① 7 & 3 ② 66 & 29 ③ 6.25 & 37.5 ④ 4/5 & 1/5 ⑤ -25 & -4.5

Key

6.1 Graphing Systems of Equations

$\begin{cases} x+y=12 \\ 2x-y=18 \end{cases}$ (10, 2)

- System – set of sentences joined by the word "and"
- Solution to a System – pair of numbers that makes the system true

#s that make both sent true

Example

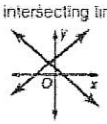
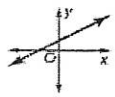
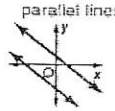
Show that (-4, -2) is the solution to the system: (x, y)

$\begin{cases} y=\frac{1}{2}x \\ y=x+2 \end{cases}$

$-2=\frac{1}{2}(-4)$ $-2=-2$ ✓

$-2=-4+2$ $-2=-2$ ✓

Special Situations:

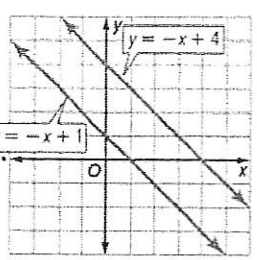
Graph of a System	intersecting lines 	same line 	parallel lines 
Number of Solutions	exactly one solution	infinitely many solutions	no solution
Terminology	consistent and independent	consistent and dependent	inconsistent

Examples – No Calculator

Use the graph to determine whether each system is consistent or inconsistent and if it is independent or dependent.

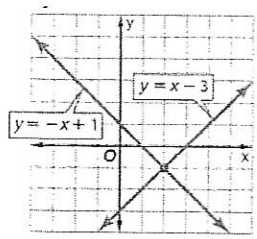
$y=-x+1$
$y=-x+4$

inconsist

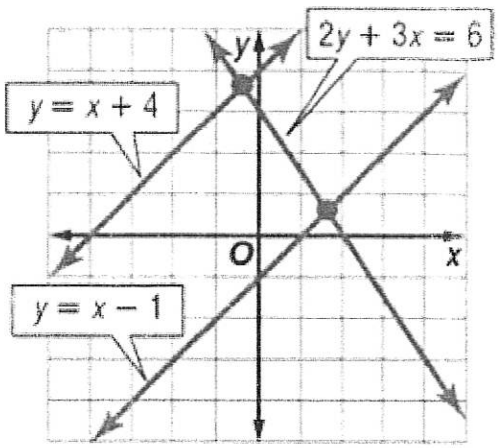


$y=x-3$
$y=-x+1$

consist & indep.



Use the graph at the right to determine if each system is consistent or inconsistent and if it is independent or dependent.



1) $\begin{cases} 2y + 3x = 6 \\ y = x - 1 \end{cases}$ consistent & indep. $\approx (1.8, .7)$

2) $\begin{cases} y = x + 4 \\ y = x - 1 \end{cases}$ Inconsistent (no sol)

Examples – No Calculator

Graph each system and determine the number of solutions. If it has one solution, name it.

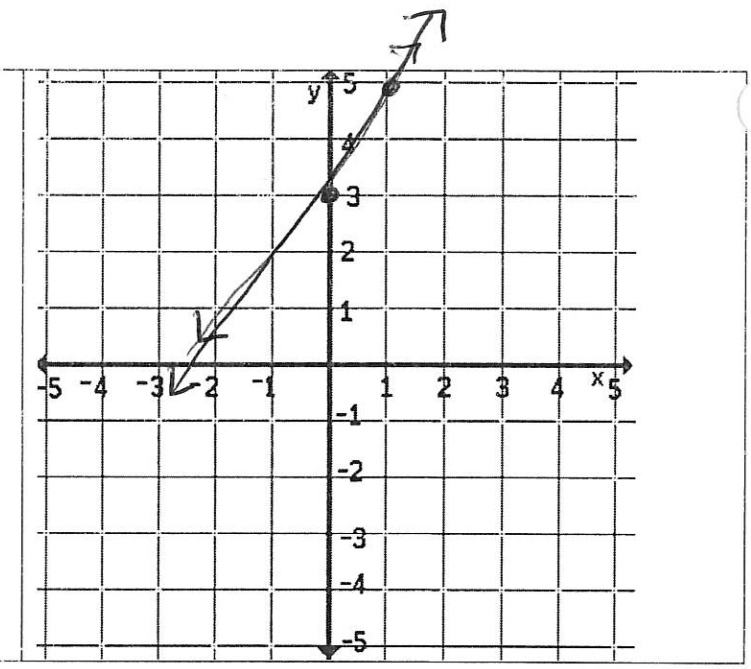
1) $\begin{cases} y = 2x + 3 \\ 8x - 4y = -12 \end{cases}$

$\frac{-4y}{-4} = \frac{-12 - 8x}{-4}$

$y = 3 + 2x$

consistent & dependent

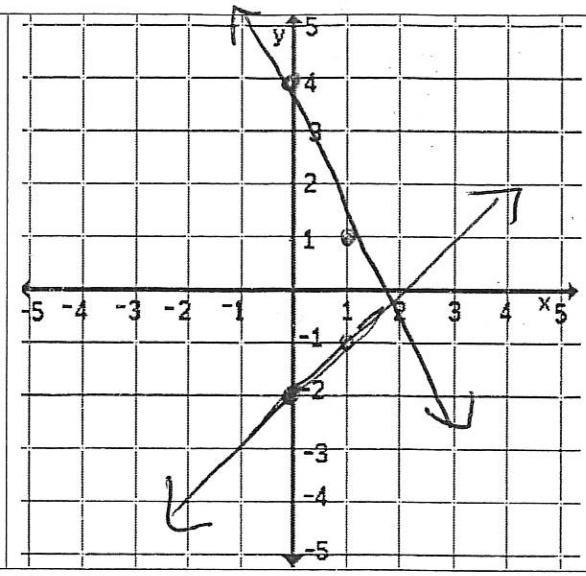
(Inf. Sol)



2) $\begin{cases} y = -3x + \cancel{10}\ 4 \\ y = x - 2 \end{cases}$

1 sol

(1.5, -0.5)



Day 2

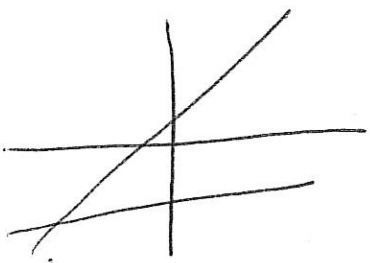
Examples

① $Y_1 =$
$Y_2 =$

② 2nd-Trace
Intersect
Enter 3X

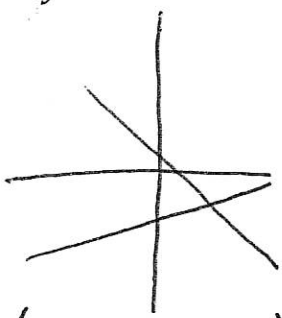
Solve the following systems using your graphing technology.

$\begin{cases} y = 2x + 3 \\ y = x - 5 \end{cases}$



(-8, -13)

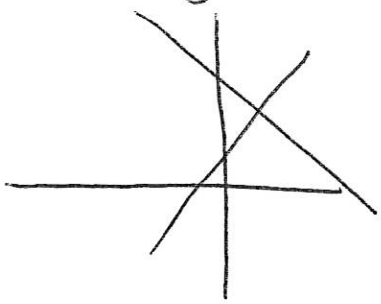
$\begin{cases} y = \frac{1}{3}x - 4 \\ y = -4x + 5 \end{cases}$



(2.07, -3.31)

$\begin{cases} y = 3x + 1 \\ 2x + y = 9 \end{cases}$

$y = 9 - 2x$



(1.6, 5.8)

Examples

Natalie rode 20 miles last week and plans to ride 35 miles per week. Diego rode 50 miles last week and plans to ride 25 miles per week. Predict the week in which Natalie and Diego will have ridden the same number of miles.

$\begin{cases} 20+35x=y & \text{Natalie} \\ 50+25x=y & \text{Diego} \end{cases}$ $(3, 125)$ ↓ 3 weeks

Alex and Amber are both saving money for a summer vacation. Alex has already saved $100 and plans to save $25 per week until the trip. Amber has $75 and plans to save $30 per week. In how many weeks will Alex and Amber have the same amount of money?

$\begin{cases} 100+25x=y & \text{Alex} \\ 75+30x=y & \text{Amber} \end{cases}$ $(5, 225)$ ↓ 5 weeks

Josh and Joe each want to buy a video game. Joe has $14 and saves $10 a week. Josh has $26 and saves $7 a week. In how many weeks will they have the same amount?

$\begin{cases} 14+10x=y & \text{Joe} \\ 26+7x=y & \text{Josh} \end{cases}$ $(4, 54)$ ↑ 4 weeks

6.2 Substitution

Sometimes the exact answers to a system are not whole numbers. So graphing is not the best option.

- Substitution – replacing a variable in the equation with an equivalent equation

$\begin{cases} x = y \\ y = 2x + 3 \end{cases}$

$x = 2x + 3$
$-x = 3$
$x = -3$

Example

Solve by substitution.

$\begin{cases} y = 5x + 9 \\ y = -3x + 37 \end{cases}$

$5x + 9 = -3x + 37$
$8x = 28$
$x = 3.5$

$5(3.5) + (9)$
$= 26.5$

$(3.5, 26.5)$

Solve by substitution.

$\begin{cases} x + y = 14 \\ x = 6y \end{cases}$

$6y + y = 14$
$7y = 14$
$y = 2$
$x = 12$

$(12, 2)$

Solve by substitution.

$\begin{cases} 3x + 4y = -15 \\ Y = 2x - 3 \end{cases}$

$3x + 4(2x - 3) = -15$
$3x + 8x - 12 = -15$
$11x = -3$
$x = -3/11$

$\left(-3/11, \frac{-39}{11}\right)$

Example

Mrs. Janeski wants to have a clown deliver helium balloons to her daughter's birthday party. Two companies offer this service. Company A charges 80 cents a balloon, plus $6 delivery fee. Company B charges 95 cents per balloon but has no delivery fee. For how many balloons is the cost the same? (Let x = the number of balloons and y = cost of the balloons)

Comp A $.80x + 6 = y$

Comp B $.95x = y$

$.80x + 6 = .95x$

$6 = .15x$

$x = 40$

Example

The Wolff family bought two chairs. One cost $15 less than the other. Together the chairs cost $374. Find the price of each chair.

$y = x - 15$

$x + y = 374$

$x + x - 15 = 374$

$2x = 389$

$x = 194.50$

194.50 & 179.50

6.3 Elimination Using Addition and Subtraction

So far we have used graphing and sub. to solve systems. You can also use elimination

- Elimination –

$2x+3y=12$
$-2x+y=36$
$4y=48$
$y=12$

$-2x+12=36$
$-2x=24$
$x=-12$

Examples – No Calculator

Solve the system.

$x+y=63$
$x-y=12$
$2x=75$
$x=37.5$

$37.5+y=63$
$y=25.5$
$(37.5, 25.5)$

$-4x+3y=-3$
$4x-5y=5$
$-2y=2$
$y=-1$

$-4x-3=-3$
$-4x=0$
$x=0$
$(0, -1)$

Examples

Four times one number minus three times another number is 12. Two times the first number added to three times the second number is 6. Find the numbers.

$4x-3y=12$
$2x+3y=6$
$6x=18$
$x=3$

$12-3y=12$
$-3y=0$
$y=0$

English is spoken as the primary language in 78 more countries than Farsi is spoken as the first language. Together, English and Farsi are spoken as first languages in 130 countries. In how many countries are each spoken as a first language?

$E+F=130$
$E=78+F$

$78+2F=130$
$F=26$

$E=104$

Day 2

Examples

Solve the following systems.

$4x + 13y = 40$

$x-1\,(4x - 5y = -4)$

$4x+13y=40$

$-4x+5y=4$

$18y=44$

$y=2.\overline{4}$ $\boxed{\frac{22}{9}}$

$4x+13\left(\frac{22}{9}\right)=40$

$x=\frac{37}{18}$

$\left(\frac{37}{18}, \frac{22}{9}\right)$

$\begin{cases} 3x + 2y = -1 \\ -4x + 2y = +6 \end{cases}$

$-1x = 5$

$x=-5$

$-15+2y=-1$

$2y=14$

$y=7$

$(-5, 7)$

$\begin{cases} 3(x + 1y) = 12 \\ 3x - 5y = 20 \end{cases}$

$3x+3y=12$

$-3x+5y=-20$

$8y=-8$

$y=-1$

$3x+5=20$

$3x=15$

$x=5$

$(5, -1)$

Example

A hardware store earned $956.50 from renting ladders and power tools last week. The store charged customers for a total of 36 days for ladders and 85 days for power tools. This week the store charged 36 days for ladders and 70 days for power tools, and earned $829. How much does the store charger per day for ladders and for power tools?

ladders tools

$36x + 85y = 956.50$

$-36x + 70y = -829$

$15y = 127.50$

$y = 8.5$

$8.50 for tools

$6.50 for ladder

$36x + 85(8.50) = 956.50$

$36x = 234$

$x = 6.5$

Going with the wind, a blimp flies 360 miles to an air show. The trip takes 4 hours. The return trip, flying against the wind, takes 9 hours. How fast is the blimp flying in still air? What is the speed of the wind?

$b + w = 360/4$

$b - w = 360/9$

$b + w = 90$

$b - w = 40$

$2b = 130$

$b = 65$

blimp 65

wind 25

6.4 Elimination using Multiplication

Examples

Solve the following systems:

not same

$-2(5x + 8y = 21)$

$10x - 3y = -15$

$-10x - 16y = -42$

$10x - 3y = -15$

$-19y = -57$

$y = 3$

$10x - 9 = -15$

$10x = -6$

$x = -.6$

$(-.6, 3)$

$5x + 2y = 11$

$-5(x + 6y = 19)$

$5x + 2y = 11$

$-5x - 30y = -95$

$-28y = -84$

$y = 3$

$5x + 6 = 11$

$x = 1$

$(1, 3)$

$5a + 3b = -15$

$(a + .5b = -3)(-5)$

$5a + 3b = -15$

$-5a - 2.5b = 15$

$.5b = 0$

$b = 0$

$5a = -15$

$a = -3$

$(-3, 0)$

Example

The accounting department of a company bought 4 staples and 5 boxes of paper for $51.50. The planning department bought 10 staplers and one box of paper for $65.50. What is the cost of one stapler and one box of paper?

$4x + 5y = 51.50$

$(10x + 1y = 65.50)\cdot -5$

$4x + 5y = 51.50$

$-50x - 5y = -327.50$

$-46x = -276$

$x = 6$ stapler

$24 + 5y = 51.50$

$y = 5.50$ paper

- Recall systems can have special situations such as No Solutions and Infinite Solutions!

No Sol. $2 = 3$ $5 = 9$

Inf. Sol. $5 = 5$ $100 = 100$

Examples

Solve the following systems.

$y = 6x - 7$

$24x - 4y = 28$

$24x - 4(6x - 7) = 28$

$24x - 24x + 28 = 28$

$28 = 28$

Inf. Sol

$y = 5x - 7$

$20x - 4y = 9$

$20x - 4(5x - 7) = 9$

$20x - 20x + 28 = 9$

$28 = 9$

No Sol

6.5 Applying Systems of Linear Equations

1. **MONEY** Veronica has been saving dimes and quarters. She has 94 coins in all, and the total value is $19.30. How many dimes and how many quarters does she have?

$(d+q=94)\cdot -10$

$.10d+.25q=19.30$

$-.10d+-.10q=-9.40$

$.15q=9.9 \quad q=66$

$d+66=94$

$d=28$

dimes 28

quarters 66

2. **CHEMISTRY** How many liters of 15% acid and 33% acid should be mixed to make 40 liters of 21% acid solution?

Concentration of Solution	Amount of Solution (L)	Amount of Acid
15%	x	$.15x$
33%	y	$.33y$
21%	40	8.4

$26\frac{2}{3}$

$13\frac{1}{3}$

$.21\times 40$

$.15x+.33y=8.4$

$(x+y=40)\times -.15$

$.15x+.33y=8.4$

$-.15x-.15y=-6$

$.18y=2.4$

$y=13.\overline{3}$

3. **BUILDINGS** The Sears Tower in Chicago is the tallest building in North America. The total height of the tower t and the antenna that stands on top of it a is 1729 feet. The difference in heights between the building and the antenna is 279 feet. How tall is the Sears Tower?

$t+a=1729$

$t-a=279$

$2t=2008$

$t=1004$

$1004+a=1729$

$a=725$

4. PRODUCE Roger and Trevor went shopping for produce on the same day. They each bought some apples and some potatoes. The amount they bought and the total price they paid are listed in the table below.

	Apples (lb)	Potatoes (lb)	Total Cost ($)
Roger	8	7	18.85
Trevor	2	10	12.88

What was the price of apples and potatoes per pound?

$8a+7p=18.85$

$x-4$ $(2a+10p=12.88)$

$8a+7p=18.85$

$-8a-40p=-51.52$

$-33p=-32.67$

$p=.99$

$8a+7(.99)=18.85$

$a=1.49$

Day 2 – Matrices

- Matrix – 4th way to solve systems

rows → columns ↓

$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ 2 rows x 2 columns

$\begin{bmatrix} A & D & G & J \\ B & E & H & K \\ C & F & I & L \end{bmatrix}$ 3 x 4

- Augmented Matrix –

Linear System:

$x - 2y = 7$

$-3x + 5y = -4$

Augmented Matrix:

$\left[\begin{array}{cc|c} 1 & -2 & 7 \\ -3 & 5 & -4 \end{array}\right]$

- 2nd matrix
- Edit

Example

Write the augmented matrix and solve using technology.

$\begin{cases} 2x + 3y = -11 \\ -8x - 5y = 9 \end{cases}$

$\left[\begin{array}{cc|c} 2 & 3 & -11 \\ -8 & -5 & 9 \end{array}\right]$

$\begin{bmatrix} 2 & 3 \\ -8 & -5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -11 \\ 9 \end{bmatrix}$

$(2, -5)$

Examples

Write an augmented matrix for each system and then solve the system.

$\begin{cases} -3x + y = -3 \\ 4x + 2y = 14 \end{cases}$

$\begin{bmatrix} -3 & 1 \\ 4 & 2 \end{bmatrix}^{-1} \begin{bmatrix} -3 \\ 14 \end{bmatrix}$

$(2, 3)$

$\begin{cases} 2x + 6y = -8 \\ x - 3y = 8 \end{cases}$

$\begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}^{-1} \begin{bmatrix} -8 \\ 8 \end{bmatrix}$

$(2, -2)$

$\begin{cases} 3x + 2y = 10 \\ 6x + 4y = 20 \end{cases}$

$\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ 20 \end{bmatrix}$

No Sol.

$\begin{cases} -\frac{1}{2}x + 4y = 6 \\ 3x - 5y = 10 \end{cases}$

$\begin{bmatrix} -\frac{1}{2} & 4 \\ 3 & -5 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ 10 \end{bmatrix}$

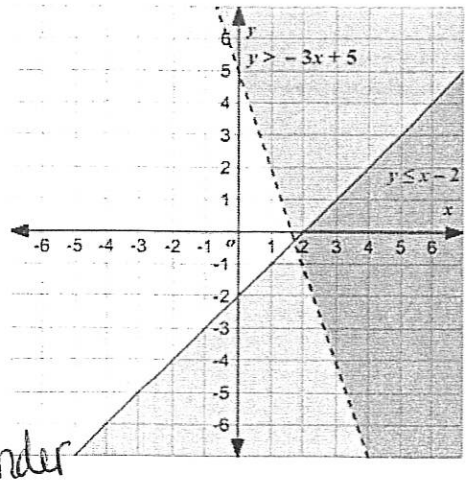
$(7.36, 2.42)$

6.6 Systems of Inequalities

- System of Inequalities

< > - - - -

≤ ≥ ——



X = # ↕

y = # ↔

y < under

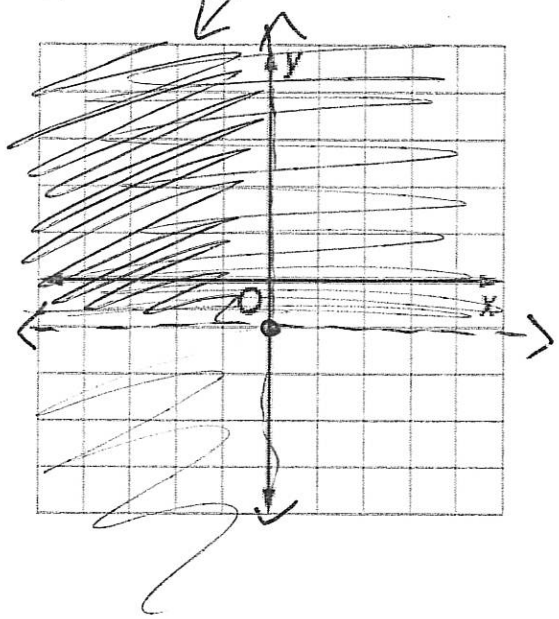
y > above

Examples

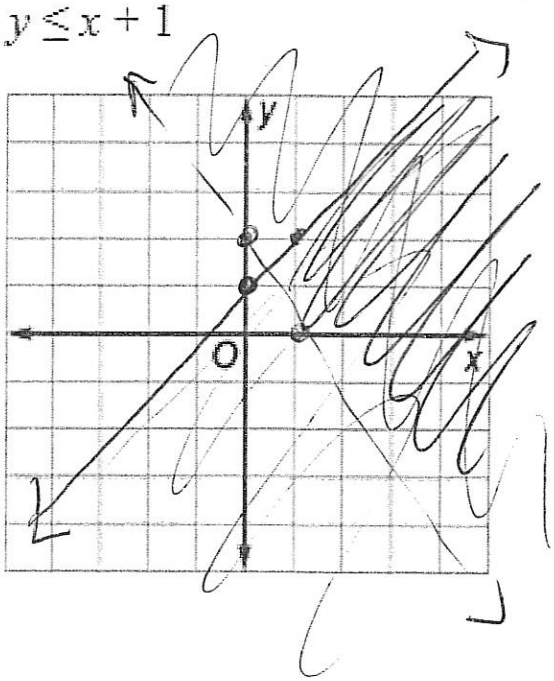
Solve each system by graphing.

1. $y > -1$
   $x < 0$

solutions



2. $y > -2x + 2$
   $y \leq x + 1$

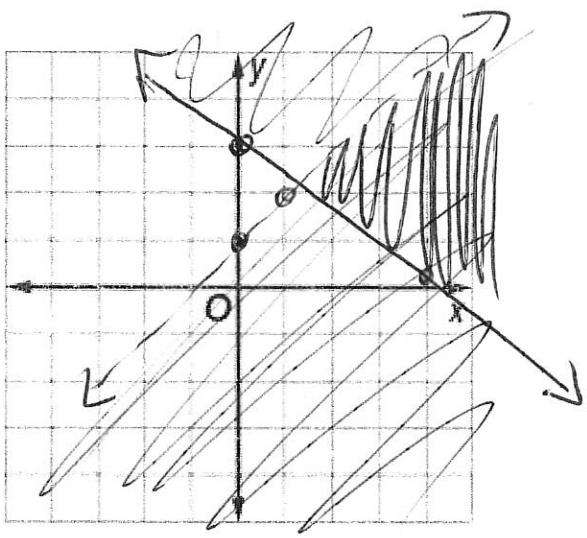


$4y \geq 12 - 3x$

$y \geq 3 - \frac{3}{4}x$

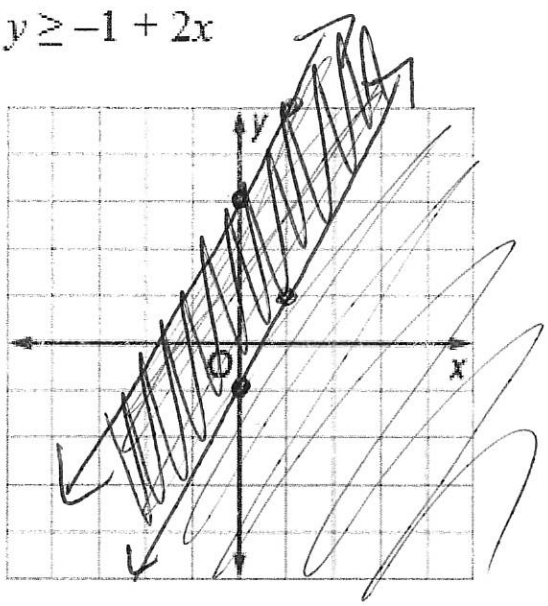
3. $y < x + 1$

$3x + 4y \geq 12$



$y \leq 2x + 3$

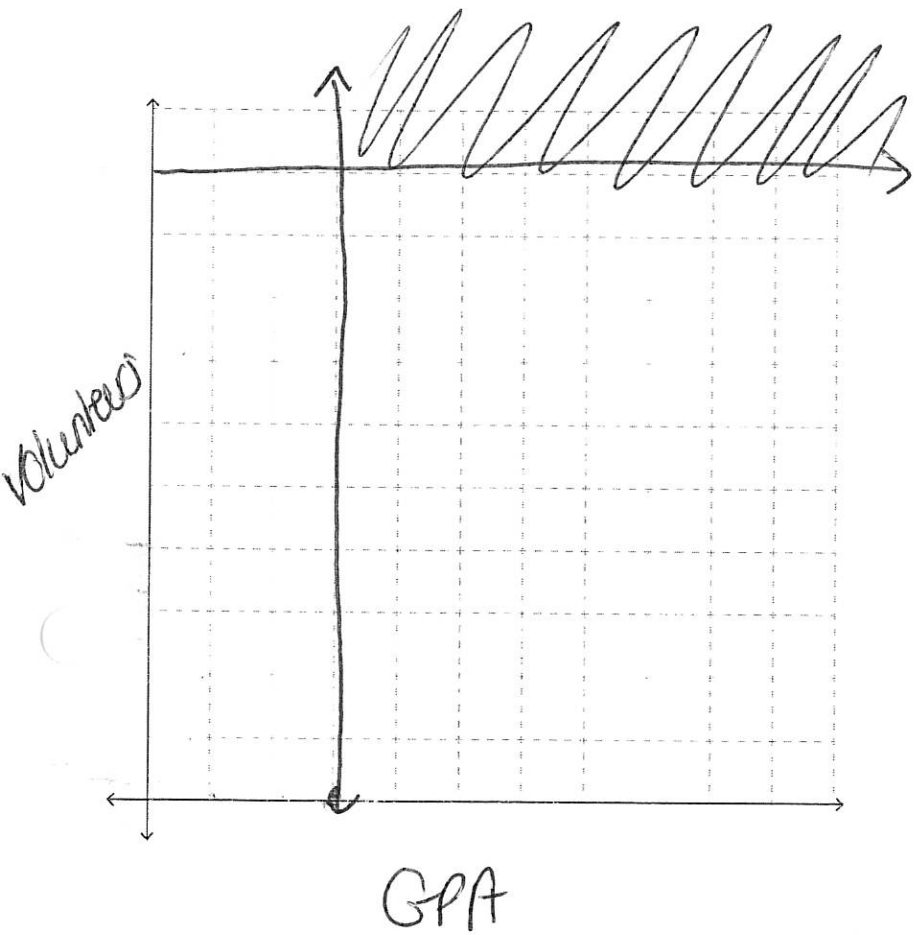
$y \geq -1 + 2x$



Example

A college service organization requires that its members maintain at least a 3.0 grade point average and volunteer at least 10 hours a week.

a) Define the variables and write a system of inequalities to represent this situation. Then graph the system.



x = GPA

$x \geq 3.0$

y = volunteer hrs

$y \geq 10$

b) Name one possible solution.

(3.2, 12)
GPA hrs