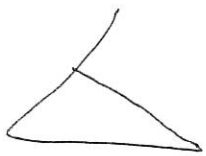
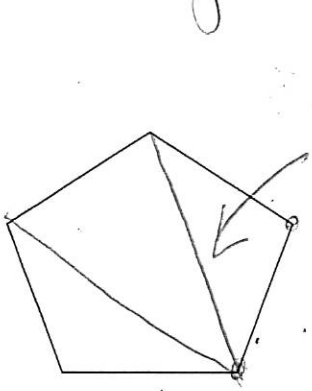
2nd

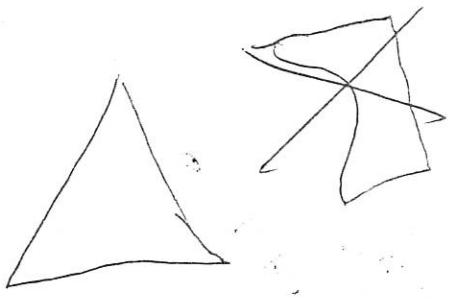
6.1

~~8.1~~ Angles of Polygons



- **Polygon** – closed figure with at least 3 sides



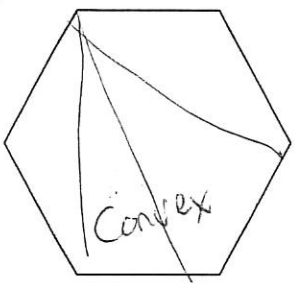


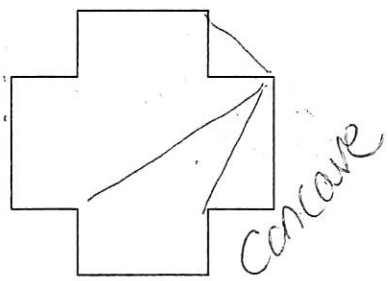
- **Diagonal** – segment that connects any two nonconsecutive vertices

Two types:

Convex – **has no diagonal with points outsides the polygon**

Concave – **has at least one diagonal with points outside the polygon**

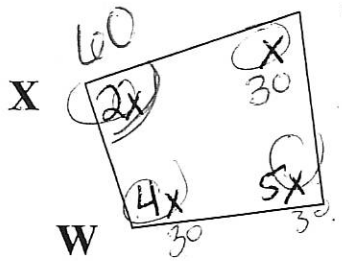




Explore

- **Interior Angle Sum Theorem** – **The sum of the measures of the angles of an n-gon is** $180(n-2)$

Example – Find the m ∠X is the quadrilateral XYZW.



$180(4-2) =$

$180 \cdot 2 = 360$

$x + 2x + 4x + 5x = 360$

$12x = 360$

$x = 30$

1 interior measures 108° Find # of sides

$180(n-2)=108n$

$180n-360=108n$ $-360=-72n$

$-180n \quad -180n$ $n=5$

Example

The measure of an interior angle of a regular polygon is 135. find the number of sides in the polygon.

$180(n-2)=135n$

$180n-360=135n$

$-180n \quad -180n$

$-360=-45n$

$\frac{-360}{-45}=\frac{-45n}{-45}$

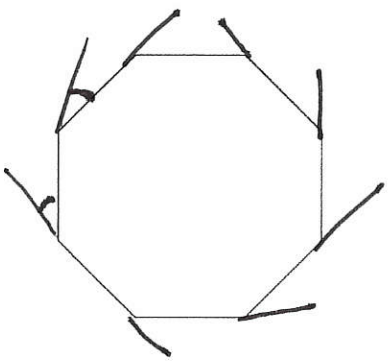
$n=8$

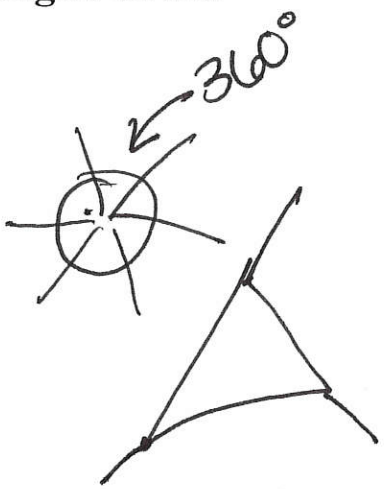
- **Exterior Angle-Sum Theorem – If a polygon is convex, the sum of the measures of the exterior angles of the polygon is** 360°

Ex Reg hexagon find each int. angle.

$180(n-2)$

$720/6$





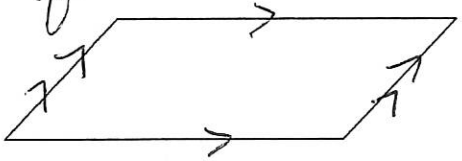
What is the measure of an exterior angle for a regular octagon?

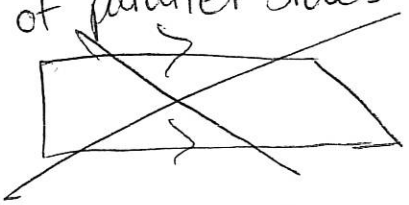
$360/8=45°$

6.2

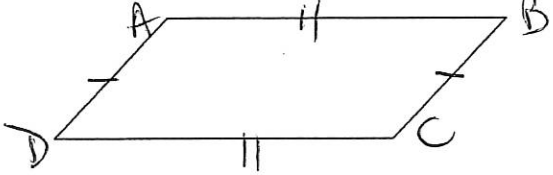
~~8.2~~ Parallelograms

Parallelogram – quadrilateral w/ 2 sets of parallel sides





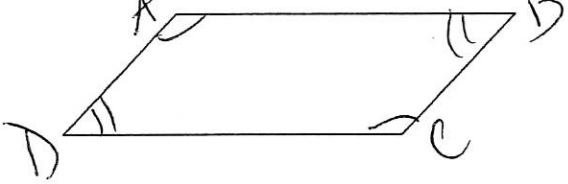
Theorem – Opposites sides of a parallelogram are congruent



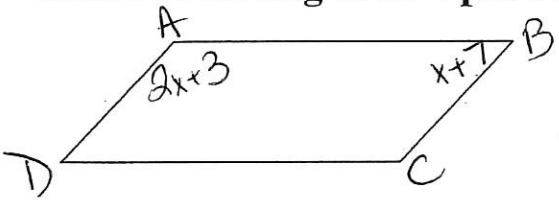
$AD = 2x + 1$

$BC = 3x - 4$

Theorem – Opposite angles in a parallelogram are congruent

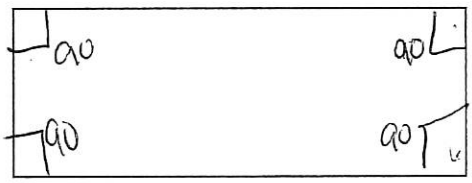


Theorem – Consecutive angles in a parallelogram are Supplementary

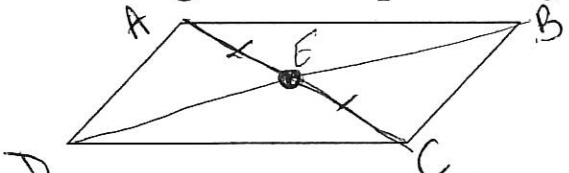


$m\angle A + m\angle B = 180$ (180)

Theorem – If a parallelogram has one right angle, then 4 rt. angles



Theorem – The diagonals of a parallelogram bisect each other



$\overline{AE} \cong \overline{EC}$

$\overline{BE} \cong \overline{DE}$

$A(2,5)$ $C(-2,-3)$

$B(3,3)$ $D(-3,-1)$

$\frac{x+x}{2}, \frac{y+y}{2}$

$(0,1)$

6.3

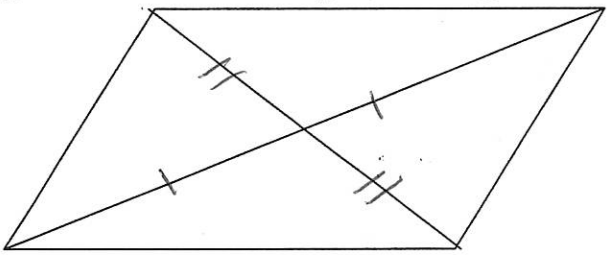
~~8.3~~ Tests for Parallelograms

Measure dist. between

Theorem – If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram

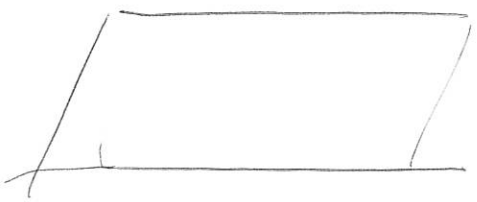
Theorem - If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram

Theorem - If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.



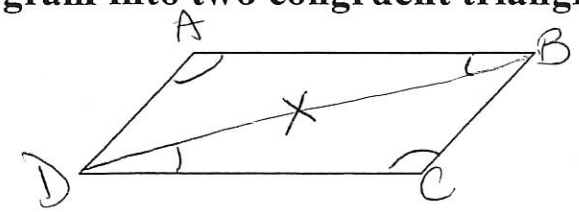
Theorem - If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.

Test w/ Straws



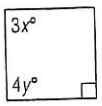
<u>Theorem</u> – Each diagonal of a parallelogram separates the parallelogram into two congruent triangles

$\triangle ABD \cong \triangle CDB$

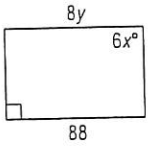


Examples

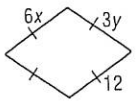
Find x and y in each parallelogram.

1. 

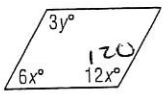
$x = 30$
$y = 22.5$

2. 

$x = 15$
$y = 11$

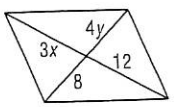
3. 

$x = 2$
$y = 4$

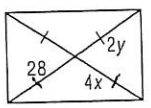
4. 

$6x + 12x = 180$
$x = 10$
$y = 40$

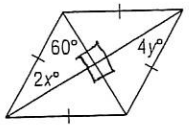
Find x and y in each parallelogram. $2y = 28$

1. 

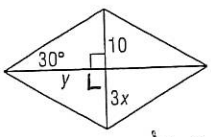
$x = 4$
$y = 2$

2. 

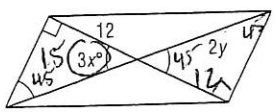
$x = 7$
$y = 14$

3. 

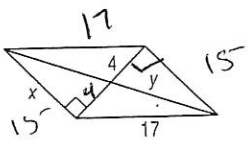
$x = 15$
$y = 7.5$

4. 

$x = 3.3$
$y = 10\sqrt{3}$

5. 

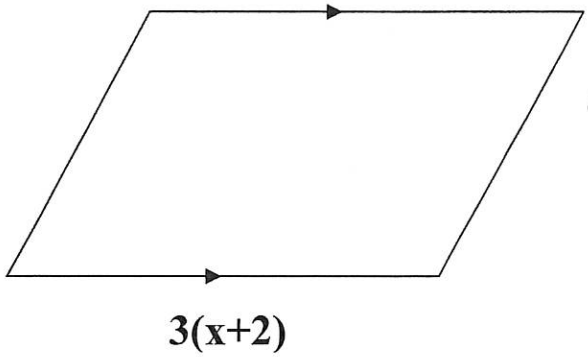
$x = 15$
$y = 6\sqrt{2}$

6. 

$x = 15$
$y = 15.5$

Example

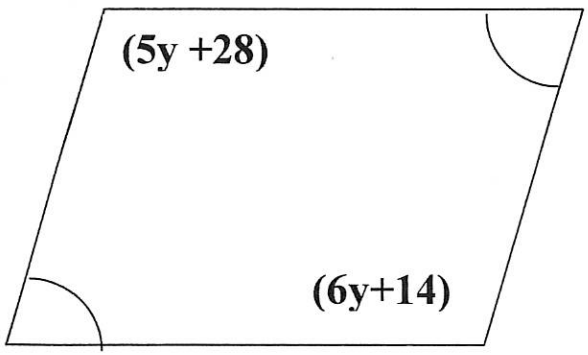
Find x and y so that each quadrilateral is a parallelogram.



$4x-1=3x+6$

$x-1=6$

$x=7$



$5y+28=6y+14$

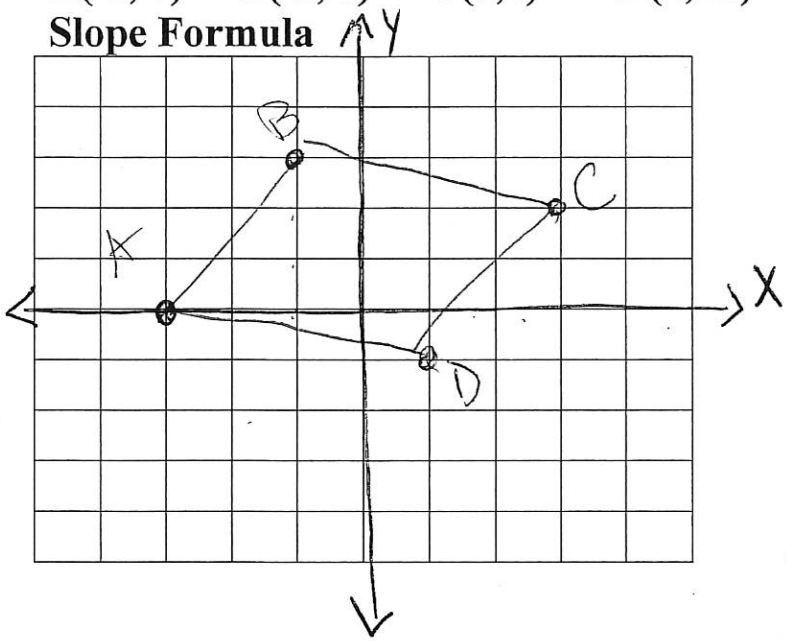
$5y+14=6y$

$14=y$

- Slope - verifies parallel
- Dist - verifies cong
- S+D same side

Determine whether the figure with the given vertices is a parallelogram. Use the method indicated.

A(-3, 0) B(-1, 3) C(3,2) D(1, -1)
Slope Formula



BC $\frac{3-2}{-1-3} = \frac{1}{-4}$

AD $\frac{-1}{4}$

CD $\frac{2+1}{3-1} = \frac{3}{2}$

AB $\frac{3}{-1+3} = \frac{3}{2}$

Yes

6.4

~~8.4~~ Rectangles

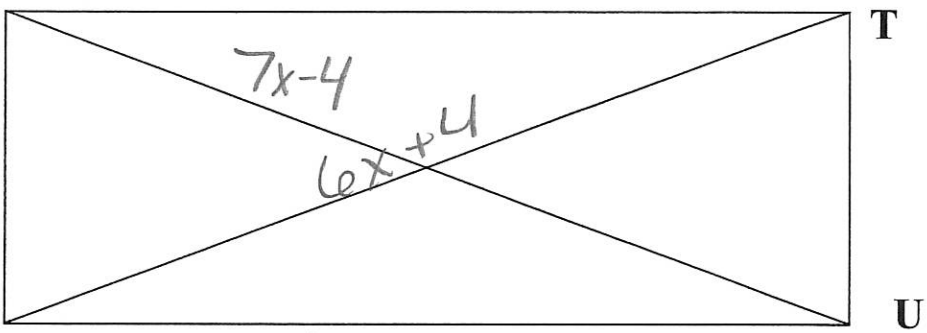
➢ **Rectangle –** quad w/ 4 right angles

Theorem – If a parallelogram is a rectangle, then the diagonals are congruent

$\overline{AC} \cong \overline{BD}$

Example

Quadrilateral RSTU is a rectangle. If RT = 6x+4 and SU = 7x-4, find x.



$6x+4=7x-4$

$4=x-4$

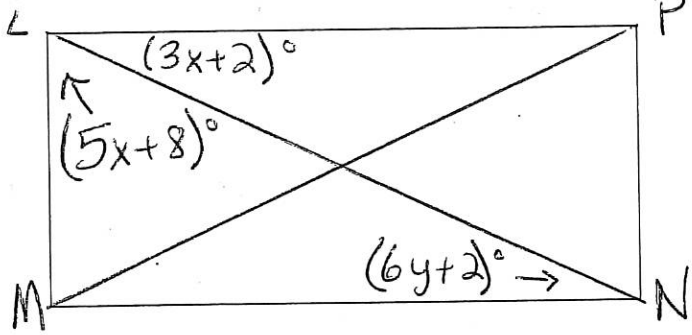
$x=8$

Theorem – If the diagonals of a parallelogram are congruent then the parallelogram is a rectangle.

Example

Quadrilateral LMNP is a rectangle.

Find x and find y.



$5x+8+3x+2=90$

$8x+10=90$

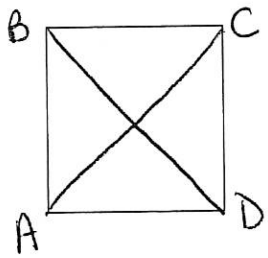
$8x=80$

$x=10$

$3x+2=6y+2$

Example

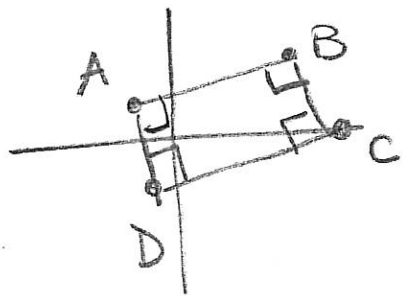
Kyle is building a barn for his horse. He measures the diagonals of the door opening to make sure they bisect each other and they are congruent. How does he know that the measure of each corner is 90?



b/c $\overline{AC} \cong \overline{BD}$ must be rectangle, so all angles $= 90°$

Example

Quadrilateral ABCD has vertices A (-2, 1) B (4, 3), C (5, 0) and D(-1, -2). Determine whether ABCD is a rectangle using the Slope formula.



show perp $\times -1$

$AB = \frac{1}{3}$] $\frac{1}{3} \cdot -3 = -1$

$BC = -3$

$CD = \frac{1}{3}$] $\frac{1}{3} \cdot -3 = -1$

$AD = -3$

Yes

6.5

~~8.5~~ Rhombi and Squares

- **Rhombus** – quad w/ 4 congruent sides
- **Square** – quad that is rhombus & rect.

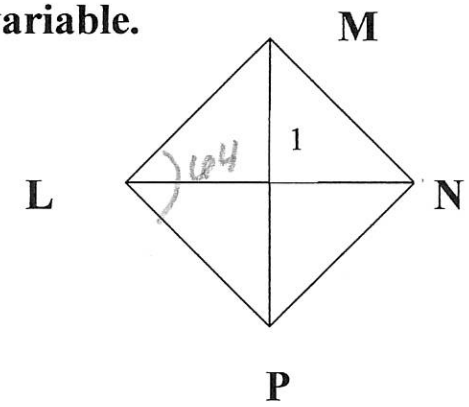
Theorem – diagonals of a rhombus are perpendicular

Theorem – If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Theorem – each diagonal of a rhombus bisects a pair of opposite angles

Example

Use rhombus LMNP and the given information to find the value of each variable.



Find y if m $\angle 1 = y^2 - 54$

$y^2 - 54 = 90$

$y^2 = 144$

$y = 12$

Find m $\angle$ PNL if m $\angle$ MLP = 64. 32

Example

Determine whether parallelogram ABCD is a rhombus, a rectangle, or a square for
A(-2, -1), B (-1, 3), C(3,2) and D(2, -2).

- Perp Diag only Rhombus or
- Cong Diag only Rect.

Square
Both perp & cong.

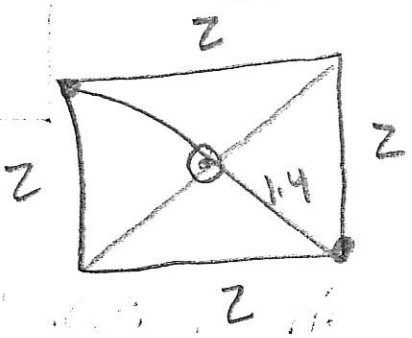
Square

$AC = \sqrt{34}$
$BD = \sqrt{34}$

Slope $AC \frac{2+1}{3-2} = \frac{3}{5}$
Slope $BD \frac{-2-3}{2+1} = -\frac{5}{3}$
Perp.

Example

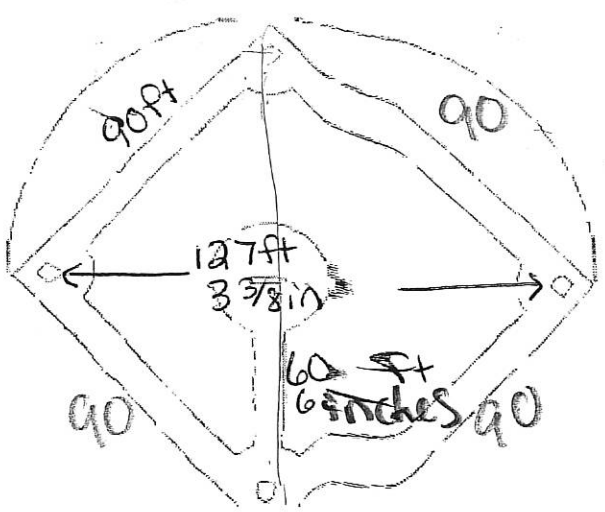
A square table has four legs that are 2 feet apart. The table is placed over an umbrella stand so that the hole in the center of the table lines up with the hole in the stand. How far away from a leg is the center of the hole?



2^2+2^2
$8 = x^2$
$x = 2.8$
1.4ft

Example

The infield of a baseball diamond is a square as shown at the right. Is the pitcher's mound located in the center of the infield? Explain.



$90^2+90^2 = 127.3$

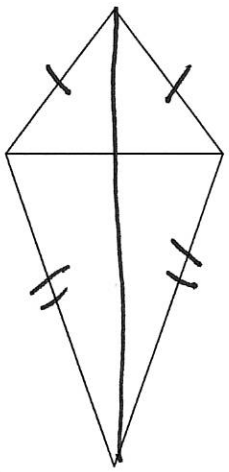
63.6

no 3 ft closer to home.

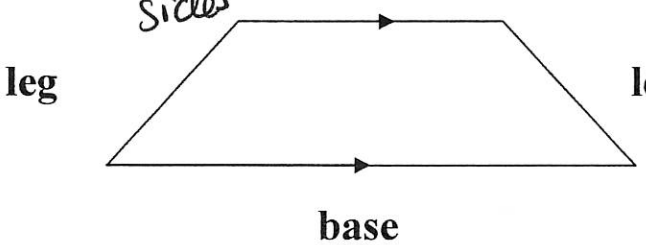
6.6

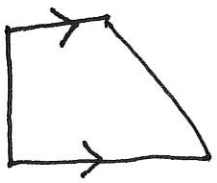
~~8.6~~ Kites and Trapezoids

- **Kite** – quad w/ 2 pairs of adjacent congruent sides.



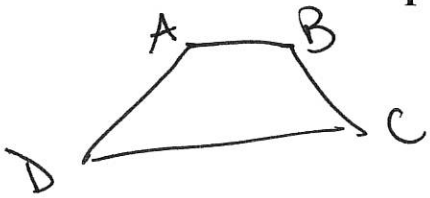
- **Trapezoids** – quad w/ 1 pair of parallel sides





- **Theorem** – both pairs of base angles in an isos. trap are congruent

- **Theorem** – The diagonals of an isosceles trapezoid are ≅ congruent



$\overline{AC} \cong \overline{BD}$