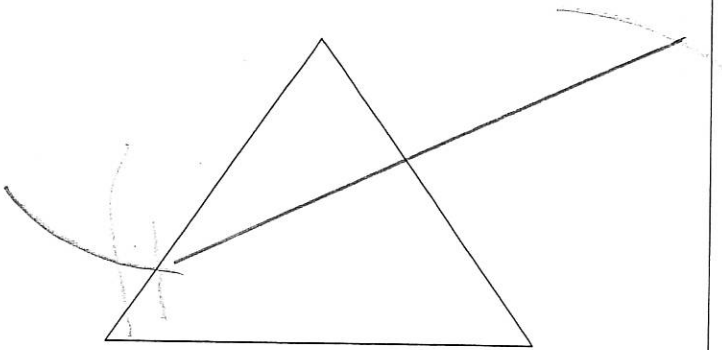
5.1 Bisectors, Medians, and Altitudes

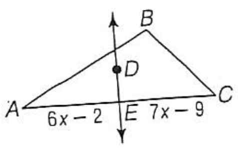
- Concurrent Lines – when three or more lines intersect at one point
- Point of Concurrency – the point where three or more lines intersect

Triangles have four sets of lines that are concurrent

1. Perpendicular Bisectors
2. Angle Bisectors
3. Medians
4. Altitudes

Concurrent Lines	**Point of Concurrency**
Perpendicular Bisectors	Circumcenter
Theorem: The circumcenter is equidistant from the vertices	**Construction** 

Example



$\overline{DE}$ is the perpendicular bisector of $\overline{AC}$.

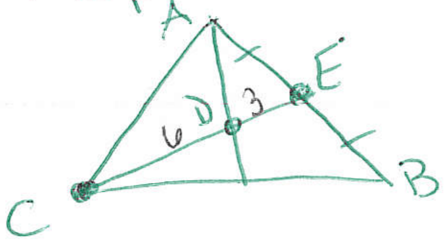
$6x-2=7x-9$

$-2=x-9$

$7=x$

Concurrent Lines

Median – segment whose endpoints are vertex & midpt of opp side



Point of Concurrency

Centroid

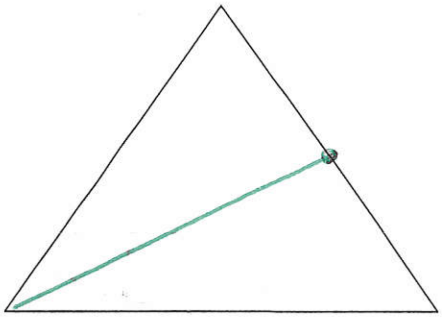
pt of balance

Theorem

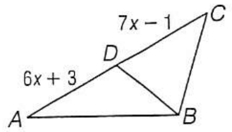
The medians of a triangle are concurrent at a point that is 2/3 the distance from each vertex to the midpoint of the opposite side

$CD = \frac{2}{3} CE$

Construction



Example

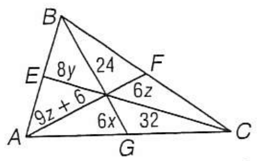


$\overline{BD}$ is a median.

$6x+3=7x-1$

$3=x-1$

$x=4$



D is the centroid of $\triangle ABC$.

$\frac{2}{3}(32+8y)=32$

$\frac{64}{3}+\frac{16}{3}y=32$

$y=2$

$24=\frac{2}{3}BG$

$BG=36$

$24+6x=36$

$\boxed{x=2}$

$\boxed{y=2}$

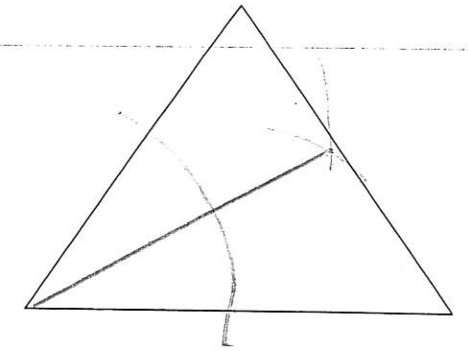
$\frac{2}{3}(6z+9z+6)=9z+6$

$\frac{2}{3}(15z+6)=9z+6$

$10z+4=9z+6$

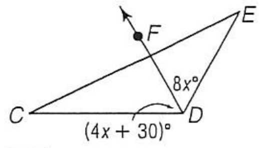
$z+4=6$

$\boxed{z=2}$

Concurrent Lines	Point of Concurrency
Angle Bisectors	incenter
Theorem The incenter is equidistant from the sides a. *Note: You can inscribe a circle with the incenter.*	**Construction** 

GSP

Example

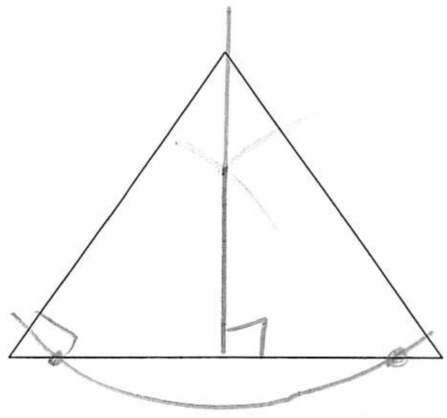


$\overline{DF}$ bisects $\angle CDE$.

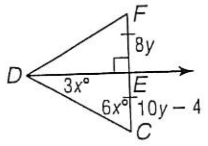
$4x+30=8x$

$30=4x$

$x=7.5$

Concurrent Lines **Altitude – perpendicular segment from a vertex to the line containing the opposite side**	**Point of Concurrency** Orthocenter
Theorem **The lines that contain the altitudes of a triangle are concurrent**	**Construction** 

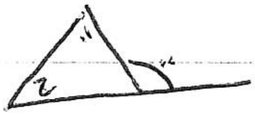
Example



$\triangle CDF$ is equilateral.

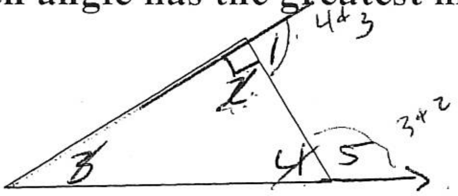
5.2 Inequalities in Triangles

- Exterior Angle Inequal Theorem – **if an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles.**

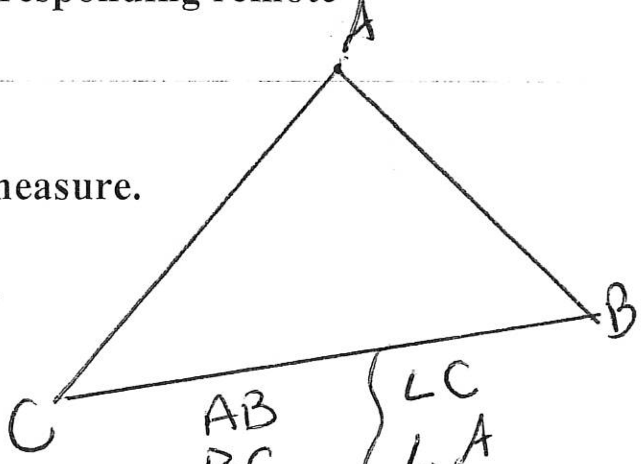


Example

Determine which angle has the greatest measure.



$\angle 5$

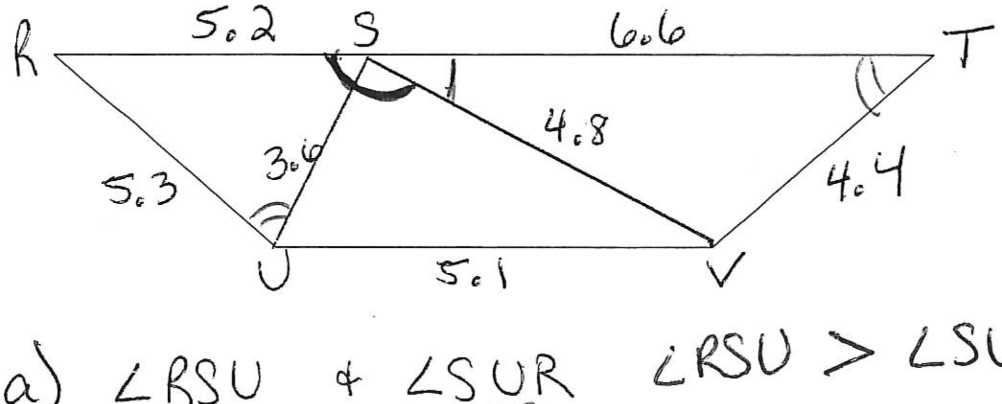


AB $\quad \angle C$
BC $\quad \angle A$
AC $\quad \angle B$

- **One side of a triangle is longer than another side if and only if the angle opposite the** longer side has a greater measure than the angle opposite the smaller side.

Example

Determine the relationship between the measures of the given angles.



a) $\angle RSU$ & $\angle SUR$ $\quad \angle RSU > \angle SUR$
5.3 $\quad$ 5.2

b) $\angle TSV$ & $\angle STV$ $\quad \angle STV > \angle TSV$
4.4 $\quad$ 4.8

c) $\angle RSV$ & $\angle RUV$ $\quad \angle RSV > \angle RUV$

5.3 Indirect Proof

- **Indirect Reasoning** – assume conclusion is false
- **Steps for Indirect Proofs**:

1) Assume conclusion is false

2) Show contradiction

3) Point out because contradiction, original conclusion must be true.

Example

State the assumption you would make to start an indirect proof of each statement.

a) **Line EF is not a perpendicular bisector.** Line EF is perp. bisector.
b) **$3x = 4y + 1$** $3x \neq 4y + 1$
c) **Angle 1 is less than or equal to angle 2** Angle 1 is greater than angle 2.
d) **If B is the midpoint of segment LH and LH = 26, then segment BH is congruent to segment LB** not

Example

Given : $\frac{1}{2y+4} = 20$

Prove: $y \neq -2$

Assume $y = -2$

$\frac{1}{2(-2)+4} = 20$

$\frac{1}{-4+4} = 20$

$\frac{1}{0} \neq 20$ Contradiction

$y \neq -2$

Try it:

Given: $2x - 3 > 7$

Prove: $x > 5$

Assume $x < 5$

$2 \cdot 2 - 3 > 7$

$1 > 7$ Contradiction

$x > 5$

5.3 (day 2)

Example

Marta signed up for three classes at a community college for a little under $156. There was an administration fee of $15, but the class costs varied. How can you show that at least one class cost less than $47?

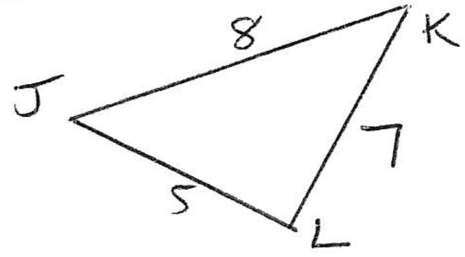
Assume $x \geq 47$

$47+47+47+15 \geq 156$ Contradiction

So one class must be less than \$47.

Example

Given triangle JKL with side lengths 5, 7, and 8 has shown.



Prove:

$m\angle K < m\angle L$

Assume $m\angle K > m\angle L$

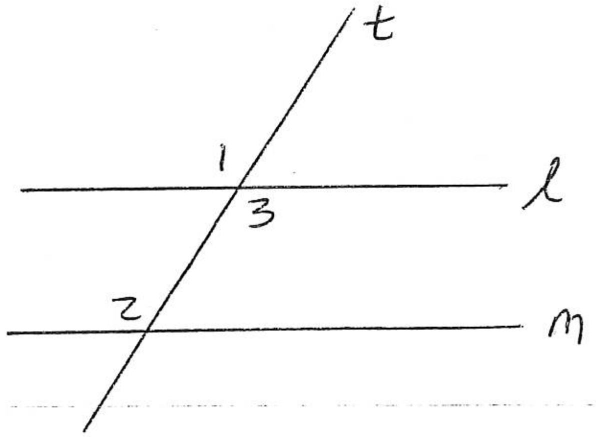
$\overline{JL} > \overline{JK}$ by angle side relation.

Contradiction

$\therefore m\angle K < m\angle L$

Example

Given: $m\angle 2 \neq m\angle 1$



Prove: $l \nparallel m$

Assume $l \parallel m$

$m\angle 1 \cong m\angle 2$ Cooresponding $\angle$s

Contradiction!

$\therefore l \nparallel m$

Example

Given: n is odd

Prove: n^2 is odd

Assume n^2 is even.

N is odd so $2a+1 = n$

$n^2 = (2a+1)^2$

$= (2a+1)(2a+1)$

$= 4a^2+4a+1$

$\therefore n^2$ is even. $= 2(2a^2+2a)+1$ odd Contradiction!

5.4 The Triangle Inequality

- **Triangle Inequality Theorem** – The sum of the lengths of any two sides of a triangle is greater than the length of the third side

Example

Determine whether the given measures can be the lengths of the sides of a triangle.

a) 6.5, 6.5, 14.4 NO $6.5 + 6.5 \not> 14.5$

b) 6.8, 7.2, 5.1 Yes $6.8 + 5.1 > 7.2$

Example

In triangle PQR, PQ = 7.2 and QR = 5.2. Which measure cannot be PR?

A) 7 B) 9 C) 11 D) 13

Example

Find the range for the measures of the third side of a triangle given the measures of two sides.

A) 14 and 23 x

$14 + 23 > x$ $14 + x > 23$ $23 + x > 14$

$37 > x$ $x > 9$ $x > -9$

$\boxed{37 > x > 9}$

B) 15 and 18

$15 + x > 18$ $18 + x > 15$ $15 + 18 > x$

$x > 3$ $\cancel{x > 3}$ $33 > x$

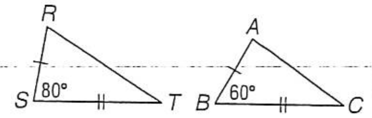
$33 > x > 3$

NAME ______________________________ DATE __________ PERIOD _____

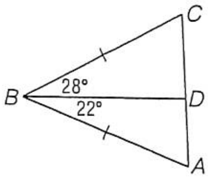
5-5 Study Guide and Intervention

Inequalities Involving Two Triangles

SAS Inequality The following theorem involves the relationship between the sides of two triangles and an angle in each triangle.

SAS Inequality/Hinge Theorem	If two sides of a triangle are congruent to two sides of another triangle and the included angle in one triangle has a greater measure than the included angle in the other, then the third side of the first triangle is longer than the third side of the second triangle.	 If $\overline{RS} \cong \overline{AB}$, $\overline{ST} \cong \overline{BC}$, and $m\angle S > m\angle B$, then $RT > AC$.

Example **Write an inequality relating the lengths of $\overline{CD}$ and $\overline{AD}$.**

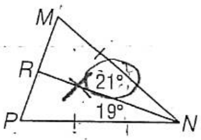


Two sides of $\triangle BCD$ are congruent to two sides of $\triangle BAD$ and $m\angle CBD > m\angle ABD$. By the SAS Inequality/Hinge Theorem, $CD > AD$.

Exercises

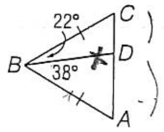
Write an inequality relating the given pair of segment measures.

1.



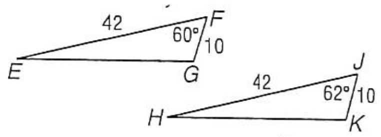
MR, RP $MR > RP$

2.



AD, CD $AD > CD$

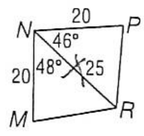
3.



EG, HK $HK > EG$

60 62

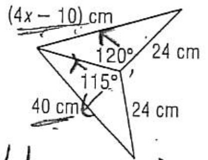
4.



MR, PR $MR > PR$

Write an inequality to describe the possible values of x.

5.

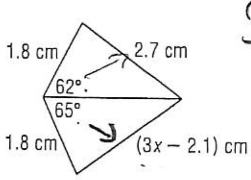


$4x - 10 > 40$

$4x > 50$

$\boxed{x > 12.5}$

6.



$3x - 2.1 > 2.7$

$+2.1 \quad +2.1$

$3x > 4.8$

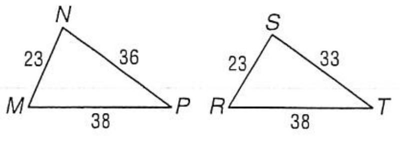
$\boxed{x > 1.6}$

Lesson 5-5

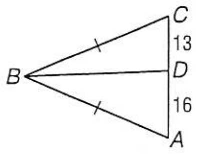
5-5 Study Guide and Intervention *(continued)*

Inequalities Involving Two Triangles

SSS Inequality The converse of the Hinge Theorem is also useful when two triangles have two pairs of congruent sides.

SSS Inequality	If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.	 If $NM = SR$, $MP = RT$, and $NP > ST$, then $m\angle M > m\angle R$.

Example **Write an inequality relating the measures of $\angle ABD$ and $\angle CBD$.**

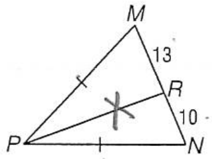


Two sides of $\triangle ABD$ are congruent to two sides of $\triangle CBD$, and $AD > CD$. By the SSS Inequality, $m\angle ABD > m\angle CBD$.

Exercises

Write an inequality relating the given pair of angle measures.

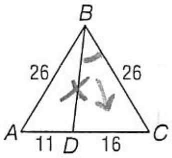
1.



$m\angle MPR$, $m\angle NPR$

m∠MPR > m∠NPR

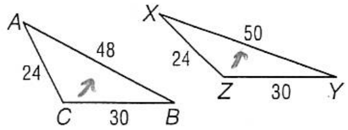
2.



$m\angle ABD$, $m\angle CBD$

m∠CBD > m∠ABD

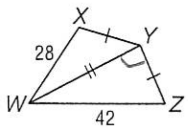
3.



$m\angle C$, $m\angle Z$

m∠Z > m∠C

4.

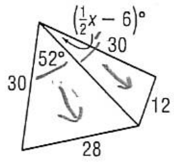


$m\angle XYW$, $m\angle WYZ$

m∠WYZ > m∠XYW

Write an inequality to describe the possible values of x.

5.



$52 > \frac{1}{2}x - 6$

$58 > \frac{1}{2}x$

$116 > x$

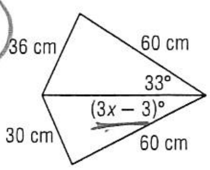
$\frac{1}{2}x - 6 = 0$

$\frac{1}{2}x = 6$

$x = 12$

x between 12 + 116

6.



$33 > 3x - 3$

$36 > 3x$

$12 > x$

$\boxed{\text{between } 1 + 12}$

$3x - 3 = 0$

$3x = 3$

$x = 1$