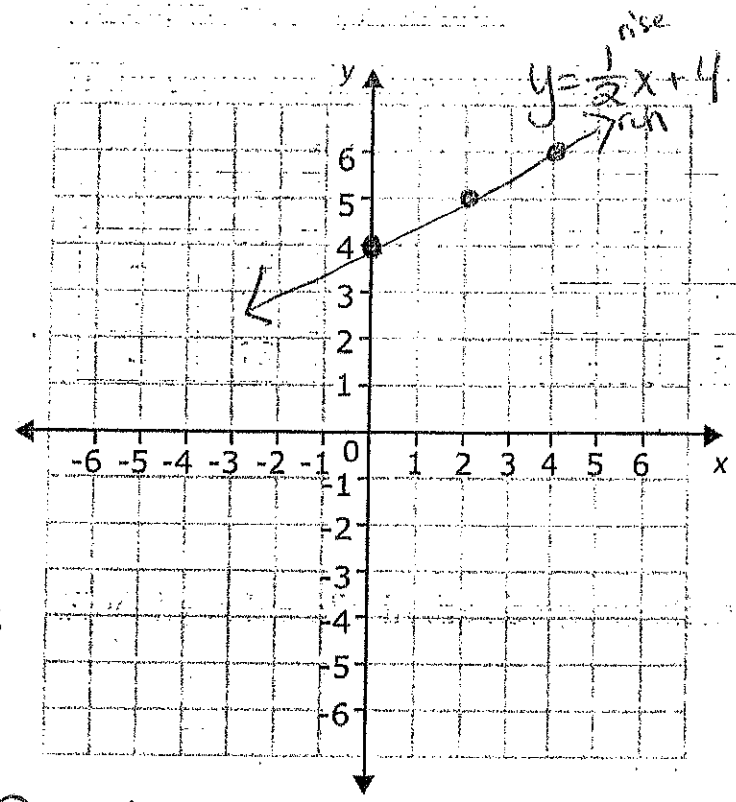
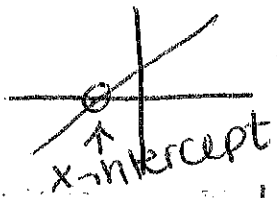
Seg lines

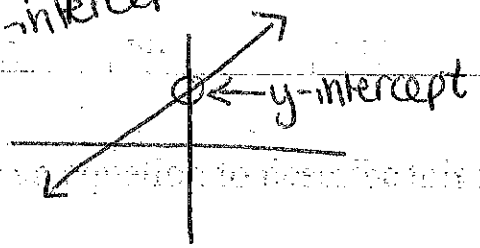
3.1 Graphing Linear Equations



- Linear Equation –
  Straight line
  - Equation where x is raised to first power.
  $y=2x$ $3x+2y=10$
  $\cancel{y=2x^2}$
- Slope-Intercept Form
  $y=mx+b$
  m ↑ slope $\frac{\text{rise}}{\text{run}}$ steep
  b ↑ y-intercept
- Standard Form
  $Ax+By=C$ $3x+5y=10$
  $\cancel{\frac{1}{2}x+9y=-10}$
  Integers
- X-Intercept – crosses x-axis
  (#, 0)



- Y-Intercept – crosses y-axis
  (0, #)



Example

Determine whether each equation is linear and write the equation in standard form if it is linear.

a) $3x^2+7=10$ No squared

b) $5x+3=xy+2$ No

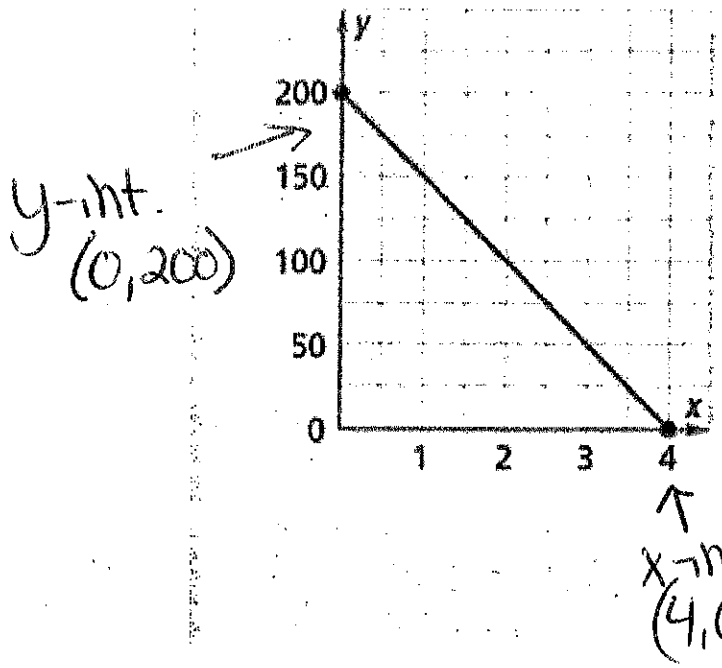
c) $\frac{3}{4}x^1=y+8$ Yes $(\frac{3}{4}x-y=8)4$ $\boxed{3x-4y=32}$

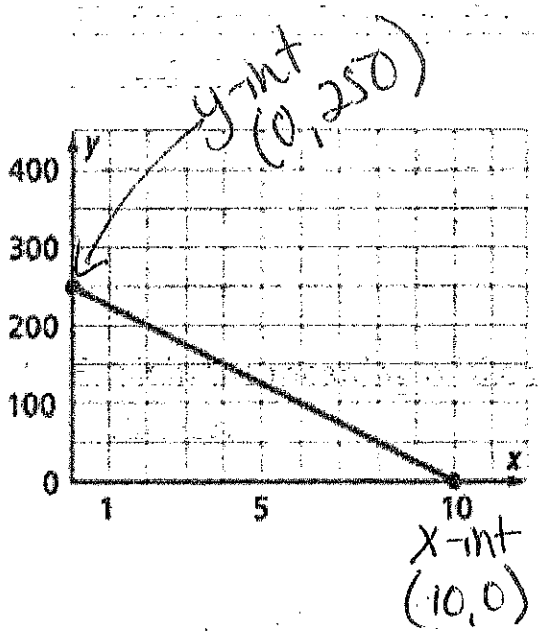
d) $y=4-3x$ Yes

e) $x=\frac{1}{3}y-2$ Yes $(x-\frac{1}{3}y=-2)3$ $3x-1y=-6$

Examples

Find the x and y intercepts of the segments graphed below.





Examples

A box of peanuts is poured into bags at a rate of 4 ounces per second. The table shows the function relating the weight of peanuts in the box and the time in seconds the peanuts have been pouring out of the box. What are the x and y-intercepts? What does this mean in terms of the situation?

Pouring Peanuts	
Time (s) X	Weight (oz) Y
0	2000
125	1500
250	1000
375	500
500	0

X-int: (500, 0) 500sec, 0oz empty bag

y-int: (0, 2000) 0sec, 2000oz, ~~full~~ 2000 oz

ANALYZE TABLES Jules has a gas card for a local gas station. The table shows the function relating the amount of money on the card and the number of times he has stopped to purchase gas.

Number of Gas Stops	Amount of Money ($)
0	125
1	100
2	75
3	50
4	25
5	0

What are the x and y-intercepts? What does this mean in terms of the situation?

X-int (5, 0), 5 stops, $0

y-int (0, 125) 0 stops, $125

Examples – No Calculator

(#, 0) (0, #)

Graph the following equations using x and y-intercepts.

a) $4x - 1y = 4$

(1, 0)

0 $\frac{-1y}{-1} = \frac{4}{-1}$ (0, -4)

b) $-x + 2y = 3$

(-3, 0)

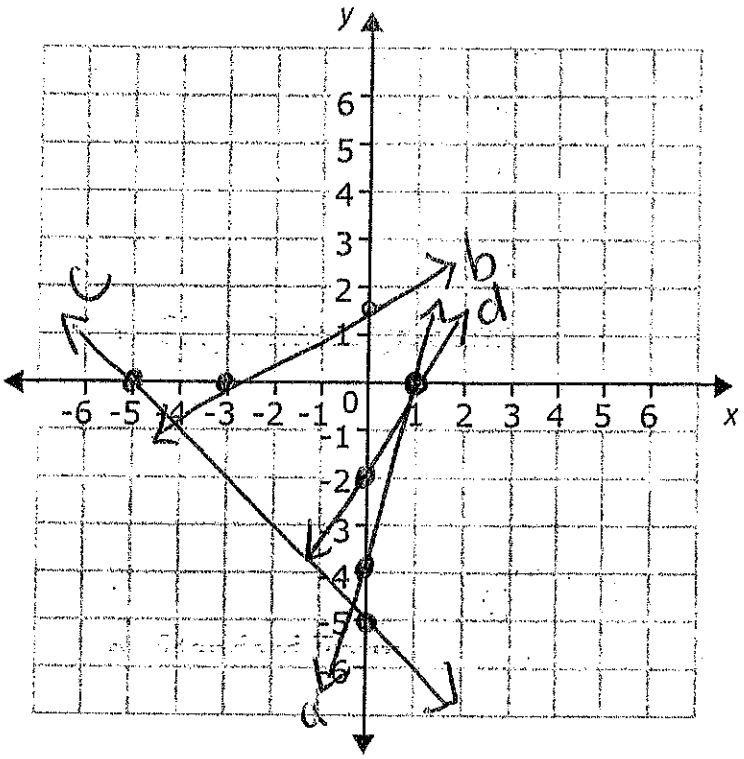
(0, 1.5)

c) $y = -x - 5$

$0 = -x - 5$ (-5, 0)

$5 = -1x$

(0, -5)



d) $2x - y = 2$

$2x = 2$ (1, 0)

$-y = 2$ (0, -2)

Graph the following equations.

a) $x = 3$ (3, -4) x-int (3, 0)

vertical line (3, 5) y-int no

b) $2x - y = 2$ (1, 0)

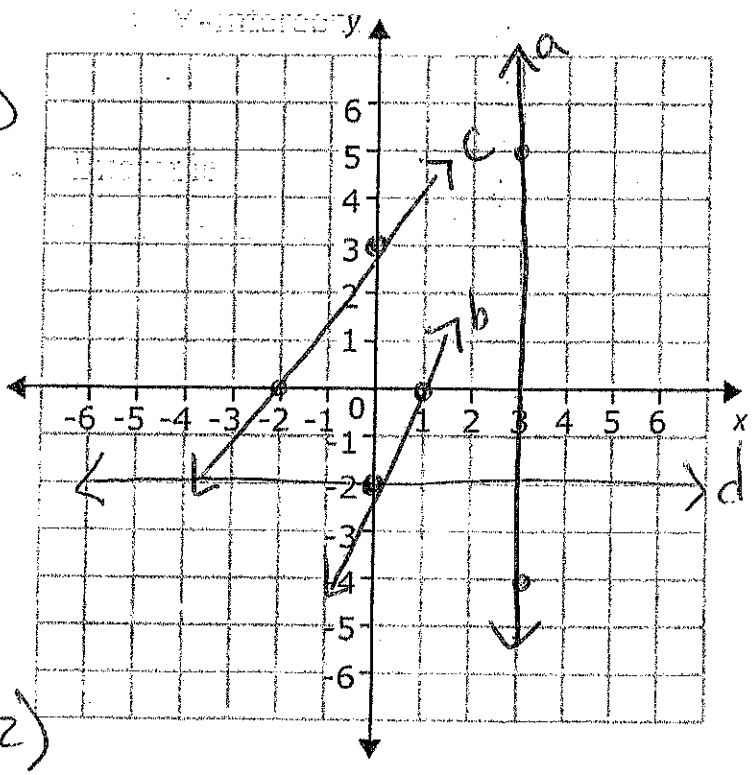
(0, -2)

c) $-3x + 2y = 6$ (-2, 0)

(0, 3)

d) $Y = -2$ (4, -2) x-int No

Horiz line (7, -2) y-int (0, -2)



3.2 Solving Linear Equations by Graphing

Recall how to solve: $2x - 8 = 0$

$y = \frac{2}{1}x - 8$ ← y-int

$y = 0$

Algebraically

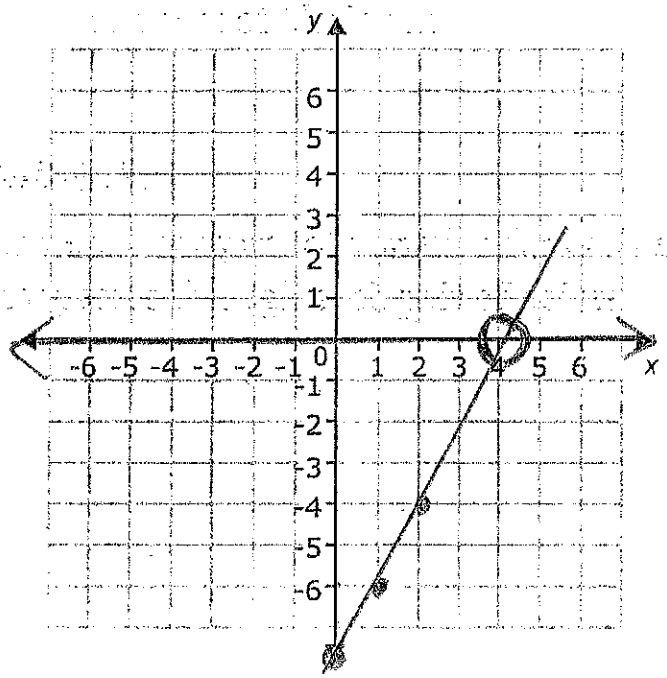
$2x - 8 = 0$

$+8 \quad +8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

Graphically



- Root – x-intercept, zero (plug in 0 for y)

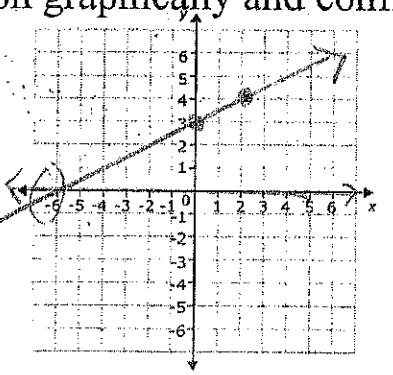
Examples

Solve each equation graphically and confirm algebraically.

1) $0 = \frac{1}{2}x + 3$

$y = 0$

$y = \frac{1}{2}x + 3$



$0 = \frac{1}{2}x + 3$

$-3 \quad -3$

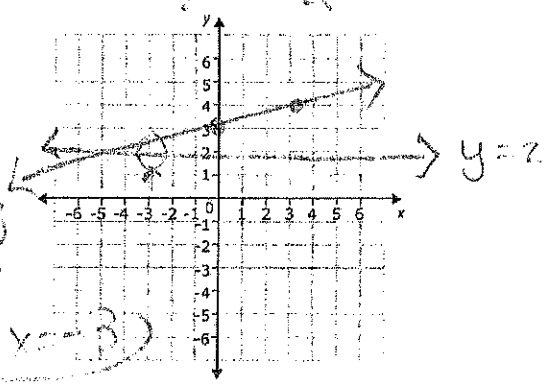
$\frac{-3}{\frac{1}{2}} = \frac{\frac{1}{2}x}{\frac{1}{2}}$

$x = -6$

2) $2 = \frac{1}{3}x + 3$

$y = 2$

$y = \frac{1}{3}x + 3$



$y = 2$

$\frac{1}{3}x + 3$

$-3 \quad -3$

$\frac{-1}{\frac{1}{3}} = \frac{\frac{1}{3}x}{\frac{1}{3}}$

$x = -3$

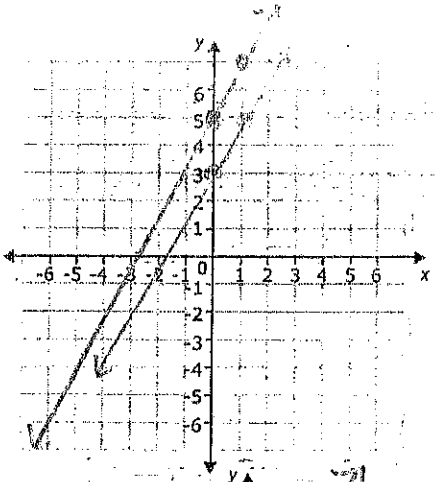
3) $2x + 5 = 2x + 3$

$-5 \quad -3$

$\frac{2}{-2x} = \frac{2x}{-2x}$

$2 = 0$

No Sol.



Inf. Sol

4) $3x + 7 = 3(x - 1)$

$2x + 1 = -\frac{1}{3}x + 4$

$-1 \quad -1$

$y = 2x + 1$

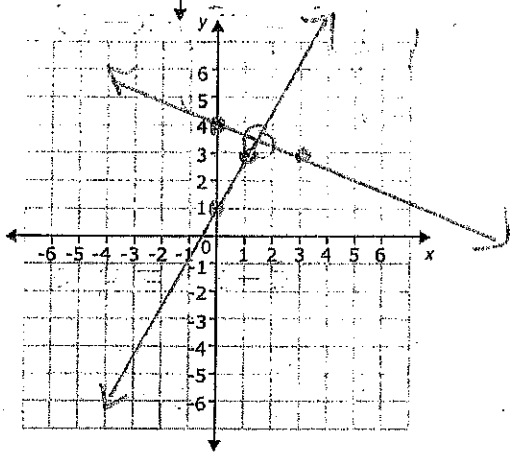
$y = -\frac{1}{3}x + 4$

$2x = \frac{1}{3}x + 3$

$-\frac{1}{3}x \quad -\frac{1}{3}x$

$\frac{2\frac{1}{3}x}{2\frac{1}{3}} = \frac{3}{2\frac{1}{3}}$

$x = 1.3$



Example

Kendra's class is selling greeting cards to raise money for new soccer equipment. They paid \$115 for the cards, and they are selling each card for \$1.75. The function $y = 1.75x - 115$ represents their profit y for selling x greeting cards. Find the zero of this function. Describe what this value means in this context.

profit

$0 = y$

$0 = 1.75x - 115$ cards

$+115 \quad +115$

$\frac{115}{1.75} = \frac{1.75x}{1.75}$

$x = 65.7$

66 cards, 0 profit

break even pt.

3.3 Rate of Change and Slope

The Daredevil Drop at Wet 'n Wild Emerald Pointe in Greensboro North Carolina is a thrilling ride that drops you 76 feet down a steep water chute. The rate of change of the ride might describe the distance a rider has fallen over a length of time.

- Rate of Change
  AKA Slope

$\frac{\text{change y values}}{\text{change x values}}$ $\frac{y_2 - y_1}{x_2 - x_1}$

$(2, 5)$ $(3, 10)$ $\frac{10-5}{3-2}$
x_1 y_1 x_2 y_2

$\frac{5}{1}$

Example

Use the table to find the rate of change. Then explain its meaning.

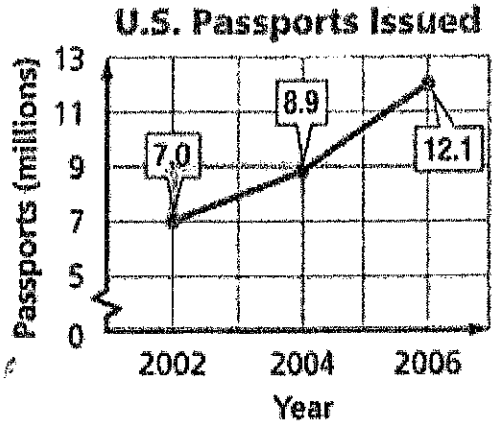
Time Driving (h) x	Distance Traveled (mi) y
2	76
4	152
6	228

$(2, 76)$ $(6, 228)$
x_1 y_1 x_2 y_2

$\frac{228-76}{6-2} = \frac{152}{4}$ miles / hrs.

38 miles/hr
averaging

The graph below shows the number of US passports issued in 2002, 2004, and 2006.



a) Find the rate of change for 2002-2004 and 2004-2006.

2002 to 2004 (2002, 7) (2004, 8.9)

$\frac{8.9-7}{2004-2002} = \frac{1.9}{2} = .95$

2004 to 2006 (2004, 8.9) (2006, 12.1)

$\frac{12.1-8.9}{2006-2004} = \frac{3.2}{2} = 1.6$

b) Explain the meaning of the rate of change in each case.

2002 to 2004 approx. .95 million passports/yr.

2004 to 2006 1.6 million passports/yr

c) How are the rates of change shown in the graph?

Steepness of graph

Steeper 2004 to 2006

- In order for a graph to be linear the rate of change must stay the same the entire time.

X	Y
1	6
2	12
3	18
4	24

Yes

6

x	Y
-10	5
-2	1
6	-4
14	-10

-4

-5

No

x	Y
2	11
4	13
6	15
8	17

Yes

-2

x	Y
-5	7
-4	7
-3	7
-2	7

Yes

0

- Slope

AKA Rate of Change

If you're given two points (x_1, y_1) and (x_2, y_2)

Pos	Neg	Zero	Undefined
$\frac{4}{1}$		$\frac{0}{\#}$	$\frac{\#}{0}$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ memorize

Examples

Find the slope of the line that passes through each pair of points.

a) (-3, 2) and (5, 5)

$x_1\ y_1 \quad x_2\ y_2$

$\frac{5-2}{5+3} = \frac{3}{8}$

b) (-3, -4) and (-2, -8)

$\frac{-8+4}{-2+3} = \frac{-4}{1} = -4$

c) (-3, 4) and (4, 4)

$\frac{4-4}{-3-4} = \frac{0}{-7}$ zero

d) (6, 3) and (6, 7)

$\frac{7-3}{6-6} = \frac{4}{0}$ Undefined

Example

Find the value of r so that the line through (6, 3) and (r, 2) has a slope of ½ .

$\frac{2-3}{r-6} = \frac{1}{2}$

$\frac{-1}{r-6} = \frac{1}{2}$

$-2 = 1(r-6)$ $\boxed{r=4}$

Ex (r, 4) (8, 3) Slope $\frac{2}{3}$

$\frac{3-4}{8-r} = \frac{2}{3}$

$\frac{-1}{8-r} = \frac{2}{3}$

$-3 = 16 - 2r$

$-16 \quad -16$

$\frac{-19}{-2} = \frac{-2r}{-2}$ $(r = 9.5)$

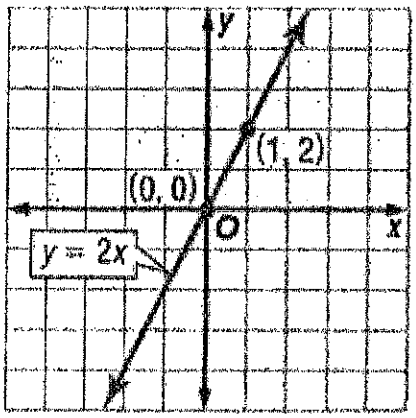
3.4 Direct Variation

- Direct Variation $y = Kx$ where $K \neq 0$ $y = 5x$ $y = \frac{1}{3}x$ $y = -2x$
- Constant of Variation K, number repeatedly using AKA Slope

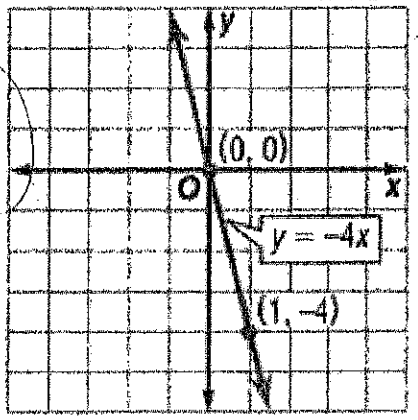
Example

Name the constant of variation for each equation. Then find the slope of the line that passes through each pair of points.

Const. 2
Slope 2

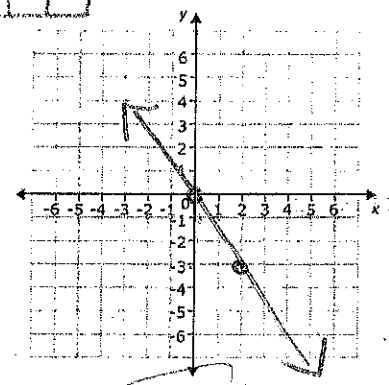


Const -4
Slope -4



Example

Graph $y = -\frac{3}{2}x$



Direct Variation goes through origin

Example

Suppose y varies directly as x and y = 9 when x = -3.

a) Write a direct variation equation that relates x and y.

① Solve for K

$y = Kx$

$\frac{9}{-3} = \frac{K \cdot -3}{-3}$

$K = -3$

② Plug K in

$y = -3x$

b) Use the direct variation equation to find x when y = 15

$y = -3x$

$\frac{15}{-3} = \frac{-3x}{-3}$

$x = -5$

Example

Suppose y varies directly as x and y = 6 when x = 10.

a) Write a direct variation equation that relates x and y.

$y = Kx$

$\frac{6}{10} = \frac{K \cdot 10}{10}$ $K = \frac{6}{10}$ $y = .6x$

b) Use the direct variation equation to find x when y = 15.

$\frac{15}{.6} = \frac{.6x}{.6}$

$x = 25$

Example

The Ramirez family is driving cross-country on vacation. They drive 330 miles in 5.5 hours.

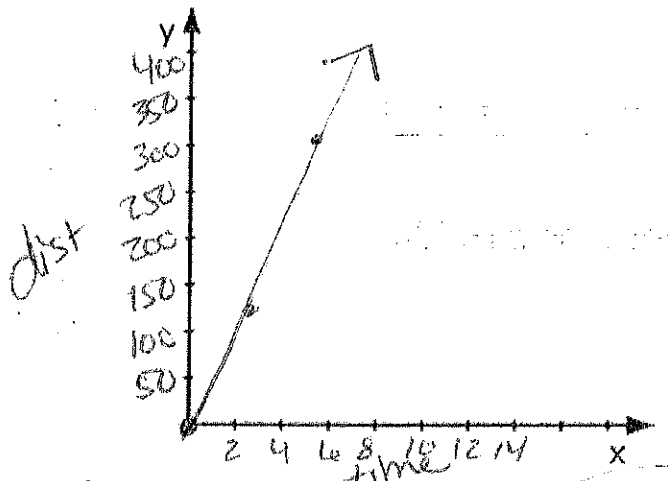
$y = Kx$

a) Write a direct variation equation to find the distance d driven in time t.

$d = rt$ (dist = rate x time)

$\frac{330}{5.5} = \frac{r \cdot 5.5}{5.5}$ $r = 60$ $y = 60x$

b) Graph the equation



(2, 120)

(5, 300)

c) Estimate how many hours it would take to drive 500 miles.

$\frac{500}{60} = \frac{60x}{60}$

$x = 8.3$ hrs

3.5 Arithmetic Sequences as Linear Functions

Complete the table for the following linear functions. What do you notice?

$y = 2x + 0$

x	y
1	2
2	4
3	6
4	8

$y = -3x + 0$

x	y
1	-3
2	-6
3	-9
4	-12

$y = \frac{1}{4}x + 0$

x	y
1	.25
2	.5
3	.75
4	1

- Sequence – 3, 5, 7, 9, ...

  1st term, 2nd term, 3rd term, 4th term; three dots means goes on forever (infinite)

  ("Term" "sequence" or "Number" mean the same thing)

x	y
1	3
2	5
3	7
⋮	⋮

- Arithmetic Sequence – formed using repeated addition or subtraction

  (Linear)

Example

Determine whether each sequence is an arithmetic sequence and justify your thinking.

a) $-15, -13, -11, -9, \ldots$ Yes, Add 2

b) $\frac{7}{8}, \frac{5}{8}, \frac{1}{8}, -\frac{5}{8}, \ldots$ No

c) $-26, -22, -18, -14, \ldots$ Yes, Add 4

d) $1, 4, 9, 25, \ldots$ No

Examples

Find the next three terms of the arithmetic sequences and find an equation for the sequence.

$y = mx + b$

a) -8, -11, -14, -17, ... -20, -23, -26

$-3 \quad -3 \quad -3 \quad -3$

$y = -3x - 5$ check

b) 9.5, 11, 12.5, 14, ... 15.5, 17, 18.5

$+1.5 \quad +1.5$

$y = 1.5x + 8$

c) 6, 8, 10, 12, ... 14, 16, 18

$+2$

$y = 2x + 4$

d) 20, 17, 14, 11, ... 8, 5, 2

$-3 \quad -3 \quad -3$

$y = -3x + 23$

e) -12, -8, -4, 0, ... 4, 8, 12

$+4 \quad +4$

$y = 4x - 16$

Example

0 1 2 3 4

Write an equation for the nth term of the arithmetic sequence 1, 10, 19, 28, 37

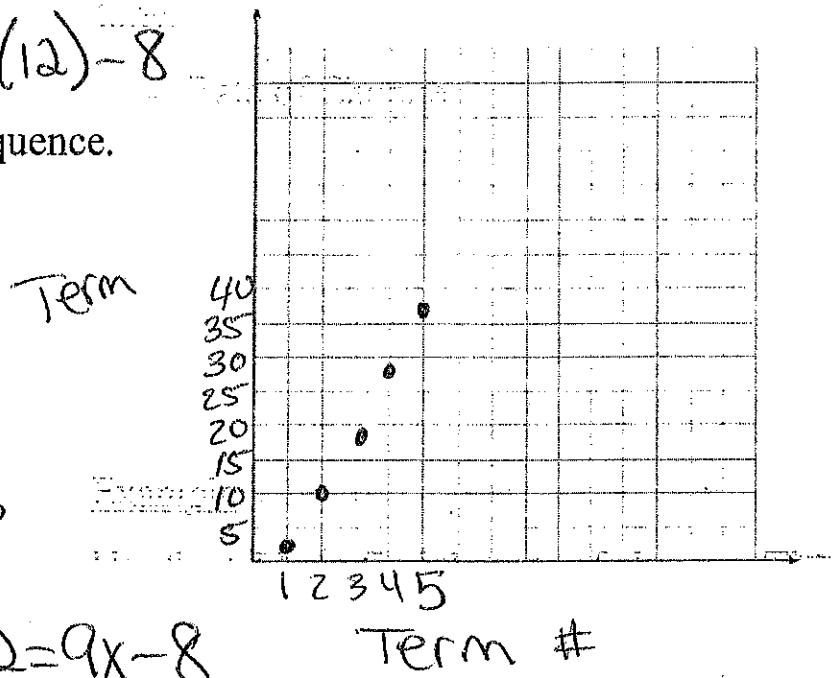
+9 +9 +9

$y = 9x - 8$

Find the 12th term of the sequence.

100 $\quad 9(12) - 8$

Graph the first five terms of the sequence.



Which term of the sequence is 172?

20

$172 = 9x - 8$

$+8 \quad +8$

$\frac{180}{9} = \frac{9x}{9}$

Example

−11 +11 +11

The arithmetic sequence 12, 23, 34, 45, … represents the total number in ounces that a bag weights after each additional newspaper is added.

a) Write a function to represent this sequence.

$y = 11x + 1$

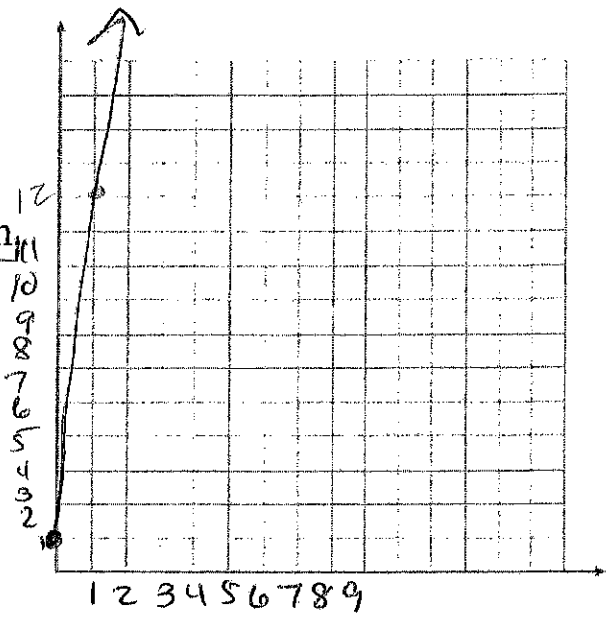
b) Graph the function and determine the domain.

X-values

newspapers

0, 1, 2, 3, ...

whole #s

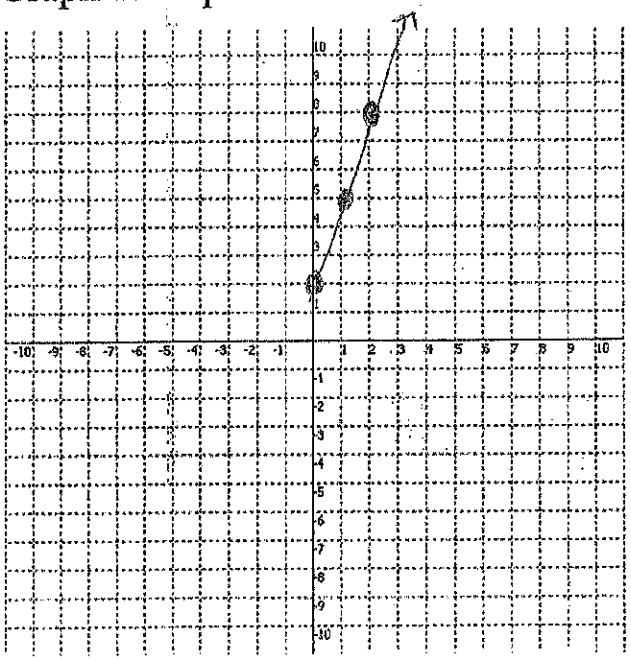


3.6 Linear Relationships

Recall you can write equations for arithmetic sequences such as: 5, 8, 11, 14, ...

$3x+2$ Graph to check

Graph the equation above.



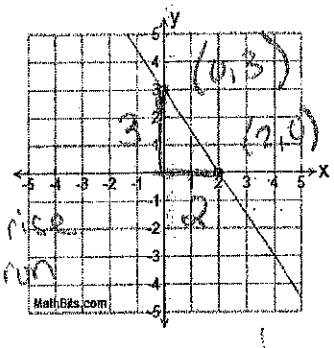
$\frac{3}{1}$ rise / run

(1, 5)
(2, 8)
(3, 11)
(4, 14)

Examples – No Calculator

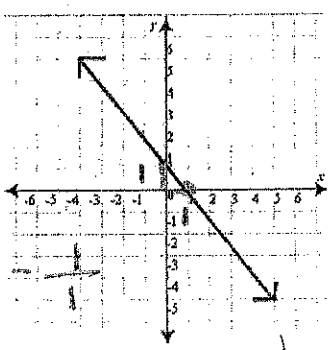
Write an equation in function notation for the given graphs.

$y = mx + b$ ← slope, ← y-int



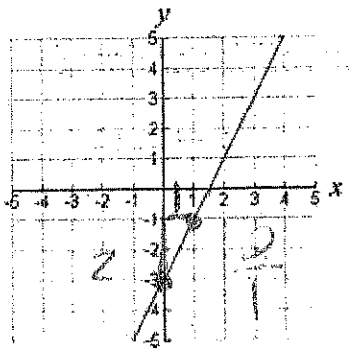
$-\frac{3}{2}$ rise / run

$y = mx + b$
$y = -\frac{3}{2}x + 3$



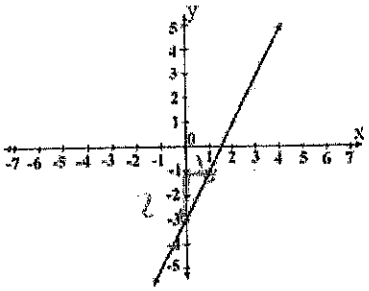
$-\frac{1}{1}$

$y = mx + b$
$y = -1x + 1$



2 $\frac{2}{1}$

$y = mx + b$
$y = 2x - 3$



2

$y = mx + b$
$y = 2x - 3$

Examples -- No Calculator

If the sequence is linear write the equation to describe this relationship.

x	y
1	9
2	12
3	15
4	18

x	y
1	-3
2	-1
3	1
4	3

x	y
0	4
1	8
2	12
3	16

$y = 3x + 6$

$y = 2x - 5$

$y = 4x + 4$

Find the value of y in the above equations when x = 10.

$3(10) + 6$

36

$2(10) - 5$

15

$4(10) + 4$

44

Example

The table shows the number of miles driven for each hour of driving.

Hours	1	2	3	4
Miles	50	100	150	200

Write an equation to describe this relationship.

Use this equation to predict the number of miles driven in 8 hours.

Lines

Graphing on Calc

- Punch in eqns $y=$
- Window $x\ [-10, 10]$
  $y\ [-10, 25]$

Standard window $[-10, 10]$
$[-10, 10]$

$y = 9x + 100$

$\uparrow$ Slope rate of chng.

$[-25, 7]$

$[-20, 125]$