5.1 Solving Inequalities by Addition and Subtraction

- Inequality

$<$ Less than 0 | $>$ Greater than 0 | $\leq$ Less than or = to ● | $\geq$ Greater than or = to ●

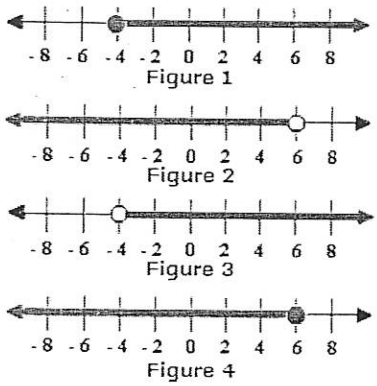
filled in b/c includes value

$x \geq -4$

$x < 6$

$x > -4$

$x \leq 6$



The data in the table shows the recommended daily allowance of Calories for girls ages 11-14 is less than that of girls between 15-18 years old. If both a 13 year old and a 16 year old girl eat 200 fewer calories than recommended, will the 13 year old still each fewer calories than the 16 year old? Yes

Girls 11-14	Girls 15-16
1845	2110

13 1645 16 1910

- Set-Builder Notation ← such that

$\{x \mid x > 5\}$

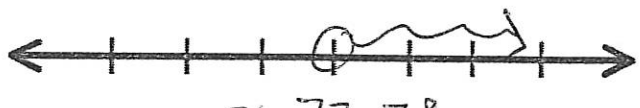
6, 10, 12, 100 .... all are solutions

Example

Solve $c - 12 > 65$ and graph your solution.

$c - 12 > 65$

$c > 77$

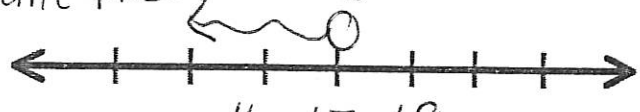


Note the arrows point the same way

Solve $x + 23 < 40$ and graph your solution.

$-23 \quad -23$

$x < 17$



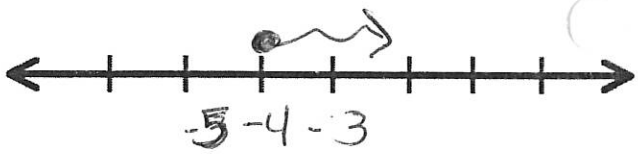
Examples

Solve $12z - 4 \leq 13z$ and graph the solution.

$-4 \leq 1z$

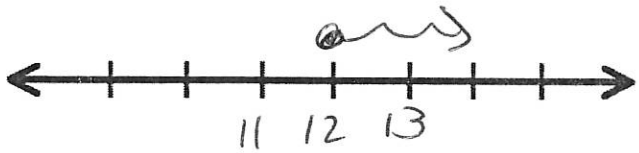
$-4 \leq z$

$z \geq -4$



Solve $5x \geq 12 + 4x$ and graph the solution.

$x \geq 12$



Example

Paul wants to buy season passes to two theme parks. If one season pass costs $54.99 and Paul has $100 to spend on both passes, the second season pass must cost no more than what amount?

$54.99 + x \leq 100$

$x \leq 45.01$

45.01

Write and solve and inequality for the problem: the sum of twice a number and 5 is at most 3 less than the number.

$2x + 5 \leq x - 3$

$x \leq -8$

5.2 Solving Inequalities by Multiplication and Division

$3 \cdot 4 > 2 \cdot 3$

$12 > 6$

$-3 \cdot 4 > 2 \cdot -3$

$-12 < -6$

- mult by neg flip sign

$\frac{4}{-2} > \frac{2}{-2}$

$-2 < -1$

- divide by neg flip sign

- Multiplication Property of Inequality - mult both sides by neg. flip inequality
- Division Property of Inequality - if divide both sides by a neg. flip inequality

Example

Matt walks at a rate of ¾ mile per hour. He knows that it is at least 9 miles to Onyx Lake. How long will it take Matt to get there? Write an inequality to find the time and solve it.

$r \cdot t = d$

$\frac{3}{4}x \geq 9$

$x \geq 12$

at least 12 hours

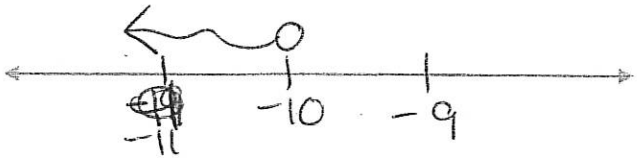
Examples

Solve and graph the following:

$-\frac{3}{5}x > 6$

$\frac{-3/5}{} \quad \frac{-3/5}{}$

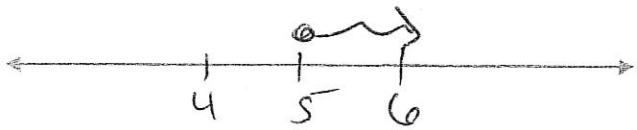
$x < -10$



$12k \geq 60$

$\frac{12}{12} \quad \frac{60}{12}$

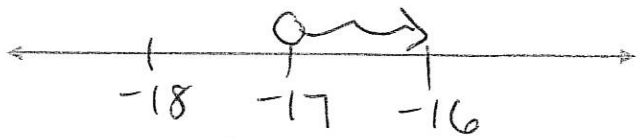
$k \geq 5$



$-8q < 136$

$\frac{-8}{} \quad \frac{136}{-8}$

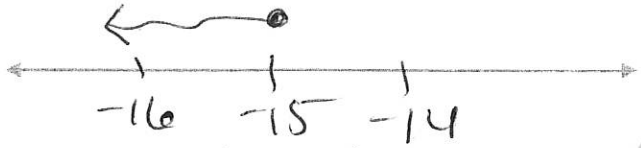
$q > -17$



$\frac{1}{5}m \leq -3$

$\frac{1}{5}$ $\frac{1}{5}$ (Don't Flip)

$m \leq -15$



5.3 Solving Multi-Step Inequalities

Recall: When solving equations you do so using reverse order of operations!

Solve $11x - 13 = 42$

$+13 \quad +13$

$11x = 55$

$x = 5$

Example

Adriana has a budget of \$115 for scanning. The scan service she uses charges \$25 to activate an account and \$0.08 per page to scan and send. How many pages can she send and stay within her budget? Use the inequality $25 + 0.08x \leq 115$.

$25 + .08x \leq 115$

$.08x \leq 90$

$x \leq 1125$

Examples

Solve the following.

$13 - 11x \geq 79$

$-13 \quad -13$

$\frac{-11x}{-11} \geq \frac{66}{-11}$

$x \leq -6$

$6x + 3(2 - x) > -2x + 1$

$6x + 6 - 3x > -2x + 1$

$3x + 6 > -2x + 1$

$5x > 5$

$x > 1$

$-7(x + 4) + 11x \geq 8x - 2(2x + 1)$

$-7x - 28 + 11x \geq 8x - 4x - 2$

$-28 + 4x \geq 4x - 2$

$-28 \geq -2$

No Solution

Recall

- No Sol. $5 = 3$, $9 < 8$ } Never true
- Inf. Sol. $8 = 8$, $10 > 8$ } Always true

$2(4x + 3) < 22 + 8(x - 2)$

$8x + 6 < 22 + 8x - 16$

$8x + 6 < 8 + 8x$

$6 < 8$

Inf. Sol.

$6(5z - 3) \leq 36z$

$30z - 18 \leq 36z$

$-18 \leq 6z$

$-3 \leq z$

Example

Define a variable, write an inequality, and solve the problem.

a) Four times a number plus twelve is less than the number minus three.

n $<$

$4n + 12 < n - 3$

$3n < -15$

$n < -5$

b) Two more than half a number is greater than twenty seven.

n $>$

$2 + \frac{1}{2}n > 27$

$\frac{1}{2}n > 25$

$n > 50$

5.4 Solving Compound Inequalities

- Compound Inequality $9 < x+3 < 12$
- Set { }
- Elements parts of a set

Example: List elements of the set{Baseball positions}

Pitcher, Catcher, 1st Base

- Intersection of Sets $\cap$ "and" $\{2,3,4\} \cap \{3,4,6,7\}$

#s in both $\{3,4\}$

- Union of Sets $\cup$ "or" $\{2,3,4\} \cup \{3,4,6,7\}$

all #s $\{2,3,4,6,7\}$

Example

$A = \{1,2,3,5,7\}$ *and* $B = \{1,2,3,4\}$

A $\cap$ B

$\{1,2,3\}$

A $\cup$ B

$\{1,2,3,4,5,7\}$

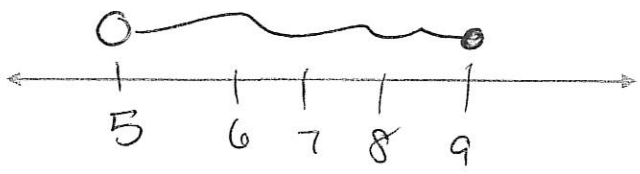
Examples

Solve and graph the following solutions.

a) $7 < z + 2 \leq 11$

$-2 \quad -2 \quad -2$

$5 < z \leq 9$



b) $6 \leq r + 7 < 10$

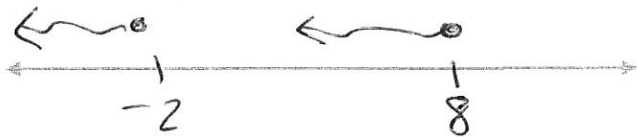
$-7 \quad -7 \quad -7$

$-1 \leq r < 3$

c) $4x - 7 \leq 25$ or $12 - 9x \geq 30$

$4x \leq 32 \qquad -9x \geq 18$

$x \leq 8$ or $x \leq -2$



Example

A ski resort has several types of hotel rooms and cabins. The hotel rooms cost at most \$89 per night, and the cabins cost at least \$109 per night. Write and graph a compound inequality that describes the amount a guest would pay per night at the resort.

$x \leq 89$ or $x \geq 109$

5.5 Inequalities Involving Absolute Value

Recall: $|x| = 4$ has two solutions. 4 or −4

The same is true with inequalities!

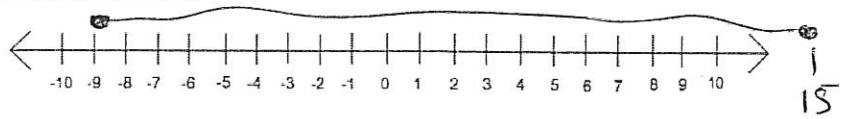
Solve each inequality. Then graph the solution set.

$|n - 3| \leq 12$

because neg flip

$n - 3 \leq 12$ $\quad$ $n - 3 \geq -12$

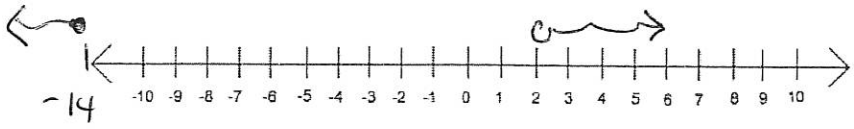
$n \leq 15$ $\quad$ $n \geq -9$



$|x + 6| < -8$

$x + 6 < -8$ $\quad$ $x + 6 > 8$

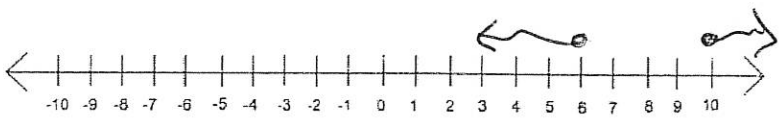
$x < -14$ $\quad$ $x > 2$



$|n - 8| \geq 2$

$n - 8 \geq 2$ $\quad$ $n - 8 \leq -2$

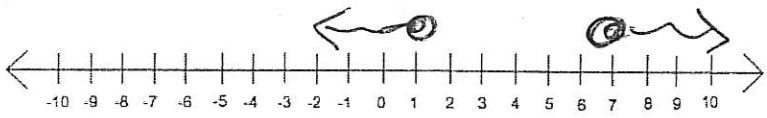
$n \geq 10$ $\quad$ $n \leq 6$



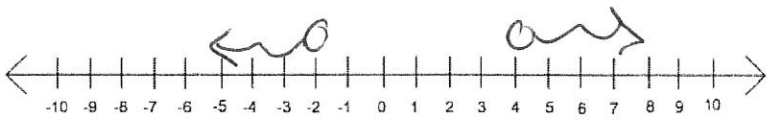
$|k - 4| > 3$

$k - 4 > 3$ $\quad$ $k - 4 < -3$

$k > 7$ $\quad$ $k < 1$



$|3y - 3| > 9$

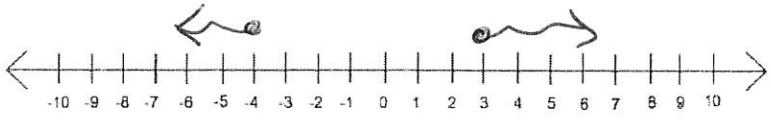


$3y-3>9$ $\quad$ $3y-3<-9$

$3y>12$ $\quad$ $3y<-6$

$y>4$ $\quad$ $y<-2$

$|2x + 1| \geq 7$



$2x+1\geq 7$ $\quad$ $2x+1\leq -7$

$2x\geq 6$ $\quad$ $2x\leq -8$

$x\geq 3$ $\quad$ $x\leq -4$

Examples

The average annual rainfall in California for the last 100 years is 23 inches. However, the annual rainfall can differ by 10 inches from the 100 year average. What is the range of annual rainfall for California?

$|x-23| \leq 10$

$x-23\leq 10$ $\quad$ $x-23\geq -10$

$+23 \quad +23$

$x\leq 33$ $\quad$ $x\geq 13$

13-33

Jeremy bought stock in his favorite fast food restaurant chain at $70.85. However, it has fluctuated up to $.50 in a day. Find the range of prices for which the stock could trade in a day.

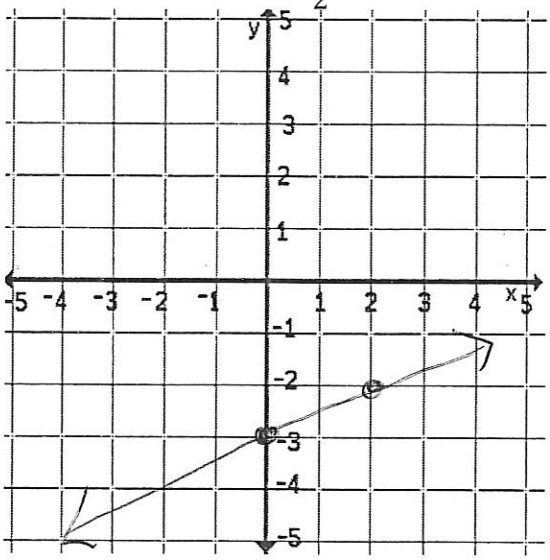
$|x-70.85| \leq .50$ $\quad$ $|x-70.85|\geq .50$

70.35 - 71.35

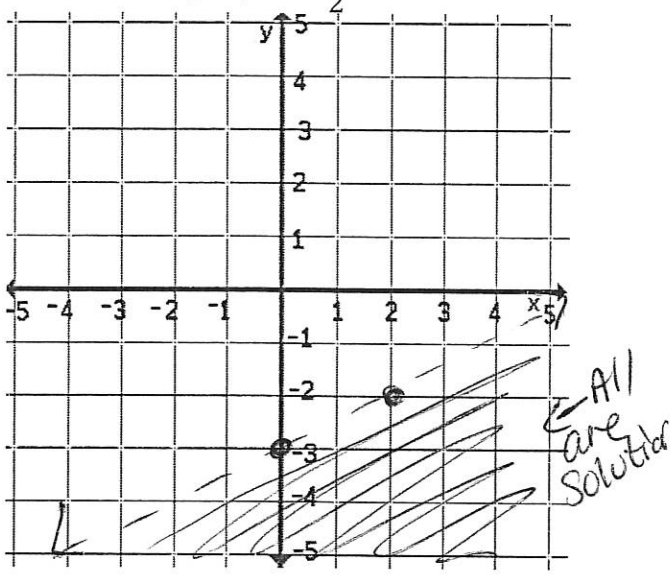
5.6 Graphing Inequalities in Two Variables

Graphing inequalities is very similar to graph linear equations

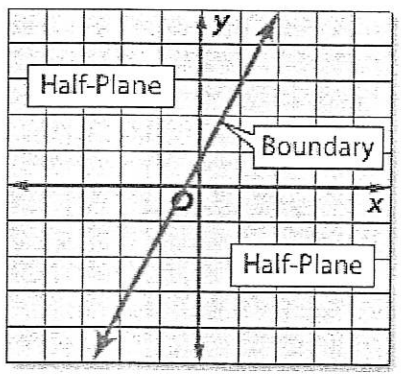
Graph $y = \frac{1}{2}x - 3$



Graph $y < \frac{1}{2}x - 3$



- Boundary – Line that divides plane
- Half Plane
- Closed Half-Plane ——— $\leq$ $\geq$
- Open Half-Plane - - - - - $<$ $>$

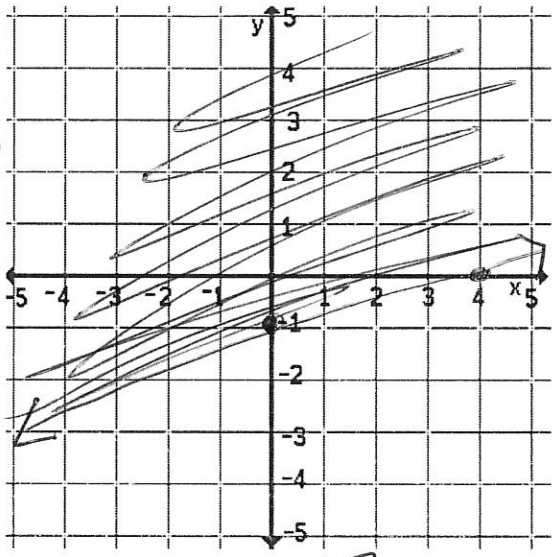


Examples – No Calculator

Graph the following inequalities and state whether the three points given are solutions to the inequalities.

1) $y \geq \frac{1}{4}x - 1$

- Shade above
- Solid line



(0, 0) Yes

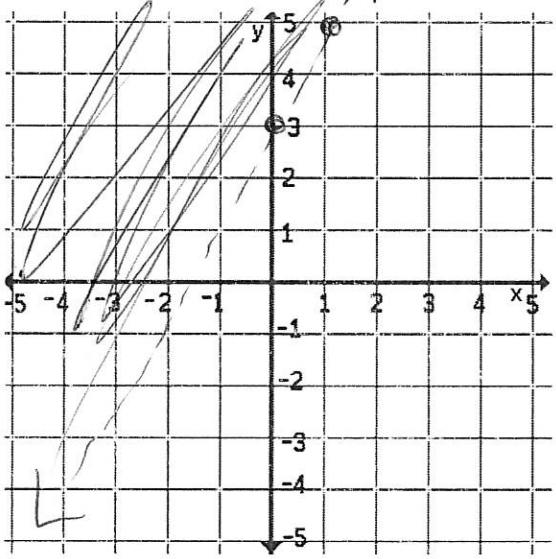
(2, 4) Yes

(-3, 2) Yes

2) $2y - 4x > 6$

$2y > 6 + 4x$

$y > 3 + 2x$



(0, 3) No, dashed line

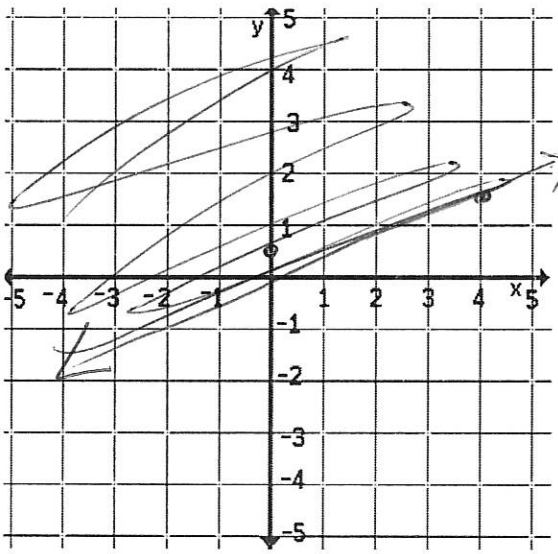
(-3, 2) Yes

(-3, 5) Yes

3) $x + 4y \geq 2$

$\frac{4y}{4} \geq \frac{2 - x}{4}$

$y \geq \frac{1}{2} - \frac{1}{4}x$



(0,0) No

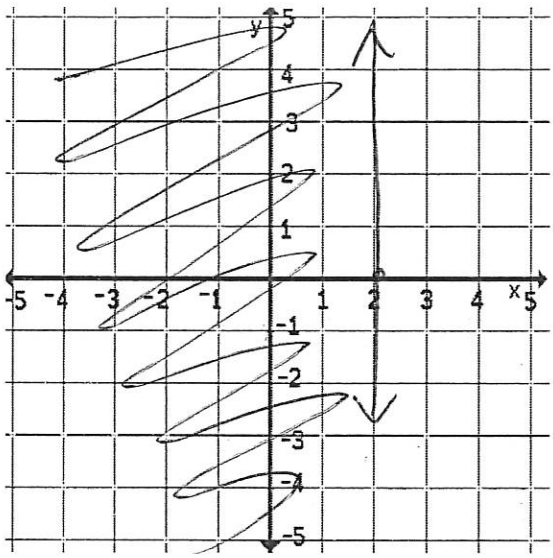
(2, 0) No

(-3, -4) No

4) $2x + 3 \leq 7$

$2x \leq 4$

$x \leq 2$



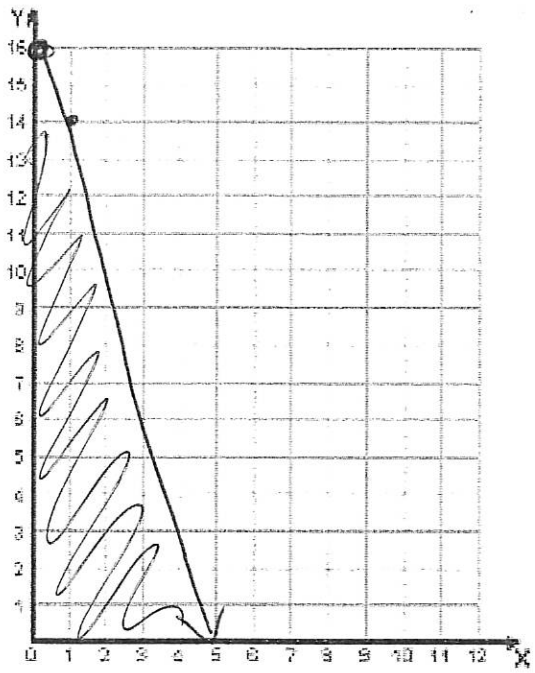
(-1, 8) Yes

(-1, 4) Yes

(2, 3) Yes

Example

Ryan writes and edits short articles for a local newspaper. It takes him about an hour to write an article and about a half-hour to edit an article. If Ryan works up to 8 hours a day, how many articles can he write and edit in one day?



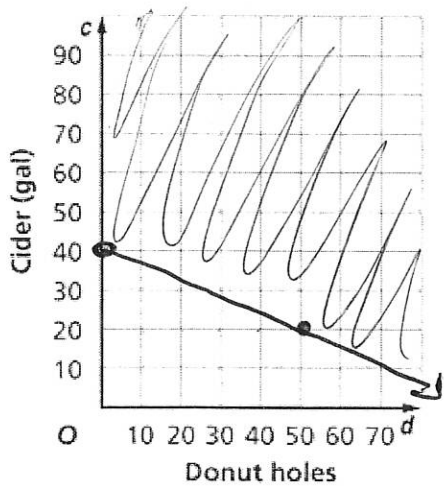
write edit

$1x + \frac{1}{2}y \leq 8$

$\frac{\frac{1}{2}y}{\frac{1}{2}} \leq \frac{8 - x}{\frac{1}{2}}$

$y \leq 16 - 2x$

The Class of 2020 is selling donuts and cider to raise money for prom. They sold donut holes for $1.00 a box and cider for $2.50 a gallon. In order to cover their expenses, they need to raise at least $100. Write and graph an inequality that represents this situation.



$1d + 2.50c \geq 100$

$\frac{2.50c}{2.50} \geq \frac{100 - d}{2.50}$

$c \geq 40 - \frac{2}{5}d$