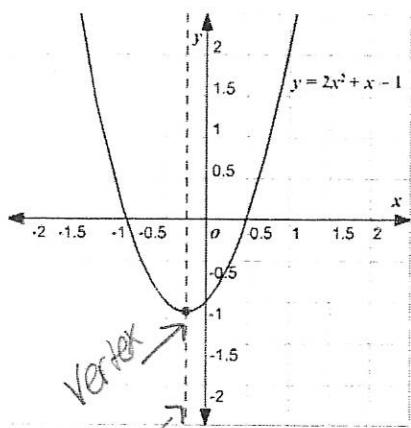
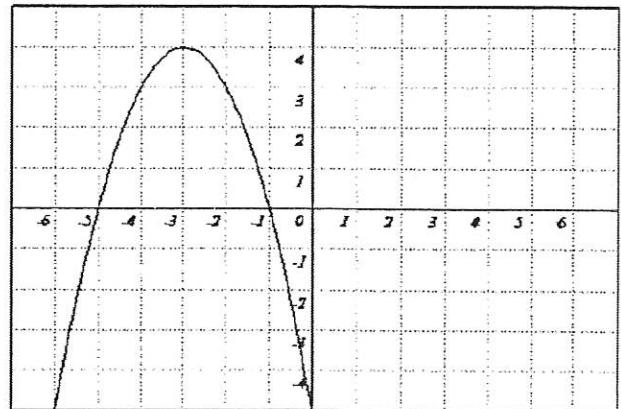


9.1 Graphing Quadratic Functions



Parabolas
 $y = x^2$

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

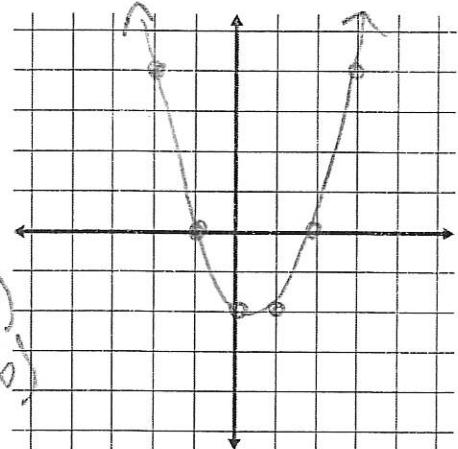


Examples

Use a table of values to graph $y = x^2 - x - 2$. State the domain and range.

x	-2	-1	0	1	2	3
y	4	0	-2	-2	0	4

Domain $(-\infty, \infty)$
Range $[-2, \infty)$

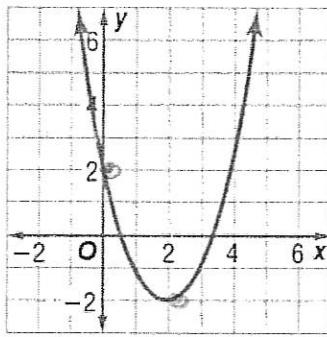


A. Find the vertex, the equation of the axis of symmetry, and y-intercept of the graph.

Vertex: $(2, -2)$

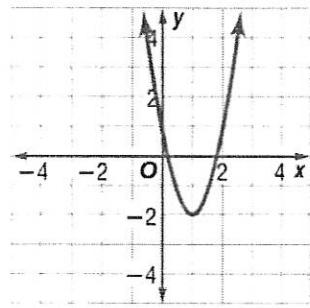
Axis: $x = 2$

y-int: $(0, 2)$



Find the vertex, the equation of the axis of symmetry, and the y-intercept of the graph.

$$\begin{aligned} &\text{Vertex } (1, -2) \\ &\text{Axis } x=1 \\ &y\text{-int } (0, 1) \end{aligned}$$



Examples

Find the vertex, the equation of the axis of symmetry, and the y-intercept for the following.

a) $y = -2x^2 - 8x - 2$

$$\begin{array}{l} \text{Vertex} \\ (-2, 6) \end{array}$$

$$\begin{array}{l} \text{Axis} \\ x=-2 \end{array}$$

$$\begin{array}{l} \text{y-int.} \\ (0, -2) \end{array}$$

b) $y = 3x^2 + 6x - 2$

$$(-1, -5)$$

$$x=-1$$

$$(0, -2)$$

c) $y = 2x^2 + 2x + 2$

$$(-\frac{1}{2}, 1.5)$$

$$x=-\frac{1}{2}$$

$$(0, 2)$$

Explore

Draw a rough sketch of each of the following graphs. What do you notice?

$y = 3x^2$	$y = 6x^2$	$y = 10x^2$	$y = -2x^2$	$y = -3x^2$	$y = -5x^2$
\cup	\cup	\cup	\cap	\cap	\cap

Example

Consider the equation $f(x) = -x^2 - 2x - 2$

- Determine whether the function has a maximum or a minimum value.
- State the maximum or minimum value of the function. $(-1, -1)$
- State the domain and range of the function.

$D: \mathbb{R}$
Range: $(-\infty, -1]$

Examples – Real World

Ben shoots an arrow. The height of the arrow can be modeled by $y = -16x^2 + 100x + 4$, where y represents the height in feet of the arrow x seconds after it is shot into the air.

- a) Graph the height of the arrow.



- b) At what height was the arrow shot? 4

- c) What is the maximum height of the arrow?

160.25

The equation $h = -.005x^2 + x + 3$ describes the path of a baseball hit into the outfield, where h is the height and x is the horizontal distance the ball travels.

vertex (100, 53)

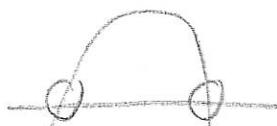
- a) What is the equation of the axis of symmetry? $x=100$

- b) What is the maximum height reached by the ball? 53

- c) An outfielder catches the ball three feet above the ground. How far has the ball traveled horizontally when the outfielder catches it? 200 ft

$$y = -.005x^2 + x + 3$$

$$y = 3$$

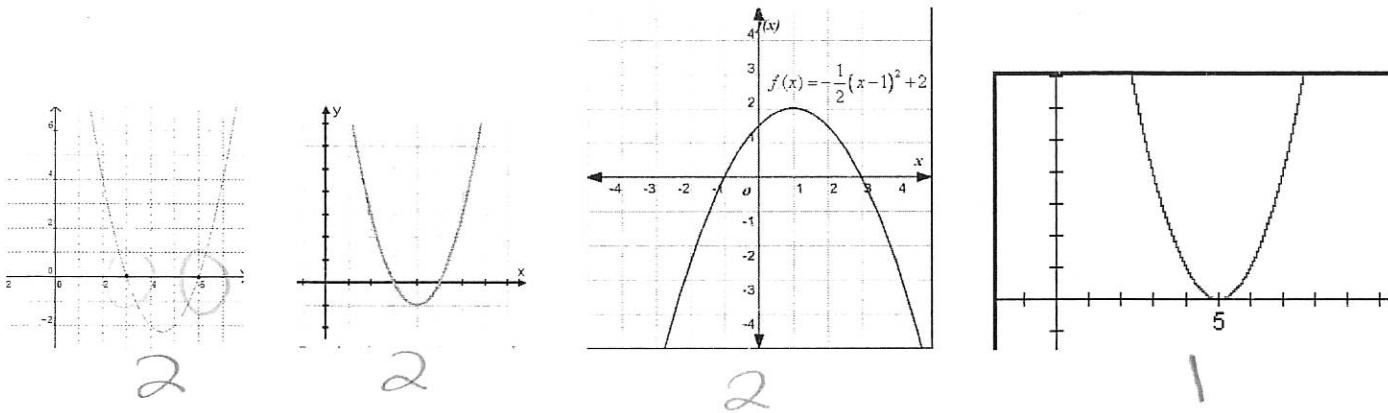


2nd Calc Intersect

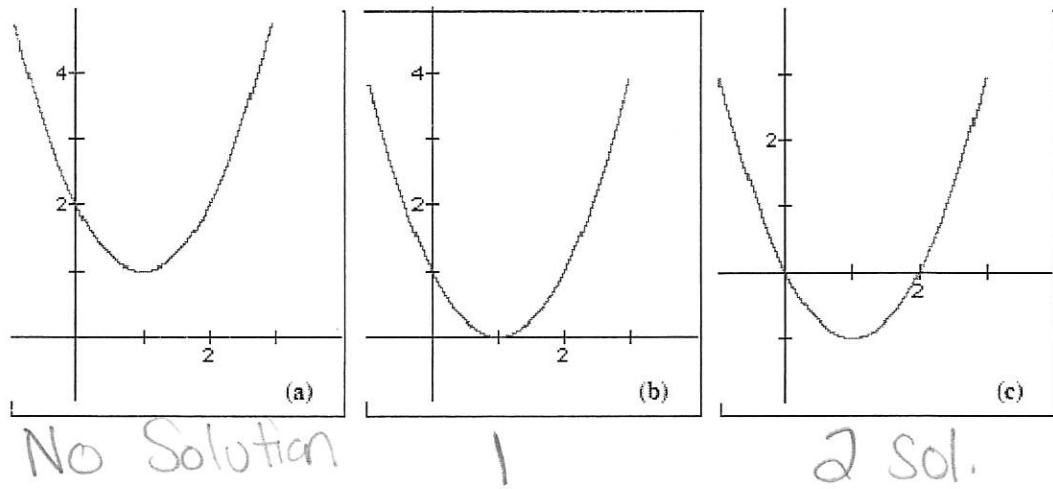
9.2 Solving Quadratic Equations by Graphing

- Roots / Zeros / X-Intercepts

Recall $x^2 + 8x + 7$
 $(x+7)(x+1)$
 $\textcircled{-7} \quad \textcircled{-1}$



The number of solutions to the quadratic equations depends on the graph.



Examples

Solve the following by graphing. If the roots are not integers, round to the nearest tenth.

a) $y = x^2 - 3x - 10$ 5 + -2

b) $x^2 + 8x = -16$ + $\textcircled{-4}$
 $x^2 + 8x + 16 = 0$

c) $f(x) = x^2 + 2x + 3$

No Sol



d) $x^2 - 4x + 2 = 0$

, 586 & 3.414

Example – Real World

Consuela built a model rocket for their science project. The equation $h = 16t^2 + 250t$ models the flight of the rocket launched from ground level at a velocity of 250 feet per second, where h is the height of the rocket in feet after t seconds.

Approximately how long was Consuela's rocket in the air?

$$0 = -16t^2 + 250t$$

$$2(-8t + 125)$$

$$2t = 0 \quad -8t + 125 = 0$$

$$\textcircled{0}$$

$$\textcircled{t = 15.625}$$

Labs

Explore
online
with vertex
form

direction opens

$$y = a(x - h)^2 + k$$

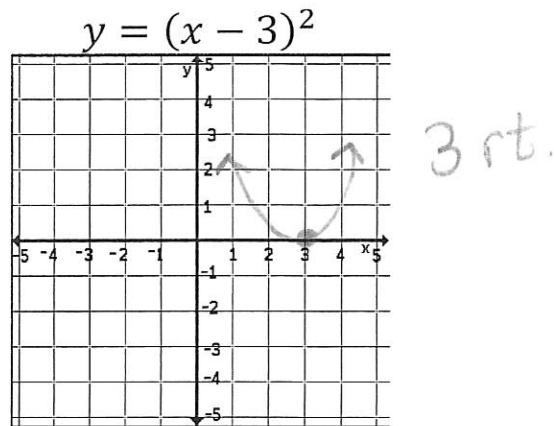
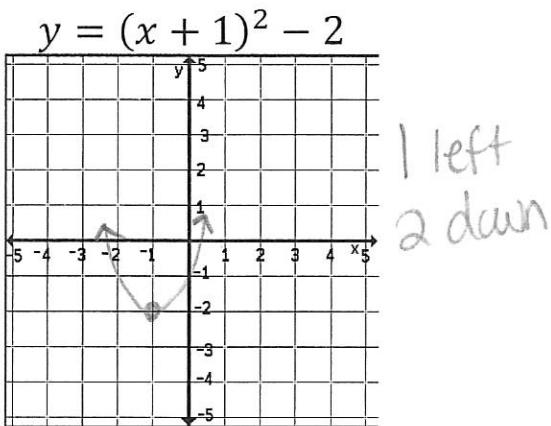
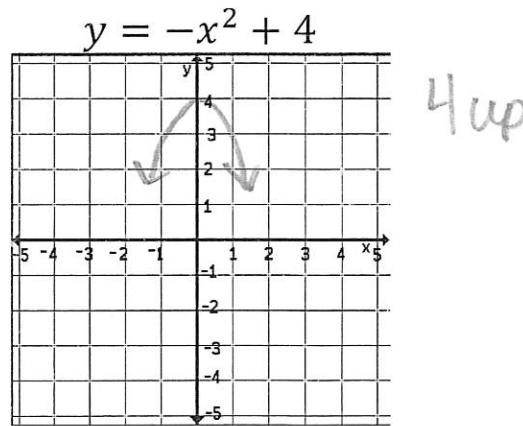
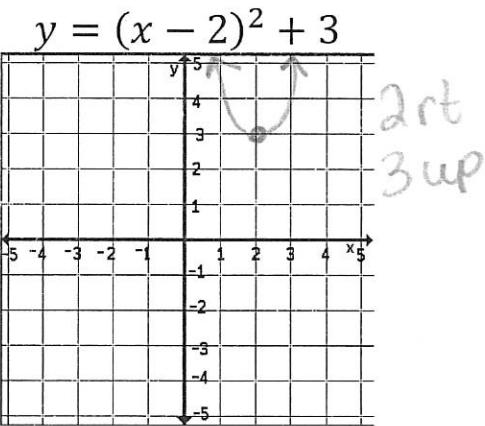
↑ shift
ct/left ↑ up/dawn

$$y = 3(x - 4)^2 + 2 \quad 4\text{rt, 2up}$$

$$y = -7(x + 3)^2 - 8 \quad 3\text{left, 8dawn}$$

Examples – Non Calculator

Graph the following parabolas and describe the transformation on the parent graph $y = x^2$.



9.3 Transformations of Quadratic Graphs

Recall: $y = a(x - h)^2 + k$

- A value - direction of opening

- Stretch $|a| > 1$



- Shrink / Compression $|a| < 1$

- H Value - rt / left

- K value - shift up / down

Examples – No Calculator

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

Think about shifts (up/down and rt/left), stretches, compressions, and reflections.

a) $g(x) = 10 + x^2$ up 10

b) $g(x) = x^2 - 8$ down 8

c) $g(x) = (x + 1)^2$ left 1

d) $g(x) = (x - 4)^2$ rt 4

e) $g(x) = (x + 1)^2 + 1$ left 1 up 1

f) $g(x) = (x - 2)^2 + 6$ rt 2 up 6

g) $g(x) = \frac{1}{2}x^2$ Shrink

h) $g(x) = 2x^2 + 1$ stretch of up 1

i) $g(x) = -3x^2 + 1$

flip, stretch, up!

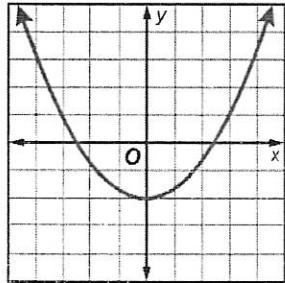
j) $g(x) = \frac{1}{5}x^2 - 7$

shrink, down ↴

Example – No Calculator

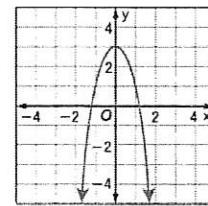
Which is an equation for the function shown in the graph?

- A. $y = \frac{1}{3}x^2 - 2$
- B. $y = 3x^2 + 2$
- C. $y = -\frac{1}{3}x^2 + 2$
- D. $y = -3x^2 - 2$



Which is an equation for the function shown in the graph?

- A. $y = -2x^2 - 3$
- B. $y = 2x^2 + 3$
- C. $y = -2x^2 + 3$
- D. $y = 2x^2 - 3$



Example

During a 4th of July celebration, a cannon is fired. The flight of the cannon ball can be modeled by $d(t) = -4(x - 5)^2 + 100$, where t is the time in seconds and $d(t)$ is the distance in meters.

How is the function of $d(t)$ related to the graph of $f(x) = x^2$?

Stretch 4, flipped, ft 5, up 100

9.5 Quadratic Formula

To solve Quadratic Equations we have learned a few different methods:

1. Factoring
2. Table
3. Graphing

- Quadratic Formula = $X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$ax^2 + bx + c = 0$$

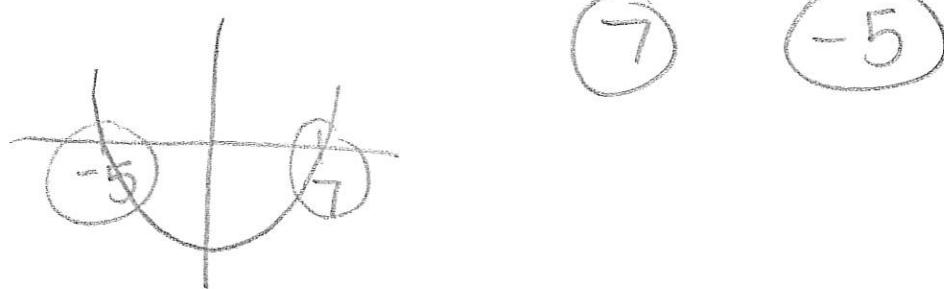
Examples

Solve the following using the quadratic formula. You can check your answers by graphing.

a) $x^2 - 2x - 35$
a b c

$$\frac{2 \pm \sqrt{(2)^2 - 4 \cdot 1 \cdot -35}}{2 \cdot 1}$$

$$\frac{2 \pm 12}{2} = \frac{14}{2} \quad \frac{-10}{2}$$



b) $2x^2 - 2x - 5$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2}$$

$$\frac{2 \pm \sqrt{44}}{4}$$

$$\frac{2 \pm 6.6}{4}$$

$$2.15$$

$$-1.15$$

c) $5x^2 - 8x = 4$

$$5x^2 - 8x - 4 = 0$$

$$\frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 5 \cdot -4}}{2 \cdot 5}$$

$$\frac{8 \pm \sqrt{144}}{10}$$

$$\frac{8 \pm 12}{10}$$

$$2$$

$$-4$$

d) $3x^2 - 5x - 12 = 0$

$$3x^2 - 5x - 12 = 0$$

$$\frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 3 \cdot -12}}{2 \cdot 3}$$

$$\frac{5 \pm \sqrt{169}}{6}$$

$$3$$

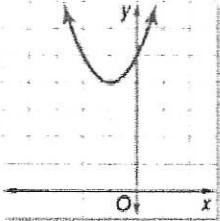
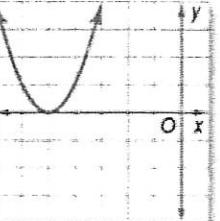
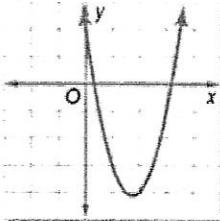
$$-1.3$$

Day 2

- Discriminant –

$$b^2 - 4ac$$

Tells you how many solutions

$x^2 + 2x + 5 = 0$	$x^2 + 10x + 25 = 0$	$2x^2 - 7x + 2 = 0$
$b^2 - 4ac = -16$ negative	$b^2 - 4ac = 0$ zero	$b^2 - 4ac = 33$ positive
		
0 x-intercepts	1 x-intercept	2 x-intercepts

Examples

- State the value of the discriminant.
- Determine how many real solutions the equation has.
- Solve the equation if possible.

$$3x^2 + 10x - 12 = 0$$

$$1) 3x^2 + 10x = 12$$

Discriminant

$$10^2 - 4 \cdot 3 \cdot 12$$

$$-44$$

of Real Solutions

$$0$$

Solutions/Roots

No Sol.

$$2) 9x^2 - 30x + 25 = 0$$

Discriminant

of Real Solutions

Solutions/Roots

$$(-30)^2 - 4 \cdot 9 \cdot 25$$

0

(1)

$$\frac{30 \pm \sqrt{0}}{2 \cdot 9}$$

1, 5

$$3) x^2 - 9x + 21 = 0$$

Discriminant

of Real Solutions

Solutions/Roots

$$(-9)^2 - 4 \cdot 1 \cdot 21$$

-3

0

No Sol.

$$4) 2x^2 + 11x + 15 = 0$$

Discriminant

of Real Solutions

Solutions/Roots

$$(11)^2 - 4 \cdot 2 \cdot 15$$

1

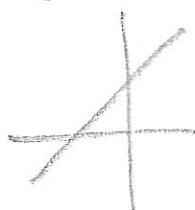
2

$$\frac{-11 \pm \sqrt{1}}{2 \cdot 2}$$

-2,5

-3

9.6 Determining if a Function is Linear, Quadratic or Exponential

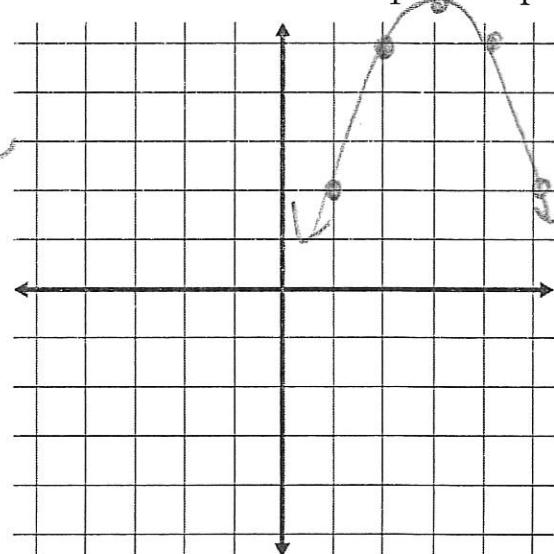
Linear	Quadratic	Exponential
$y = mx + b$  Adding or Sub $\begin{array}{ c c } \hline x & y \\ \hline 0 & 0 \\ 1 & 5 \\ 2 & 10 \\ 3 & 15 \\ 4 & 20 \\ \hline \end{array}$	$y = ax^2 + bx + c$  Break down x $\begin{array}{ c c } \hline x & y \\ \hline 0 & 0 \\ 1 & 1 \\ 2 & 4 \\ 3 & 9 \\ 4 & 16 \\ \hline \end{array}$	$y = a \cdot b^x$  mult $\begin{array}{ c c } \hline x & y \\ \hline 1 & 5 \\ 2 & 10 \\ 3 & 20 \\ 4 & 40 \\ \hline \end{array}$

Examples – No Calculator

Graph each set of ordered pairs. Determine whether the ordered pairs represent a linear, quadratic, or exponential function.

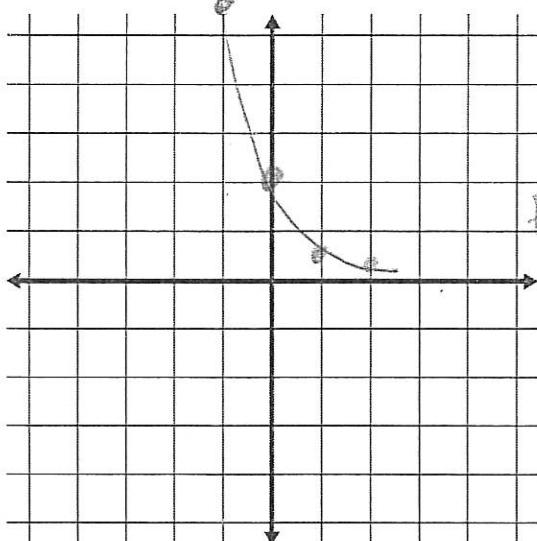
a) (1, 2), (2, 5), (3, 6), (4, 5), (5, 2)

Quadratic



b) (-1, 6), (0, 2), (1, 2/3), (2, 2/9)

Exp.



amples

Determine which type of function is given and then find equation that best models the data.

a)

X	-2	-1	0	1	2
Y	-1	1	3	5	7

Linear

+2

+2

+2

b)

X	-2	-1	0	1	2
Y	36	12	4	4/3	4/9

Exp:

$\times \sqrt[3]{3}$

$\times \sqrt[3]{3}$

c)

X	0	1	2	3	4
Y	-1	-8	-64	-512	-4096

$\times 8$

$\times 8$

$\times 8$

Exp:

Example – Real World

The table shows the number of children enrolled in a beginner's karate class for four consecutive years. Determine which model best represents the data. Then write a function that models the data.

Time (years)	0	1	2	3	4
Number Enrolled	8	11	14	17	20

$$8 + 3x$$