8.1 Adding and Subtracting Polynomials

- Monomial – #, variable or product of #s & variables

- Polynomial – Monomial or sum of monomials. Each item in a polynomial is called a term.

- Binomial – 2 monomials

$2x+3$

$5x+5+8x = 11x+5$

- Trinomial – 3 monomials

- Degree - is the greatest degree of any term in the polynomial.
  - $8x^7$ Degree 7
  - $9xy^2$ Degree 3
  - $7xy + 8x^2y - 3xyz^4$ (2, 3, 6) Degree 6

Examples

Determine whether each expression is a polynomial. If it is a polynomial, find the degree and determine whether it is a monomial, binomial, or trinomial.

a) $6x - 4$ Yes, binomial 1

b) $x^2 + 2xy - 7$ Yes, tri 2

c) $\frac{14d+19e^3}{5d^4}$ No division

d) $26b^2$ Yes mono 2

e) $5x + 4 - 3x$ Yes bi 1

f) $5^x + 3$ No exponential

- Standard form of a polynomial

places terms in decending order by degree

greatest degree

$4x^3 - 7x^2 + 9x + 1$

leading coefficient

Examples

Write each polynomial in standard form. Identify the leading coefficient.

a) $9x^2 + 3x^6 - 4x$ $3x^6 + 9x^2 - 4x$ LC

b) $12 + 5y + 6xy + 8xy^2$ $8xy^2 + 6xy + 5y + 12$ LC

- Like Terms - terms w/ same variables raised to same power

Examples

Simplify the following expressions.

a) $(7y^2 + 2y - 3) + (2 - 4y + 5y^2)$

$12y^2 - 2y - 1$

b) $(4x^2 - 2x + 7) + (3x - 7x^2 - 9)$ $-3x^2 + x - 2$

c) $(6y^2 + 8y^4 - 5y) - (9y^4 - 7y + 2y^2)$ $-1y^4 + 4y^2 + 2y$

d) $(6n^2 + 11n^3 + 2n) - (4n - 3 + 5n^2)$

$11n^3 + n^2 - 2n - 3$

Example

The total amount of toy sales T (in billions of dollars) consists of two groups: sales of video games V and sales of traditional toys R. In recent years, the sales of traditional toys and the total sales could be represented by the following equations, where n is the number of years since 2000.

$R = 0.46n^3 - 1.9n^2 + 3n + 19$

$T = 0.45n^3 - 1.85n^2 + 4.4n + 22.6$

a) Write an equation that represents the sales of video games V.

$V = T - R$

$-.01n^3 + .05n^2 + 1.4n + 3.6$

b) Use the equation to predict the amount of video game sales in the year 2020.

$n = 20$

10.32 billion

8.2 Multiplying a Polynomial by a Monomial

Recall the distributive property: $2(x+3)$

$2x+6$

Examples – No Calculator

$6y(4y^2-9y-7)$ $24y^3-54y^2-42y$

$5a^2(-4a^2+2a-7)$ $-20a^4+10a^3-35a^2$

$3(2t^2-4t-5)+6t(5t+2)$

$6t^2-12t-15+30t^2+12t$ $36t^2-15$

Examples –Real World

Admission to the Super Fun Amusement Park is \$10. Once in the park, super rides are an additional \$3 each, and regular rides are an additional \$2. Walter goes to the park and rides 15 rides, of which s of those 15 are super rides. Find the cost in dollars if Walter rode 9 super rides.

$10+3s+2r$

$9 \quad 6$

49

Katie is making triangular bandanas for the dogs and cats in her pet club. The base of the bandana is the length of the collar with 4 inches added to the end to tie it on. The height is ½ of the collar length.

a) If Katie's dog has a collar length of 12 inches, how much fabric does she need in square inches?

60

b) If Katie makes a bandana for her friend's cat with a 6-inch collar, how much fabric does Katie need in square inches?

21

Example

Solve the equation $x(12 + x) - 7 = 2x + x(-4 + x)$

$12x + x^2 - 7 = 2x + -4x + x^2$

$10x = 7 - 4x$

$14x = 7$

$x = 1/2$

8.3 Multiplying Polynomials

To multiply polynomials you can either use one of two methods: 1) The Extended Distributive Property of 2) The Box Method

Example

Multiply the following: Extended Dist

$$(n-5)(2n^2-3n+7)$$

$2n^3-3n^2+7n-10n^2+15n-35$

$2n^3-13n^2+22n-35$

$$(w^2+4w+6)(w^2+w+1)$$ Box Method

	w^2	w	1
w^2	w^4	w^3	w^2
$4w$	$4w^3$	$4w^2$	$4w$
6	$6w^2$	$6w$	6

$w^4+5w^3+11w^2+10w+6$

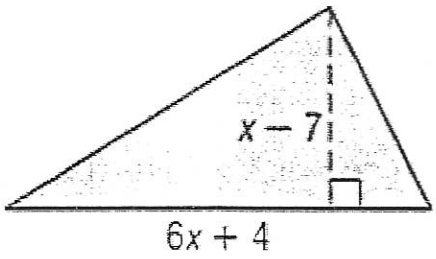
$$(2a+10b+1)(4a+b+11)$$

$8a^2+2ab+22a+40ab+10b^2+110b+4a+b+11$

$8a^2+42ab+26a+10b^2+111b+11$

Example

PATIO A patio in the shape of the triangle shown is being built in Lavali's backyard. The dimensions given are in feet. The area *A* of the triangle is one half the height *h* times the base *b*. Write an expression for the area of the patio.



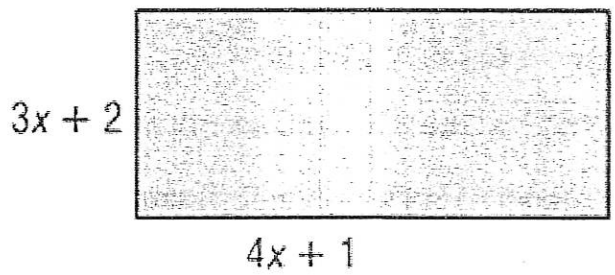
$\frac{1}{2}$ b · h

$\frac{1}{2}(6x+4)(x-7)$

$(3x+2)(x-7)$

$3x^2 - 19x - 14$

GEOMETRY The area of a rectangle is the measure of the base times the height. Write an expression for the area of the rectangle.



$(3x+2)(4x+1)$ FOIL

$12x^2 + 3x + 8x + 2$ First

Outside

Inside

$12x^2 + 11x + 2$ Last

8.4 Special Products

Explore

Problem	Multiply out	Simplify
$(x+3)(x+3)$	$x^2+3x+3x+9$	x^2+6x+9
$(x+5)(x+5)$	$x^2+5x+5x+25$	$x^2+10x+25$

Do you see a quicker way to go from the first column to the last?

- Square of a Sum formula $(a+b)^2 =$ $a^2+2ab+b^2$

Explore

Problem	Multiply out	Simplify
$(x-3)(x-3)$		x^2-6x+9
$(x-4)(x-4)$		$x^2-8x+16$

Do you see a quicker way to go from the first column to the last?

- Square of a Difference formula $(a-b)^2 =$ $a^2-2ab+b^2$

Explore

Problem	Multiply out	Simplify
$(x+3)(x-3)$		x^2-9
$(x+4)(x-4)$		x^2-16

Do you see a quicker way to go from the first column to the last?

- Product of a Sum and a Difference $(a+b)(a-b) =$ a^2-b^2

Examples

Simplify the following.

a) $(7x+2)^2$ $(7x+2)(7x+2)$ $49x^2-28x+4$

b) $(3c-4)^2$ $9c^2-24c+16$

c) $(a-2b)^2$ $a^2-4ab+4b^2$

d) $(9d+4)(9d-4)$ $81d^2-16$

e) $(3x-2)(3x+2)$ $9x^2-4$

Write an expression that represents the area of a square that has a side length of 3x+12 units.

$(3x+12)^2$

$9x^2+72x+144$

8.5 Using the Distributive Property

- Factoring – "undoing multiplication"

Ex 12 Factors 3 & 4

Ex $3x^2+9x$ $3x(x+3)$

Examples

Factor each polynomial completely,

a) $15x + 25x^2$

$5x(3+5x)$

b) $12xy + 24xy^2 - 30x^2y^4$

$6xy(2+4y-5xy^3)$

Sometimes polynomials do not have anything in common so you have to factor using another method called grouping!

Example

Factor the following.

a) $2xy + 7x - 2y - 7$ $(2y+7)(x-1)$

$(2xy-2y)+7x-7)$

$2y(x-1)+7(x-1)$

b) $15a - 3ab + 4b - 20$ $(3a-4)(5-b)$

$3a(5-b)-4(b-5)$

must be same

c) $3p - 2p^2 - 18p + 27$ $(p+9)(3-2p)$

$p(3-2p)+9(-2p+3)$

d) $x - 2xd + 8d - 4$

$x(1-2d) - 4(-2d+1)$

$(x-4)(1-2d)$

- Zero Product Property – in order for 2 items to mult to be zero at least one must be zero

$a \cdot b = 0$

$a = 0$ or $b = 0$

Examples – No Calculator

Solve each equation.

a) $(x-2)(4x-1) = 0$

$x - 2 = 0$ (2) $\quad 4x - 1 = 0$ (1/4)

b) $4y = 12y^2$

$12y^2 - 4y = 0$

$4y(3y-1) = 0$

$4y = 0$ (0) $\quad 3y - 1 = 0$ (1/3)

c) $3x(x+4) = 0$

$3x = 0$ (0) $\quad x + 4 = 0$ (-4)

d) $8x^2 - 40x = 0$

$8x(x-5) = 0$

(0) (5)

8.6 Factoring Trinomials

Recall: To simplify (x+2)(x+3)

$x^2 + 5x + 6$

Making a T-chart is helpful when factoring trinomials with a leading coefficient of 1

Examples

Factor each trinomial and check by multiplying or by using your graphing calculator.

a) $x^2 + 7x + 12$

(x+3)(x+4)

−3 −4

mult 12	add 7
1, 12	~~13~~
2, 6	~~8~~
3, 4	7

b) $x^2 - 12x + 27$

(x−3)(x−9)

3 9

c) $y^2 - 3y - 54$

(y−9)(y+6)

9 −6

d) $z^2 - 9z + 20$

(z−4)(z−5)

4 5

e) $c^2 - 16$

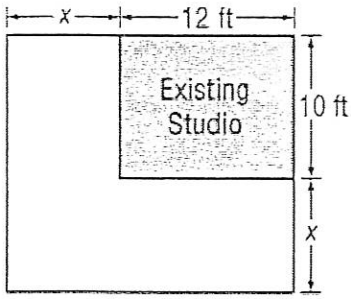
(c−4)(c+4)

4 −4

Now that we have factored we can also solve these quadratic equations. So go back and solve a-e.

Examples

Marion wants to build a new art studio that has three times the area of her old studio by increasing the length and width of the old studio by the same amount. What should be the dimensions of the new studio?



Now: 120 ft²

New $120 \times 3 = 360$

$(12+x)(10+x) = 360$

$120 + 22x + x^2 = 360$

$x^2 + 22x - 240 = 0$

$(x+30)(x-8)$ $x = 8$

20 × 18

PHOTOGRAPHY Adina has a 4 × 6 photograph. She wants to enlarge the photograph by increasing the length and width by the same amount. What dimensions of the enlarged photograph will produce an area twice the area of the original photograph?

Now $4 \times 6 = 24$

New $= 48$

$(4+x)(6+x) = 48$

$24 + 10x + x^2 = 48$

$x^2 + 10x - 24 = 0$

$(x+12)(x-2)$

2

6 by 8

8.7 Solving Quadratics by Factoring Continued

Sometimes quadratics have numbers greater than one as leading coefficients so we cannot go straight to a T-chart we have to add another step.

Factor and solve $5x^2 - 13x + 6 = 0$

30 mult add to -13

-3 & -10 Replace middle

$5x^2 - 10x - 3x + 6$ Group

$5x(x-2) - 3(x-2)$

$(5x-3)(x-2)$

- Prime Polynomial- polynomial that can't be factored

Examples

Factor $2x^2 + 5x + 3$

6 mult add to 5

2 & 3

$(2x^2+2x)+(3x+3)$

$2x(x+1)\ 3(x+1)$

$(2x+3)(x+1)$

Examples

Factor each trinomial:

a) $2n^2 - 3n - 20$

-40

$(2n^2 - 8n) + (5n - 20)$

$2n(n-4)\ 5(n-4)$

$(2n+5)(n-4)$

b) $6y^2 - 29y - 5$

-30

$6y^2 - 30y + 1y - 5$

$6y(y-5)\ 1(y-5)$

$(6y+1)(y-5)$

c) $4y^2 + 22y + 10$

40

$(4y^2 + 2y) + (20y + 10)$

$2y(2y+1)\ 10(2y+1)$

$(2y+10)(2y+1)$

Day 2

Factor and solve the following polynomials.

a) $5x^2 + 27x + 10 = 0$

$2 \cdot 25$

50

$5x^2 + 25x + 2x + 10 = 0$

$5x(x+5)\ 2(x+5)$

$(5x+2)(x+5) = 0$

$-\frac{2}{5}$ -5

b) $4x^2 + 24x + 32 = 0$

$4(x^2+6x+8) = 0$

$(x+4)(x+2)$

-4 -2

c) $2x^2 + 3x - 5 = 0$

-10 $5, -2$

$2x^2 - 2x + 5x - 5$

$2x(x-1)\ 5(x-1)$

$(2x+5)(x-1)$

$-\frac{5}{2}$ 1

Examples – No Calculator

Solve the following polynomials.

a) $x^2 + 3x + 2 = 0$

$(x+2)(x+1)$

-2 -1

b) $-3x^2 + 5x = -2$

$-3x^2 + 5x + 2 = 0$

-6 $6, -1$

$-3x^2 + 6x - 1x + 2$

$3x(-x+2)\ 1(-x+2)$

$(3x+1)(-x+2)$

$-\frac{1}{3}$ 2

c) $3x^2 - 8x - 3 = 0$ -9 and 1

-9

$3x^2 - 9x + 1x - 3$

$3x(x-3)\ 1(x-3)$

$(3x+1)(x-3)$

$-\frac{1}{3}$ 3

Example

Mr. Smith's science class built a model rocket. They launched the rocket outside. It cleared the top of a 60 foot high pole and then landed in a nearby tree. If the launch pad was 2 feet above the ground, the initial velocity of the rocket was 64 feet per second, and the rocket landed 30 feet above the ground, how long was the rocket in flight? Use the equation $h = -16t^2 + vt + h_0$.

30 64 2

$-16t^2 + 64t + 2 = 30$

$-16t^2 + 64t - 28 = 0$

$4(-4t^2 + 16t - 7) = 0$

$+28$

$+14, +2$

$(-4t^2 + 14t)(+2t - 7)$

$2t(-2t + 7) - 1(-2t + 7)$

$(2t - 1)(-2t + 7)$

$\frac{1}{2}$ $= \frac{7}{2} = 3.5$

8.8 Difference of Squares

Examples

Factor the following.

a) $x^2 - 64$ $(x-8)(x+8)$

b) $64x^2 - y^2$ $(8x-y)(8x+y)$

c) $16y^2 - 81z^2$ $(4y-9z)(4y+9z)$

d) $9y^3 - 4y$ $y(9y^2-4)$ $y(3y-2)(3y+2)$

e) $3b^3 - 27b$ $3b(b^2-9)$ $3b(b-3)(b+3)$

Sometimes you have to factor a polynomial more than once so that you can fully see the x-intercepts!

Examples

a) $y^4 - 625$ $(y^2-25)(y^2+25)$ $(y-5)(y+5)(y^2+25)$

b) $256 - n^4$ $(16-n^2)(16+n^2)$ $(4-n)(4+n)(16+n^2)$

c) $9x^5 - 36x$ $9x(x^4-9)$ $9x(x^2-3)(x^2+3)$

d) $6x^3 + 30x^2 - 24x - 120$ $6x^2(x+5)-24(x+5)$ $(6x^2-24)(x+5)$ $6(x^2-4)(x+5)$ $6(x-2)(x+2)(x+5)$

8.9 Perfect Squares

Recall Perfect Square pattern:

$(a+b)^2$ $(a-b)^2$

$a^2+2ab+b^2$ $a^2-2ab+b^2$

3 Steps to Determine if a polynomial is a perfect square:

1) Is the first term a perfect square
2) Is the last term a perfect square
3) Is the middle term equal to 2(a)(b)?

Examples

Determine whether each trinomial is a perfect square. Write yes or no. If it is a perfect square then factor it.

a) $25x^2 - 30x + 9$ 2·5·3

$(5x-3)(5x-3)$

b) $49y^2 + 42y + 36$ 2·7·6

No

c) $9x^2 + 24x + 16$ 2·3·4

$(3x+4)^2$

Examples of all types of factoring

Factor and solve each polynomial, if possible.

a) $6x^2 - 96$

$6(x^2-16)$ 4 & -4

$6(x-4)(x+4)$

b) $16x^2 + 8x - 15$

$(4y+5)(4y-3)$ -5/4 3/4

c) $4x^2 + 36x = -81$

$4x^2+36x+81=0$

$(2x+9)(2x+9)$ $\quad -\frac{9}{2}$

Example-Real World

A book falls from a shelf that is 5 feet above the floor. A model for the height in feet if an object dropped from an initial height of h_0 feet is $h = -16t^2 + h_0$, where t is the time in seconds after the object is dropped. Use this model to determine approximately how long it took for the book to reach the ground.

$0 = -16t^2 + 5$

$-5 = -16t^2$

Table

.56sec

$\sqrt{5/16} = \sqrt{t^2}$