6$

$\boxed{y = -3x + 2}$

7. m: $-\frac{1}{2}$, (−2, 5)

$\boxed{y - 5 = -\frac{1}{2}(x + 2)}$

$y - 5 = -\frac{1}{2}x - 1$

$\boxed{y = -\frac{1}{2}x + 4}$

8. m: $\frac{1}{3}$, (−3, −8)

$\boxed{y + 8 = \frac{1}{3}(x + 3)}$

$y + 8 = \frac{1}{3}x + 1$

$\boxed{y = \frac{1}{3}x - 7}$

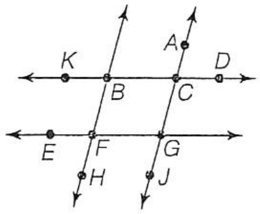
3.5 Proving Lines Parallel

Theorems

- If corresponding angles are congruent, then the lines are parallel.
- If alternate exterior angles are congruent, then the lines are parallel.
- If same-side interior angles are supplementary, then the lines are parallel.
- If alternate interior angles are congruent, then the lines are parallel.
- If two lines are perpendicular to the same line, then lines are parallel.

- **Parallel Postulate** – If you have a line and a pt. not on the line, there is EXACTLY 1 line through the pt. parallel to the other line.

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



1. $m\angle BCG + m\angle FGC = 180$

$\overleftrightarrow{KD}$ $\overleftrightarrow{EG}$ SSI

2. $\angle CBF \cong \angle GFH$

$\overleftrightarrow{KD} \parallel \overleftrightarrow{EG}$ Corresp.

3. $\angle EFB \cong \angle FBC$

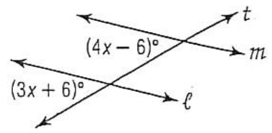
$\overleftrightarrow{BD} \parallel \overleftrightarrow{EF}$ AIA

4. $\angle ACD \cong \angle KBF$

$\overleftrightarrow{AJ} \parallel \overleftrightarrow{KH}$ Alt Ext

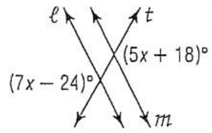
Find x so that $\ell \parallel m$.

5.



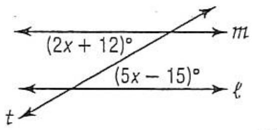
$4x-6=3x+6$
$x-6=6$
$x=12$

6.



$5x+18=7x-24$
$18=2x-24$
$42=2x$
$x=21$

7.

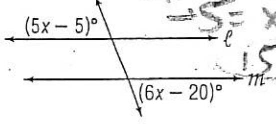


$5x-15=2x+12$
$3x-15=12$
$3x=27$
$x=9$

Try It:

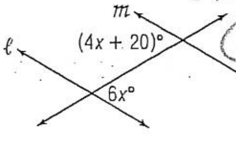
Find x so that $\ell \parallel m$.

1.



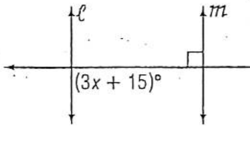
$5x-5=6x-20$
$-5=x-20$
$15=x$

2.



$6x=4x+20$
$2x=20$
$x=10$

3.



$3x+15=90$
$3x=75$
$x=25$

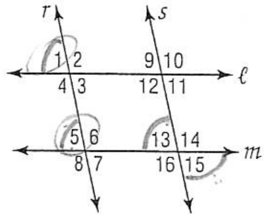
3.5 (Day 2) Proving Lines Parallel

- **Use the new theorems and postulates with the ones learned in previous chapters to prove that lines are parallel.**

For Exercises 1–6, fill in the blanks.

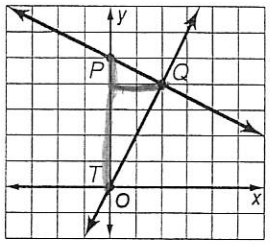
Given: $\angle 1 \cong \angle 5$, $\angle 15 \cong \angle 5$

Prove: $\ell \parallel m$, $r \parallel s$



Statements	Reasons
1. $\angle 15 \cong \angle 5$	1. Given
2. $\angle 13 \cong \angle 15$	2. Verticle ∠s are ≅
3. $\angle 5 \cong \angle 13$	3. Substitution
4. $r \parallel s$	4. Corr. ∠s are ≅
5. $\angle 1 \cong \angle 5$	5. Given
6. $\ell \parallel m$	6. If corr ∠s are ≅, then lines ∥.

7. Determine whether $\overleftrightarrow{PQ} \perp \overleftrightarrow{TQ}$. Explain why or why not.



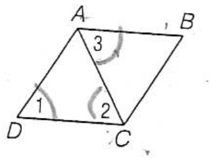
Yes

PQ $-\frac{1}{2}$

TQ $\frac{4}{2} = \frac{2}{1}$

Given: $\angle 1 \cong \angle 2$, $\angle 1 \cong \angle 3$

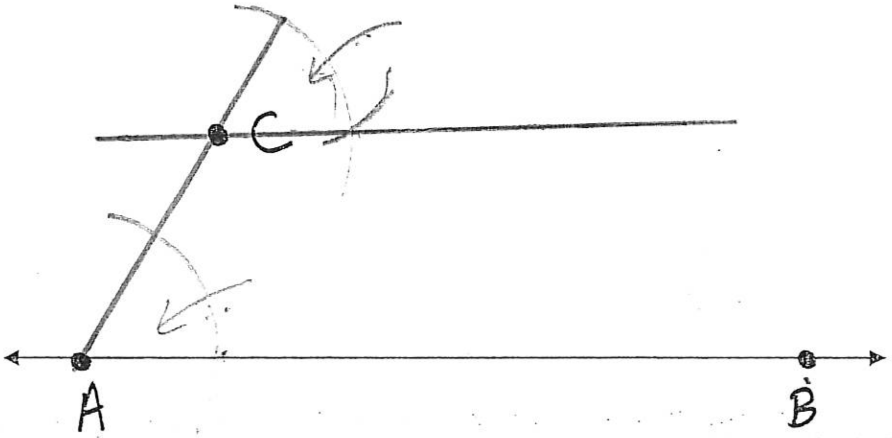
Prove: $\overline{AB} \parallel \overline{DC}$



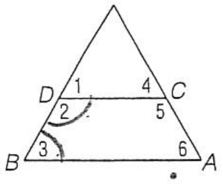
Statements	Reasons
1. $\angle 1 \cong \angle 2$ $\angle 1 \cong \angle 3$	1. Given
2. $\angle 2 \cong \angle 3$	2. Transitive Property of ≅ /Subst.
3. $\overline{AB} \parallel \overline{DC}$	3. If alt. int. angles are ≅, then the lines are ∥.

Construction

Draw a line parallel to AB and through point C.

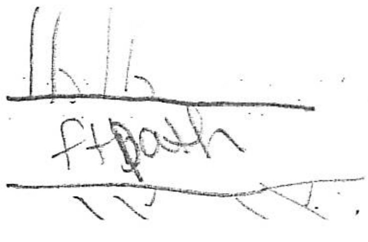


8. **PROOF** Write a two-column proof.
   Given: $\angle 2$ and $\angle 3$ are supplementary.
   Prove: $\overline{AB} \parallel \overline{CD}$



Statement	Reason
1) $\angle 2 \& \angle 3$ Suppl.	Given
2) $m\angle 2 + \angle 3 = 180$	Def. of Suppl.
3) $\overline{AB} \parallel \overline{CD}$	SS Int. are Suppl.

9. **LANDSCAPING** The head gardener at a botanical garden wants to plant rosebushes in parallel rows on either side of an existing footpath. How can the gardener ensure that the rows are parallel?



Corresponding Angles are $\cong$

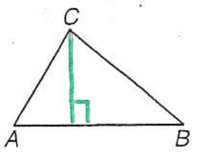
3.6 Perpendiculars and Distance

➢ **The distance from a line to a point not on the line is the length of the segment** perpendicular **to the line from the point.**

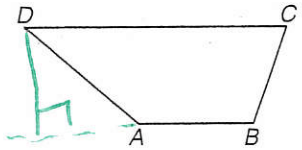
Examples

Draw the segment that represents the distance indicated.

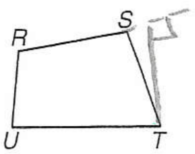
1. C to $\overleftrightarrow{AB}$



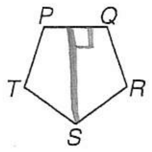
2. D to $\overleftrightarrow{AB}$



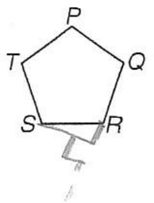
3. T to $\overleftrightarrow{RS}$



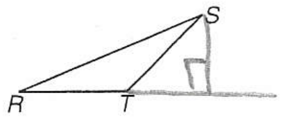
4. S to $\overleftrightarrow{PQ}$



5. S to $\overleftrightarrow{QR}$



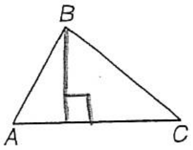
6. S to $\overleftrightarrow{RT}$



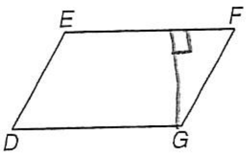
Examples

Draw the segment that represents the distance indicated.

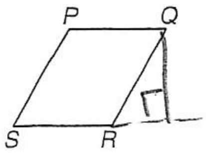
1. B to $\overleftrightarrow{AC}$



2. G to $\overleftrightarrow{EF}$

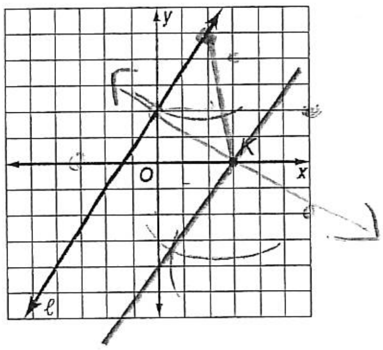


3. Q to $\overleftrightarrow{SR}$

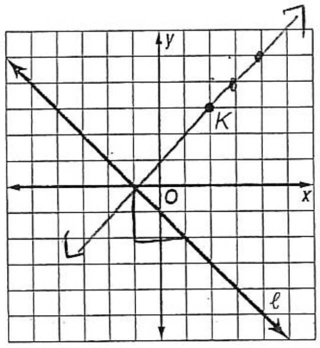


Construct a line perpendicular to ℓ through K. Then find the distance from K to ℓ.

4.



5.



-1

1

$+\frac{3}{2}$

1) Find slope

$-\frac{2}{3}$

Distance Between Parallel Lines

Example

Find the distance between the parallel lines a and b whose equations are y = 2x + 3 and y = 2x – 3 respectively.

1) Write an equation for a line perpendicular to the parallel lines.

$y = -\frac{1}{2}x - 3$

2) Solve the system
3 options:

1) Solve System
- Graph
- Sub
- Elim

$y = 2x + 3$

$-\frac{1}{2}x - 3 = 2x + 3$

$-\frac{1}{2}x = 2x + 6$

$-2.5x = 6$

$x = -2.4$

$(-2.4, -1.8)$

$(0, -3)$

3) Use the distance formula

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\sqrt{(-2.4 - 0)^2 + (-1.8 - -3)^2}$

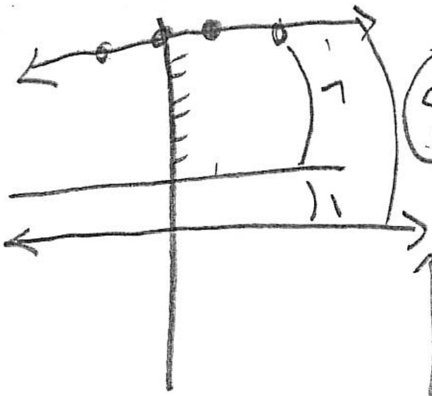
$-2.4^2 \quad 1.2^2$

$5.76 + 1.44$

$\sqrt{7.2} \approx 2.7$ units

Find the distance between each pair of parallel lines.

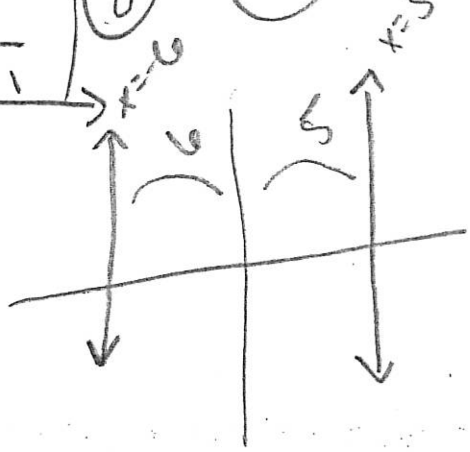
6. $y = 7$
$y = -1$



8

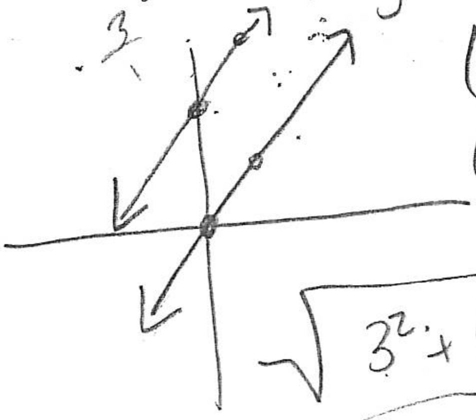
7. $x = -6$
$x = 5$

11



8. $y = 3x$ ✓
$y = 3x + 10$

$y = -\frac{1}{3}x + 10$



$(3, 9)$
$(0, 10)$

$\sqrt{3^2 + (-1)^2}$

$\sqrt{9 + 1}$

$\sqrt{10}$

3.16

9. $y = -5x$
$y = -5x + 26$

$y = \frac{1}{5}x$

$(0, 0)$
$(5, 1)$

$\sqrt{5^2 + 1^2}$

$\sqrt{25 + 1}$

$\sqrt{26}$

10. $y = x + 9$ ✓
$y = x + 3$ ✓

$y = -1x + 9$ ✓

$(3, 6)$
$(0, 9)$

$\sqrt{3^2 + (-3)^2}$

$9 + 9$

$\sqrt{18} = 4.24$

11. $y = -2x + 5$
$y = -2x - 5$

$y = \frac{1}{2}x + 5$

$(0, 5)$
$(-4, 3)$

$\sqrt{4^2 + (2)^2}$

NAME ______________________________ DATE ____________ PERIOD _____

3 Graphing Calculator Investigation

Distance Between Two Parallel Lines *(Use with Lesson 3-6.)*

A graphing calculator can be used to find the distance between two parallel lines. By graphing the parallel lines and a line that intersects them, you can identify two points of intersection. Then use the Distance Formula to find the distance.

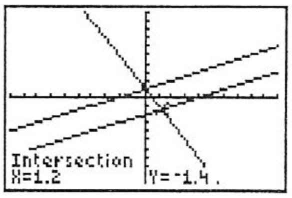
Example **Find the distance between the parallel lines l and m whose equations are $y = \frac{1}{2}x + 1$ and $y = \frac{1}{2}x - 2$, respectively.**

Step 1 Graph the two lines on the calculator.
Keystrokes: $\boxed{\text{Y=}}$ 1 $\boxed{\div}$ 2 $\boxed{\text{X,T,}\theta\text{,}n}$ $\boxed{+}$ 1 $\boxed{\text{ENTER}}$ 1 $\boxed{\div}$ 2 $\boxed{\text{X,T,}\theta\text{,}n}$ $\boxed{-}$ 2 $\boxed{\text{ENTER}}$
$\boxed{\text{ZOOM}}$ 6

Step 2 Next, graph a line that is perpendicular to the two lines. Since the slope of the parallel lines is $\frac{1}{2}$, graph a line with a slope of -2. Use $y = -2x + 1$.
Keystrokes: $\boxed{\text{Y=}}$ $\boxed{\blacktriangledown}$ $\boxed{\blacktriangledown}$ $\boxed{(-)}$ 2 $\boxed{\text{X,T,}\theta\text{,}n}$ $\boxed{+}$ 1 $\boxed{\text{ENTER}}$ $\boxed{\text{GRAPH}}$

Step 3 Now, find the points where the perpendicular line intersects the two parallel lines. Use the **intersect** function to find these two points.
Keystrokes: $\boxed{\text{2nd}}$ [CALC] 5
Select Y1 for the first curve and Y3 for the second curve. Select a point near the intersection of these two lines as your guess. Press $\boxed{\text{ENTER}}$. The point of intersection is at (0, 1).

Step 4 Do the same thing using Y2 for the first curve and Y3 for the second curve. The point of intersection is at (1.2, −1.4).



Step 5 Now use the Distance Formula to find the distance between the two points.

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$
$$\sqrt{(-1.4 - 1)^2 + (1.2 - 0)^2}$$
$$\sqrt{(-2.4)^2 + (1.2)^2}$$
$$\sqrt{5.76 + 144}$$
$$\sqrt{7.2} \text{ or about } 2.68$$

So, the distance between the two lines is approximately 2.68 units.

Exercises

Use a graphing calculator to find the approximate distance between each pair of parallel lines to the nearest hundredth.

1. $y = -\frac{1}{3}x + 2$
$y = -\frac{1}{3}x - 1$

2. $y = 2x - 3$
$y = 2x + 5$

3. $y = 3x - 4$
$y = 3x - 2$

4. $2x + y = 7$
$2x + y = -1$

5. $x - y = 2$
$x - y = 3$

6. $2x - 3y = 6$
$2x - 3y = -3$