

7.1 Multiplication Properties of Exponents

- Monomial - #, variable, or product of #s & variables

Ex 5 x x^2y $6x^2y$ Not
7+y

- Constant - monomial that is a real #

5 $5x^2$ $3x^2y$
↑ ↑

Examples

Determine whether each expression is a monomial. Write yes or no. Explain your reasoning.

a) $17 - c$ No, subtraction

b) $8f^2g$ Yes, product

c) $\frac{3}{4}$ Yes .75

d) $\frac{5}{t}$ No, dividing # & variable

Explore - Write each of the following in expanded form and see what you notice.

Problem	1 st Monomial Expanded	2 nd Monomial Expanded	Simplified
$x^2 \cdot x^5$	xx	xxxxxx	x^7
$y^4 \cdot y^3$	yyyy	yyyy	y^7
$x^4 \cdot x^6$	xxxx	xxxxxxxx	x^{10}
$y \cdot y^7$	y	yyyyyyyy	y^8

- Product of Powers Property

$$x^a \cdot x^b = x^{a+b}$$



only works with same base

$$x^3 \cdot x^8 = x^{3+8} = x^{11}$$

Examples – No Calculator

Simplify each expression.

a) $m^3(m^2)$ m^5

b) $x^5 \cdot 6x^4$ $6x^9$ (5+4)

c) $(5x^3)(6x^2y)$ $30x^5y$

d) $3x^4y(4x^6y^3)$ $12x^{10}y^4$

Explore – Write each of the following in expanded form and see what you notice.

Problem	Monomial Expanded	Power Expanded	Simplified
$(x^2)^5$	$x \cdot x$	$\overset{1}{x} \overset{2}{x} \overset{3}{x} \overset{4}{x} \overset{5}{x}$	x^{10}
$(y^3)^4$	$y \cdot y \cdot y$	$y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$	y^{12}
$(2x^3)^3$	$2x \cdot x \cdot x$	$2x \cdot x \cdot x \cdot 2x \cdot x \cdot x \cdot 2x \cdot x \cdot x$	$8x^9$
$(3x^2)^4$	$3x \cdot x$	$3x \cdot x \cdot 3x \cdot x \cdot 3x \cdot x \cdot 3x \cdot x$	$81x^8$

- Power of a Power Property

$$(x^a)^b = x^{a \cdot b}$$

Ex $(x^3)^4 = x^{12}$

$$(5x^2)^3 = 125x^6$$

Examples – No Calculator

Simplify each of the following.

a) $(4x^3)^2$ $16x^6$

b) $(-2x^2y^3h^2)^3$ $-8x^6y^9h^6$

c) $(3x^3y^6)^4$ $81x^{12}y^{24}$

Examples

Simplify the following.

a) $[(8g^3h^4)^2]^2(2gh^5)^4$

$(64g^6h^8)^2$

$4096g^{12}h^{16} \cdot 16g^4h^{20}$

$65536g^{16}h^{36}$

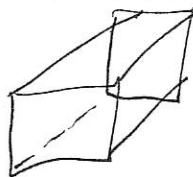
b) $(-2x^3y)(-3xy^4)^2(-xyz)^2$

$-2x^3y \cdot 9x^2y^8 \cdot 1x^2y^2z^2$

$-18x^5y^9 \cdot 1x^2y^2z^2$

$-18x^7y^{11}z^2$

c) Express the volume of a cube with side length $5xyz$ as a monomial.



$5xyz$

$(5xyz)^3$

$125x^3y^3z^3$

7.2 Division Properties of Exponents

Explore

Quotients				
Expression	Expression written as repeated multiplication	Simplified expression	Number of factors	Quotient as a power
$\frac{2^8}{2^3}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2}$	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	5	2^5
$\frac{3^5}{3^3}$	$\frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3}$	$3 \cdot 3$	2	3^2
$\frac{5^7}{5^6}$				5^1

- Quotient of Powers

$$\frac{x^a}{x^b} = x^{a-b}$$

$$\frac{7^8}{7^2} = 7^{8-2} = 7^6$$

Example – No Calculator

Simplify $\frac{x^7 y^{12}}{x^6 y^3}$ assuming that the denominator does not equal zero.

$$x^1 y^9$$

Explore

Problem	Expanded	Written with exponents	Simplified
$\left(\frac{3}{4}\right)^5$	$\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4}$	$\frac{3^5}{4^5}$	$\frac{243}{1024}$
$\left(\frac{x^2}{3}\right)^4$		$\frac{x^8}{3^4}$	$\frac{x^8}{81}$
$\left(\frac{x^2}{5}\right)^3$		$\frac{x^6}{5^3}$	$\frac{x^6}{125}$
$\left(\frac{2x^3}{4}\right)^4$		$\frac{16x^{12}}{4^4} = \frac{16x^{12}}{256}$	$\frac{x^{12}}{16}$

- Power of a Quotient

$$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$\text{Ex } \left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$$

$$\left(\frac{2x}{y}\right)^3 = \frac{8x^3}{y^3}$$

Example

Simplify the expression $\left(\frac{4c^3d^2}{5}\right)^3$

$$\frac{64c^9d^6}{5^3}$$

$$\frac{64c^9d^6}{125}$$

- Zero Exponent Property

$$\frac{x^5}{x^5} = x^0 = 1$$

(anything to zero power equals 1)

Examples – No Calculator

Simplify each expression.

a) $\left(\frac{12m^8n^7}{8m^5n^{10}}\right)^0$ |

b) $\frac{m^0n^3}{n^2} = \frac{1 \cdot n^3}{n^2} = \frac{n^3}{n^2} = n$

Explore Negative Exponents

Table 1	
Power	Value
2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{4}$
2^{-3}	$\frac{1}{8}$

Table 2	
Power	Value
3^4	81
3^3	27
3^2	9
3^1	3
3^0	1
3^{-1}	$\frac{1}{3}$
3^{-2}	$\frac{1}{9}$
3^{-3}	$\frac{1}{27}$

- Negative Exponent Property

$$x^{-n} = \frac{1}{x^n}$$

Ex $x^{-3} = \frac{1}{x^3}$

Ex $x^2 y^{-4} = \frac{x^2}{y^4}$

Examples

Simplify each expression

a) $\frac{x^{-4}y^9}{z^{-6}}$ $\frac{y^9 z^6}{x^4}$

b) $\frac{75p^3m^{-5}}{15p^5m^{-4}r^{-8}}$ $\frac{3m^4r^8}{p^2m^5} = \frac{3r^8}{p^2m}$

c) $\frac{n^{-5}p^4}{r^{-2}}$ $\frac{p^4 r^2}{n^5}$

7.3 Rational Exponents

- Rational Exponents

$5^{1/2}$ ← rational #
 $x^{1/3}$ ←

$x^{1/2}$	$x^{1/3}$	$x^{1/4}$	$x^{1/5}$	$x^{1/6}$	$x^{2/3}$	$x^{3/4}$	$x^{2/5}$
\sqrt{x}	$\sqrt[3]{x}$	$\sqrt[4]{x}$	$\sqrt[5]{x}$	$\sqrt[6]{x}$	$\sqrt[3]{x^2}$	$\sqrt[4]{x^3}$	$\sqrt[5]{x^2}$

Examples – No Calculator

Write each expression in radical form, or write each radical in exponential form.

a) $81^{1/2}$ $\sqrt{81}$

b) $\sqrt{38}$ $38^{1/2}$

c) $12m^{1/2}$ $12\sqrt{m}$

d) $\sqrt{32w}$ $(32w)^{1/2}$

- Nth Root –

$a^n = b$ a is a n^{th} root of b
 $3^5 = 243$, 3 is a 5^{th} root of 243

b) $\sqrt{38}$

Examples

Simplify the following.

a) $\sqrt[3]{64}$

$64^{1/3}$

(4)

b) $\sqrt[3]{27}$

3

c) $\sqrt[4]{256}$

4

d) $\sqrt[6]{15,625}$

5

e) $1331^{1/3}$

11

f) $2401^{1/4}$

7

g) $32^{2/5}$

4

h) $81^{5/2}$

59049

Examples

Solve each equation.

a) $9^x = 729$

(3)

Graph or
rewrite

2nd-Calc
Intersect

b) $16^{2x-1} = 8$

.875

c) $12^{2x+3} = 144$

$$2x+3 = 12$$

$$2x = 9$$

$$x = 9/2$$

- d) The population p of a culture that begins with 40 bacteria and doubles every 8 hours can be modeled by $p = 40(2)^{\frac{t}{8}}$, where t is time in hours. Find t if $p = 20,480$.

$$20480 = 40 \cdot 2^{t/8}$$

$$512 = 2^{t/8} \quad \text{Graph or}$$

$$2^9 = 2^{t/8}$$

$$9 = t/8$$

$$t = 72$$

7.4 Scientific Notation

You can use our rules of exponents on numbers in scientific notation!

- Scientific Notation - # in form

$a \times 10^n$ where $1 \leq a < 10$ & n is integer

$$2.13 \times 10^4$$

$$\begin{array}{r} 21300 \\ \hline 21300 \end{array}$$

Examples

Transfer between standard and scientific notation.

Standard Form	Scientific Notation
32,000	3.2×10^4
.0005	5×10^{-4}
4,500	4.5×10^3
.00324	3.24×10^{-3}
6450	6.45×10^3
.0018	1.8×10^{-3}
320,000	3.201×10^6 /

Examples – No Calculator

Evaluate the following. Express the result in both scientific notation and in standard form.

a) $(5 \times 10^{-6})(2.1 \times 10^{12})$ 10.2×10^6 $10,200,000$

b) $(3 \times 10^4)(6.5 \times 10^3)$ 19.5×10^7 $195,000,000$

c) $\frac{4.5 \times 10^8}{2 \times 10^3}$

2.25×10^5 225,000

d) $\frac{12 \times 10^5}{3 \times 10^2}$

4×10^3 4,000

Example

Last year Ally's state registered over 400 thousand watercraft. Boat sales in her state generated more than \$15.4 million in state sales taxes that same year.

- a) Express the number of watercraft registered and the state sales tax generated from boat sales last year in Ally's state in standard notation.

Watercraft 400,000
State tax 15,400,000

- b) Write each number in scientific notation.

4×10^5
 1.54×10^7

- c) How many watercraft have been registered in Ally's state if 12 times the number registered last year have been registered in all? Write your answer in scientific notation and standard notation.

4800,000
 4.8×10^6

7.5 Exponential Functions

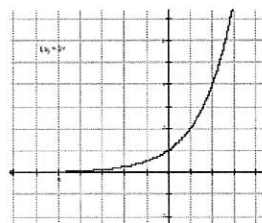
- Exponential Function

$$y = 3 \cdot 2^x$$

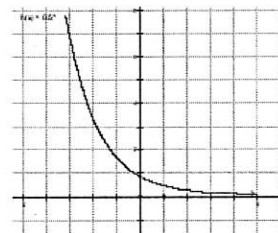
x	y
0	3
1	6
2	12
3	24

repeated mult

march madness, paper folding math



Exponential Growth
 $y = a \cdot b^x$
 $b > 1$

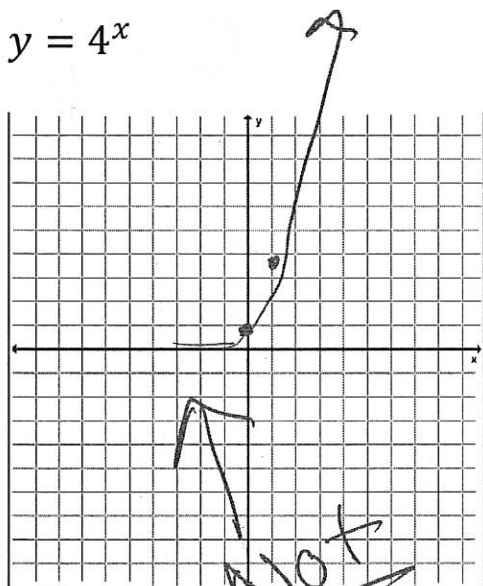


Exponential Decay
 $y = a \cdot b^x$
 $b < 1$

Examples

Graph the following functions and state the key items.

$$y = 4^x$$



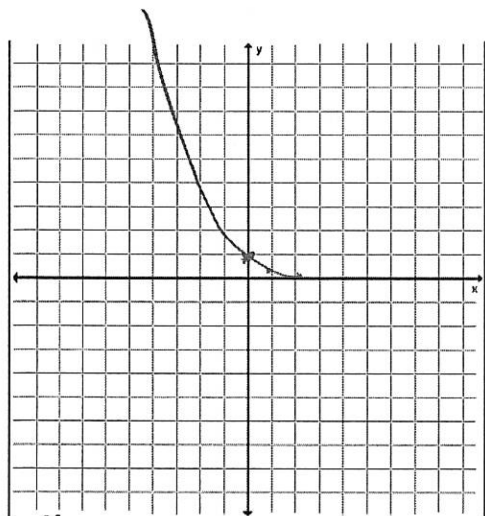
Y-Intercept 1

Domain \mathbb{R}

Range $(0, \infty)$

x	y
0	1
1	4
2	16

$$y = \frac{1}{4}^x$$



Y-Intercept

Domain

Range

1

\mathbb{R}

$(0, \infty)$

X	Y
0	1
1	$\frac{1}{4}$
2	$\frac{1}{16}$

Examples

Some people say that the value of a new car decreases as soon as it is driven off the lot. The function $y = 25,000 \cdot 0.82^t$ models the depreciation in the value of a new car that originally cost \$25,000. V represents the value of the car and t represents the time in years from the time of purchase.

- a) Graph the function. What value of V and t are meaningful in the context of the problem?

> 0 \uparrow time own the car

- b) What is the car's value after 5 years?

\$19268.50

Examples

Example

Determine whether the set of data shown below displays exponential, linear, or neither behavior. Explain your reasoning.

- a)

Linear

X	1	2	3	4
Y	5	7	9	11

+2

b)

Exponential

X	1	2	3	4
Y	4	12	36	108

$\times 3$

c)

Neither

X	1	2	3	4
Y	4	7	9	10

$+3$

$+2$

b)

d)

Exponential

X	1	2	3	4
Y	20	10	5	2.5

$\times 1/2$

c)

7.6 Growth and Decay

mike vs Ike

Exponential Growth Formula	Exponential Decay Formula
$y = a \cdot b^x$ \uparrow greater than 1	$y = a \cdot b^x$ \uparrow $0 < b < 1$

Examples

In 2008 the town of Flat Creek had a population of about 280,000 and a growth rate of 0.85% per year.

$$280,000(1 + 0.0085)^x$$

- Write an equation to represent the population of Flat Creek since 2008.
- According to the equation, what will be the population of Flat Creek in the year 2018?

$$\approx 304,731$$

A college's tuition has risen 5% each year since 2000. If the tuition in 2000 was \$10,850.

$$10,850(1 + 0.05)^x$$

- Write an equation for the amount of the tuition t years after 2000.
- Predict the cost of tuition for this college in 2015.

$$\$ 22,556.37$$

During an economic recession, a charitable organization found that its donations dropped by 1.1% per year. Before the recession, its donations were \$390,000.

- Write an equation to represent the charity's donations since the beginning of the recession.
- Estimate the amount of the donations 5 years after the start of the recession.

$$390,000(1 - 0.011)^x$$

$$369,317$$

- Compound Interest

Amount	rate of interest
$A = P(1 + \frac{r}{n})^{nt}$	time in years
Principal	number of times per year, interest is compounded

Examples

When Jing May was born her grandparents invested \$1000 in a fixed rate savings account at a rate of 7% compounded annually. Jing May will receive the money when she turns 18 to help with her college expenses. What amount of money will Jing May receive from the investment?

$$1000(1 + .07)^{18}$$

3380

Maria's parents invested \$14,000 at 6% per year compounded monthly. How much money will there be in the account after 10 years?

$$14,000\left(1 + \frac{.06}{12}\right)^{12 \cdot 10}$$

25471.55

Find the amount of an investment of \$300 which earns 3.5% compounded quarterly for 5 years.

$$300\left(1 + \frac{.035}{4}\right)^{4 \cdot 5}$$

357.10

7.7 Geometric Sequences as Exponential Functions

Recall: Arithmetic Sequences

4, 7, 10, 13, 16

$$\begin{array}{ccccccc} & \frown & & \frown & & & \\ 4 & & 7 & & 10 & & 13 & & 16 \\ & + & 3 & & + & 3 & & & \end{array}$$

$$3x + 1$$

- Geometric Sequences — repeated mult.

$$\begin{array}{ccccccc} & \nwarrow & & \nwarrow & & \nwarrow & \\ 5 & & 15 & & 45 & & 135 \\ & \div 3 & & \times 3 & & \times 3 & \end{array}$$

$$\frac{5}{3} \cdot 3^x$$

Examples

Determine whether each sequence is arithmetic, geometric, or neither. Explain.

a) 0, 8, 16, 24, 32, ... Arithmetic adding

$$\begin{array}{ccccccc} & \frown & & \frown & & & \\ 0 & & 8 & & 16 & & 24 & & 32 \\ & + & 8 & & + & 8 & & & \end{array}$$

b) 64, 48, 36, 27, ... Geo, mult

$$\begin{array}{ccccccc} & \frown & & \frown & & & \\ 64 & & 48 & & 36 & & 27 \\ & \div 1.3 & & \div 1.3 & & \div 1.3 & \end{array}$$

Examples

Find the next three terms in each geometric sequence.

a) 1, -8, 64, -512, ... 4096, -32768, 262144

b) 40, 20, 10, 5, ... $5/2$, $5/4$, $5/8$

c) 24, 36, 54, 81, ... 121.5, 182.25, 273.375

$$\times 1.5$$

Example

Write an equation for the nth term of the geometric sequence: 1, -2, 4, -8, ...

$$-\frac{1}{2} \cdot -2^x$$

$$\frac{1}{-2}$$
$$x-2$$

Write an equation for the nth term of the geometric sequence: 96, 48, 24, 12, ...

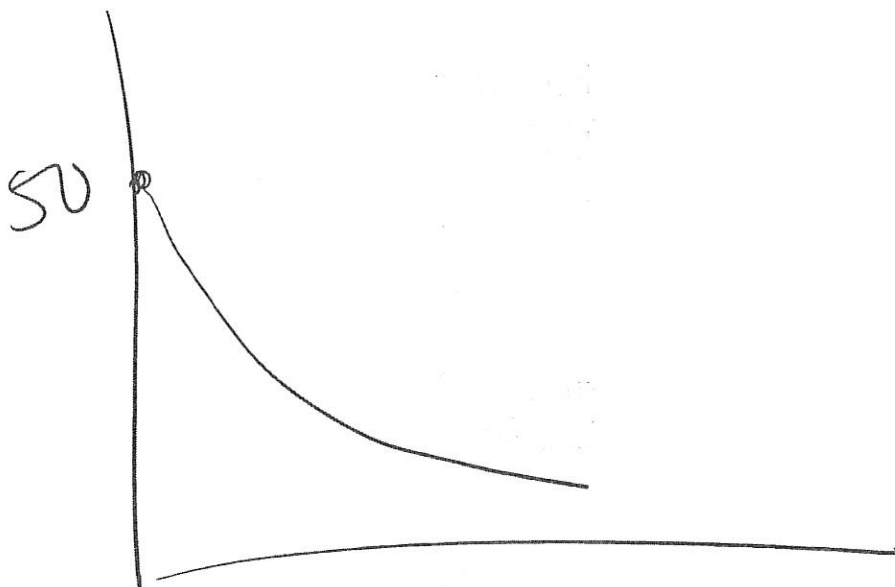
$$192 \cdot \frac{1}{2}^x$$

$$x^{1/2}$$

Example

A 50-pound ice sculpture is melting at a rate in which 80% of its weight remains each hour. Draw a graph to represent how many pounds of the sculpture is left each hour.

$$50 \cdot .80$$



7.8 Recursive Formulas

Recall: Arithmetic Sequences

4, 7, 10, 13, 16

+3

$$a_1 = 4$$

$$a_n = a_{n-1} + 4 \quad n \geq 2$$

current previous

- Recursive Formulas –

formula in terms
of previous terms

Example

Find the first five terms of the sequence in which

$$a_1 = -8 \text{ and } a_n = -2a_{n-1} + 5 \text{ if } n \geq 2$$

-8, 21, -37, 79, -153

Examples

Write a recursive formula for each sequence.

a) 23, 29, 35, 41, ... $a_1 = 23$ $a_n = a_{n-1} + 6 \quad n \geq 2$

b) 7, -21, 63, -189, ... $a_1 = 7$ $a_n = -3a_{n-1} \quad n \geq 2$

- c) The price of a car depreciates at the end of each year as shown in the chart.

Year	1	2	3	4
Price	12,000	7,200	4,320	2,592

$$a_1 = 12,000$$

$$a_n = 0.6a_{n-1}$$

- d) Ronald has a savings account that earns interest. He does not make an additional withdrawals or deposits. The accounts balance at the beginning of each year is \$10,000, \$10,300, \$10,609, \$10,927.27, and so on.

$$a_1 = 10,000$$

$$a_n = a_{n-1} \cdot 1.03 \quad n \geq 2$$

Examples

#1

- a) Write a recursive formula for $a_n = 2n - 4$.

$$a_1 = -2$$

$$a_n = a_{n-1} + 2 \quad n \geq 2$$

- b) Write an explicit formula for $a_1 = 84$, $a_n = 1.5a_{n-1}$, $n \geq 2$.

$$a_n = 84(1.5)^{n-1}$$

#2

- a) Write a recursive formula for $a_n = 4(3)^{n-1}$.

$$a_1 = 4$$

$$a_n = a_{n-1} \times 3 \quad n \geq 2$$

- b) Write an explicit formula for $a_1 = -16$, $a_n = a_{n-1} - 7$, $n \geq 2$.

$$\begin{array}{r} -16 \quad -23 \\ \quad \quad \quad \downarrow \\ \quad \quad \quad -7 \end{array}$$

$$a_n = -7n - 9$$