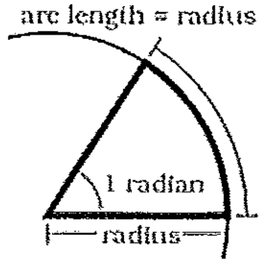
Key

4.1 Angles and their Measures

- **Central Angle –** angle whose vertex is center of circle

- **Radian –** the measure of a central angle whose intercepted arc has a length equal to the circle's radius



$2\pi r$ = Circumference of Circle

$2\pi r$

$360° = 2\pi$ rad

$\text{rad} = \frac{360}{2\pi} = \frac{180}{\pi}$

$\text{deg} = \frac{2\pi}{360} = \frac{\pi}{180}$

  - **Radians to Degrees**

$$\frac{180°}{\pi \text{ radians}}$$

  - **Degrees to Radians** $\frac{\pi \text{ radians}}{180}$

Ex

(a) How many radians are in 45°? $45 \cdot \frac{\pi}{180} = \frac{45\pi}{180} = \frac{\pi}{4}$ radians

(b) How many degrees are in $\pi/10$ radians? $\frac{\pi}{10} \cdot \frac{180}{\pi} = \frac{180}{10}$ 18°

(c) Find the length of an arc intercepted by ~~a central~~ angle of 1/8 radian in a circle of radius 24 inches. $\frac{1}{8} \cdot 24 =$ 3 inches

(d) Find the radian measure of a central angle that intercepts an arc of length s in a circle of radius r. $\frac{x \text{ rad}}{s} = \frac{1 \text{ rad}}{r \text{ units}}$ $rx = s$ $x = s/r$

- **Arc Length –**

$$\frac{\text{deg}}{360} \cdot 2\pi r = \frac{\pi r \,\text{deg}}{180}$$

Example

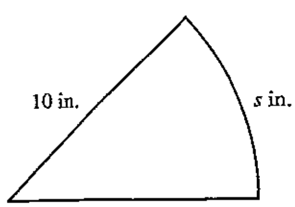


Figure 4.1 A 45° slice of a large pizza.

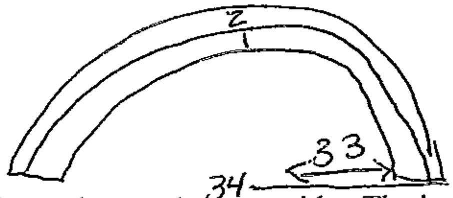
Find the perimeter of a 45° slice of a large (10 in. radius) pizza.

$P = 10 + 10 + s$ ← need to find

$\frac{45}{360} \cdot 2\pi \cdot 10 = \frac{450\pi}{180} = 7.9$

$\boxed{27.9 \text{ in}}$

Example



A running track at a College has lanes that are 1 meter wide. The inside radius of lane one is 33 meters, and the inside radius of lane 2 is 34 meters. How much longer is lane 2 than lane 1 around one turn?

Semicircle $\theta = \pi$

length $s = r\theta = r\pi$

$34\pi - 33\pi$

$= \pi$ $\boxed{3.14 \text{ meters longer}}$

Example

Denzel Murphy's truck has wheels 36 inches in diameter. If the wheels are rotating at 720 rpm (revolutions per minute), find the truck's speed in miles per hour.

36

$$\frac{720\,\cancel{\text{rev}}}{1\,\cancel{\text{min}}} \cdot \frac{60\,\cancel{\text{min}}}{1\,\text{hr}} \cdot \frac{2\pi\,\cancel{\text{rad}}}{1\,\cancel{\text{rev}}} \cdot \frac{18\,\cancel{\text{in}}}{1\,\cancel{\text{rad}}} \cdot \frac{1\,\cancel{\text{ft}}}{12\,\cancel{\text{in}}} \cdot \frac{1\,\text{mi}}{5280\,\cancel{\text{ft}}} = \frac{\text{mi}}{\text{hr}}$$

$77.11 \frac{\text{mi}}{\text{hr}}$

Example

How many nautical miles is it from Boston to San Fancisco if there is a distance of 2698 stat miles?

Stat miles = 5280ft

1 stat mile = .87 naut mile

1 naut mile = 1.15 stat miles

$1' = \left(\frac{1}{60}\right) \times \frac{\pi}{180} = \frac{\pi}{10{,}800}$ radians

$S = r\theta$

$X = \frac{2698}{1.15}$

$X = 2346.$

- **DMS Measure –**

Degree, Minutes, Seconds

, $'$, $''$

Ex Convert 37.425° to DMS

$37°$ $\left(.425 \cdot \frac{60'}{1°}\right) = 25.5'$

$\left(.5 \cdot \frac{60''}{1'}\right) = 30''$

$\boxed{37°25'30''}$

Example

(a) Convert 23.325° to DMS.

(b) Convert 30°30′18″ to degrees.

a) $23°$ $\left(.325 \cdot \frac{60'}{1°}\right) = 19.5$

$\left(.5 \cdot \frac{60''}{1'}\right) = 30$

$23°\ 19'\ 30''$

b) 1 min = $\frac{1}{60}$ deg

1 sec = $\frac{1}{3600}$ deg

$\boxed{30.505°}$

4.2 Trig Functions of Acute Angles (Day 1)

Sine $\frac{opp}{hyp}$

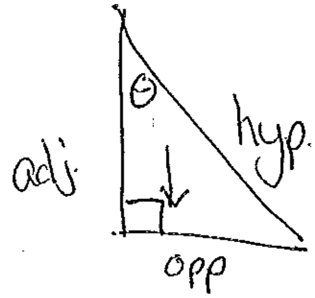
Cosine $\frac{adj}{hyp}$

Tangent $\frac{opp}{adj}$

Cosecant $\frac{hyp}{opp}$

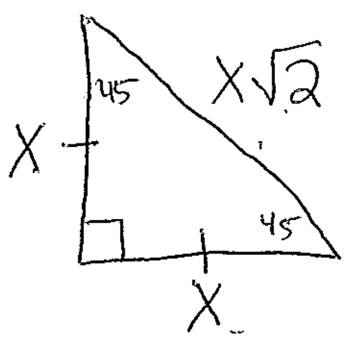
Secant $\frac{hyp}{adj}$

Cotangent $\frac{adj}{opp}$



Special Right Triangles

45-45-90



$x^2 + x^2 = c^2$

$\sqrt{2x^2} = \sqrt{c^2}$

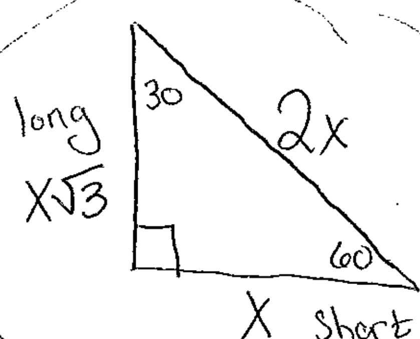
$\sqrt{2} \cdot \sqrt{x^2} = c$

$x\sqrt{2} = c$

$\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2}$

$= 5\sqrt{2}$

30-60-90



Review 4.1-4.2 Deg min Sec

Convert DMS to decimal form.

$23^0 12'$ $\quad 23 + \frac{12}{60} = \boxed{23.2^\circ}$

$118^0 44' 15''$ $\quad 118 + \frac{44}{60} + \frac{15}{3600} = \boxed{118.7375^\circ}$

Convert from decimal to DMS form.

21.2^0 $\quad .2 \times 60 = 12$ $\quad \boxed{21^\circ 12'}$

118.32^0 $\quad .32 \times 60 = 19.2$ $\quad .2 \times 60 = 12$ $\quad \boxed{118^\circ 19' 12''}$

Convert from radians to degrees.

$\Pi/6$ $\quad \frac{\pi}{6} \cdot \frac{180}{\pi} = 30^\circ$

$\Pi/10$ $\quad 18^\circ$

$7\Pi/9$ $\quad 140^\circ$

2 $\quad 114.6^\circ$

R to D $\cdot \frac{180}{\pi}$

Convert from degrees to radians.

45^0 $\quad \frac{45}{1} \cdot \frac{\pi}{180} = \frac{45\pi}{180} = \frac{\pi}{4}$

120^0 $\quad 120 \cdot \frac{\pi}{180} = \frac{120\pi}{180} = \frac{2\pi}{3}$

100^0 $\quad \frac{5\pi}{9}$

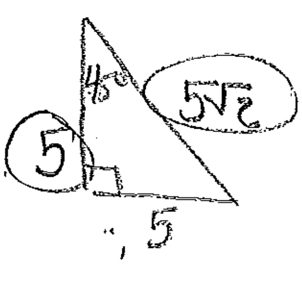
32^0 $\quad \frac{8\pi}{45}$

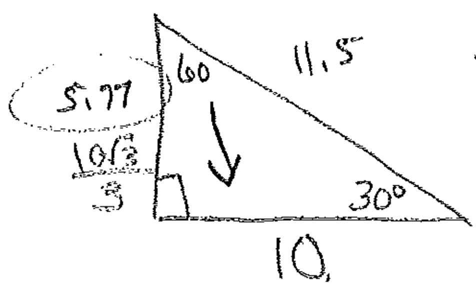
D to R $\cdot \frac{\pi}{180}$

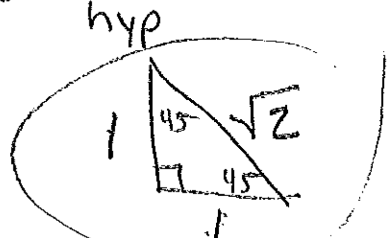
Solve for each of the following:

SOH - CAH - TOA (Sin, Cos, Tan)

$\sin\theta = \frac{opp}{hyp}$



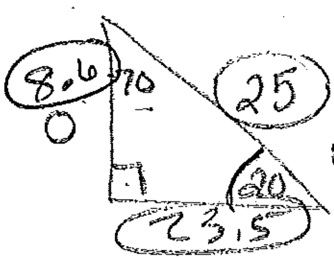




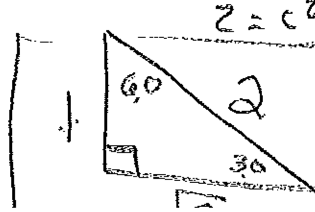
$\frac{10}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$

$1^2 + 1^2 = c^2$

$2 = c^2$



$\sin 20 = \frac{x}{25}$



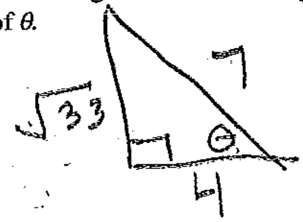
$\pi = 180°$ $\frac{\pi}{4} = 45°$ $\frac{\pi}{6} = 30°$

$\frac{\pi}{2} = 90°$ $\frac{\pi}{3} = 60°$

4.2 Trigonometric Functions of Acute Angles

Example CAH

Let θ be an acute angle such that $\cos \theta = 4/7$. Evaluate the other five trigonometric functions of θ.



$\sin \theta = \frac{\sqrt{33}}{7}$

$\tan \theta = \frac{\sqrt{33}}{4}$

$\csc \theta = \frac{7}{\sqrt{33}}$ $\frac{7\sqrt{33}}{33}$

$\sec \theta = \frac{7}{4}$

$\cot \theta = \frac{4}{\sqrt{33}}$ $\frac{4\sqrt{33}}{33}$

$7^2 - 4^2 = x^2$

$49 - 16 = x^2$

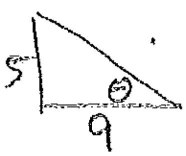
$\sqrt{33} = x$

- **Common Mistakes**
  - **Wrong Mode** mode
  - **Using Inverse Trig Keys for Cot, Sec, and Csc** $\cot(30) = (\tan 30)^{-1}$
  - **Using shorthand that the calculator does not recongnize**
  - **Not Closing parenthesis**

Ex #13

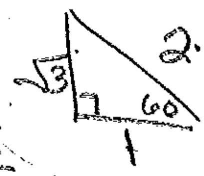
$\tan\theta = \frac{5}{9}$

$\cot\theta = \frac{9}{5}$



#19 Eval $\sin\left(\frac{\pi}{3}\right)$

$\sin 60°$ $\frac{\sqrt{3}}{2}$



Stop

Example

A right triangle with hypotenuse 6 includes a 35° angle (Figure 4.3). Find the measures of the other two angles and the lengths of the other two sides.

$6 \cdot \sin 35 = \frac{a}{6}$

$a = 3.441$

$\sin 55 = \frac{b}{6}$ $180 - 90 - 35$

$b = 4.9$

$\cos 35 = b/6$

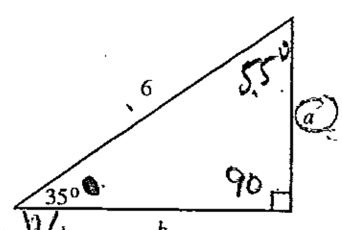
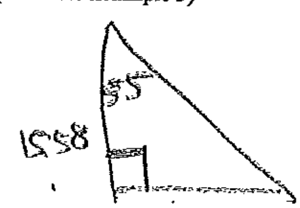


Figure 4.3 (Alternate Example 5)

Solve rt. triangle ABC for all of its unknown parts if a = 15.58 and β=55°.



$\tan 55 = \frac{b}{15.58}$

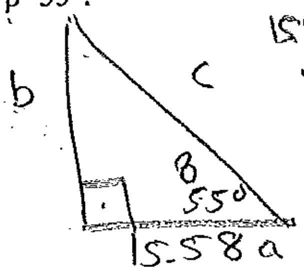
$c \cdot \cos 55 = \frac{15.58}{c}$

$\frac{15.58}{\cos 55}$

$\alpha = 35°$

$b = 22.25$

$c = 27.16$



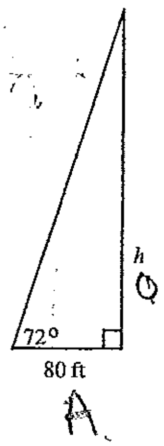
α - a

β - b

SOH CAH TOA

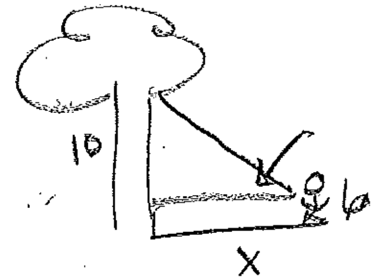
Example

From a point 80 feet away from the base of a building, the angle of elevation to the top of the building is 72°. (See Figure 4.4.) Find the height *h* of the building.



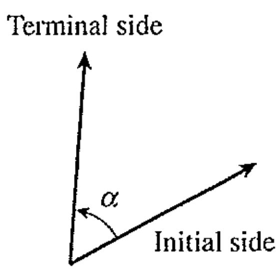
$80 \cdot \tan 72 = \frac{h}{\cancel{80}} \cancel{\cdot 80}$

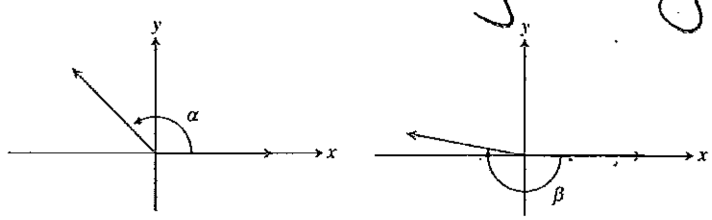
h = 246 ft



4.3 Trigonometry: The Circular Function

"rotating a ray"





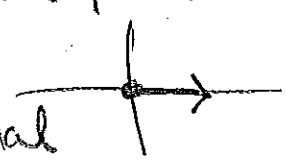
A positive angle (counterclockwise)
(a)

A negative angle (clockwise)
(b)

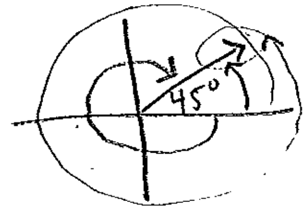
- **Initial Side**
- **Vertex**
- **Terminal Side**
- **Positive Angles**

-210

- **Negative Angles**
- **Standard Position** – vertex on origin, initial side pos x-axis
- **Coterminal Angles** –



Same initial & vertex & terminal
but diff measures



360° 2π

Example

Find and draw a positive angle and a negative angle that are coterminal with the given angle.

a) 30^0 $= -330^\circ$ (360-30) $= 390^\circ$ (360+30)

b) -150^0 210° -510

c) $2\Pi/3$ radians

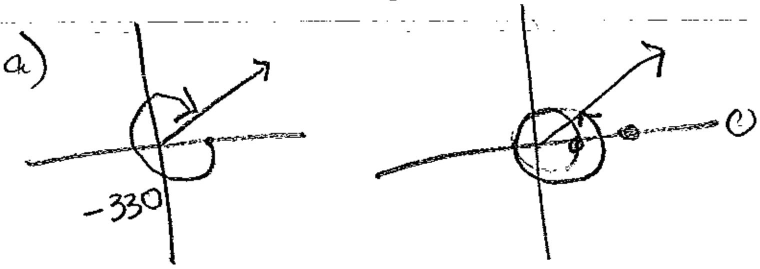
$\frac{2\pi}{3} + 2\pi$

$\frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$ 480

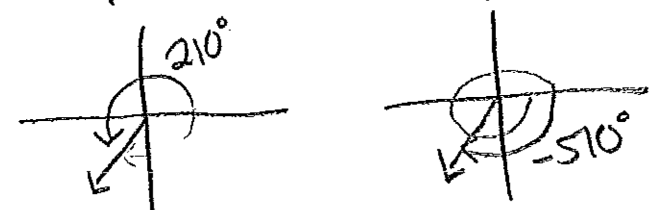
-240°

$\frac{2\pi}{3} - 2\pi$ $\quad -\frac{4\pi}{3}$

a)

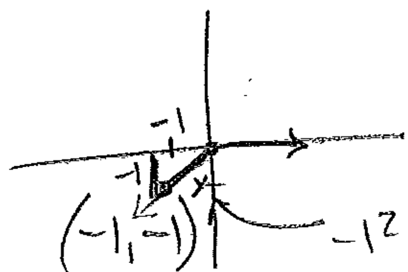


b)



Example

Evaluate the six trigonometric functions of the angle Θ whose terminal side contains the point (-1, -1).



$-1^2 + -1^2 = \sqrt{2}$

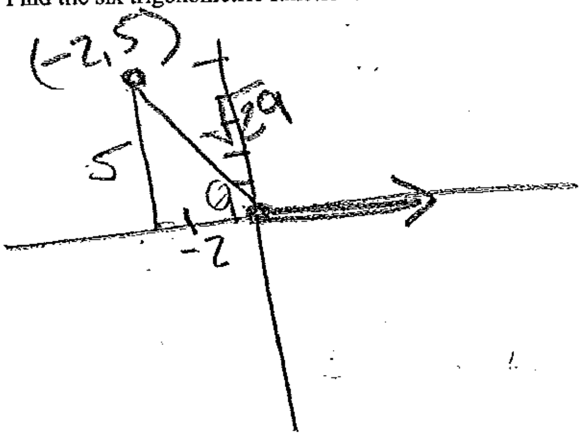
$\sin\theta = \frac{-1}{\sqrt{2}}$ $\quad \csc\theta = -\sqrt{2}$

$\cos\theta = \frac{-1}{\sqrt{2}}$ $\quad \sec\theta = -\sqrt{2}$

$\tan\theta = \frac{-1}{-1} = 1$ $\quad \cot\theta = 1$

Example

Let θ be any angle in standard position whose terminal side contains the point (–2, 5).
Find the six trigonometric functions of θ.



$\sin\theta = \frac{5}{\sqrt{29}}$ $\quad \csc\theta = \frac{\sqrt{29}}{5}$

$\cos\theta = \frac{-2}{\sqrt{29}}$ $\quad \sec\theta = -\frac{\sqrt{29}}{2}$

$\tan\theta = -5/2$ $\quad \cot\theta = -2/5$

Trigonometric Functions of Any Angle

Let θ be any angle in standard position and let $P(x,y)$ be any point on the terminal side of the angle (except the origin). Let r denote the distance from $P(x,y)$ to the origin, i.e., let $r = \sqrt{x^2+y^2}$. Then

$\sin\theta = \frac{y}{r}$ $\qquad$ $\csc\theta = \frac{r}{y}$ $\quad (y \neq 0)$ $\quad \frac{1}{\sin}$

$\cos\theta = \frac{x}{r}$ $\qquad$ $\sec\theta = \frac{r}{x}$ $\quad (x \neq 0)$ $\quad \frac{1}{\cos}$

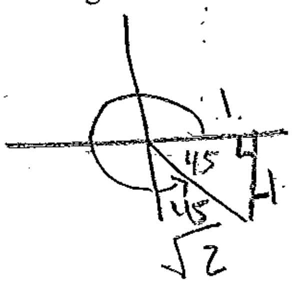
$\tan\theta = \frac{y}{x}$ $\quad (x \neq 0)$ $\qquad$ $\cot\theta = \frac{x}{y}$ $\quad (y \neq 0)$

$\frac{\sin}{\cos}$ $\qquad$ $\frac{\cos}{\sin}$

$(\sin x)^{-1}$

Example

Find the six trigonometric functions of 315°



$\sin 315 = \frac{-1}{\sqrt{2}}$

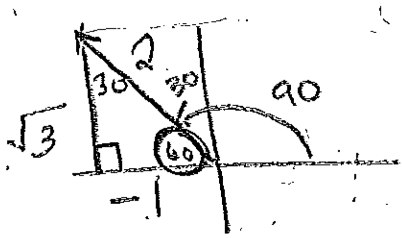
$\cos 315 = \frac{1}{\sqrt{2}}$

$\tan 315 = -1$

Example

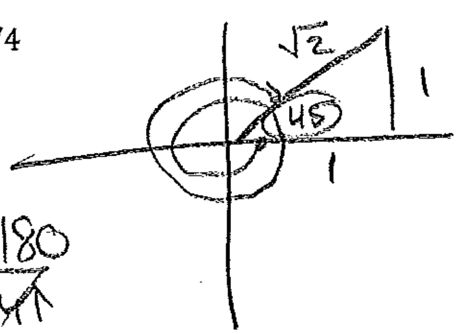
Find the following without using a calculator.

Cos 120°



$\cos \quad \frac{A}{H} \quad \frac{-1}{2} = \frac{-1}{2}$

Tan -15*Π*/4



$\frac{O}{A} \quad \frac{1}{1} = 1$

$\frac{-15\pi}{4} \cdot \frac{180}{\pi}$

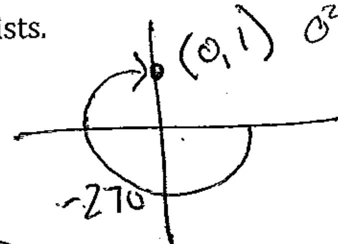
$= -675°$

➤ Quadrantal Angles – angles whose terminal sides are on coordinate axis

Example

Find each of the following, if it exists.

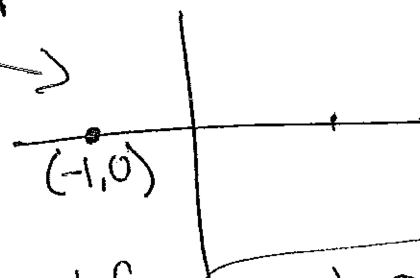
a) Sin (-270°)



$0^2+1^2=$ $\sqrt{1}=1$

$\sin \frac{1}{1} = 1$

b) Tan 3Π

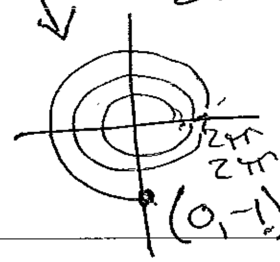


$\frac{0}{-1} = 0$

c) Sec 11Π/2

5.5π

inv. cos $\frac{x}{r}$



$\frac{1}{0}$ = undef.

$\sin \theta =$ → $\frac{1}{\sin} = \csc$

$\sin 90 = \frac{O}{H}$

$\sin = \frac{O}{H}$

Example

Find sin θ and tan θ by using the given information to construct a reference triangle.

(a) cos $\theta = -\frac{4}{5}$ and tan $\theta < 0$

(b) sec $\theta = \frac{5}{3}$ and sin $\theta > 0$

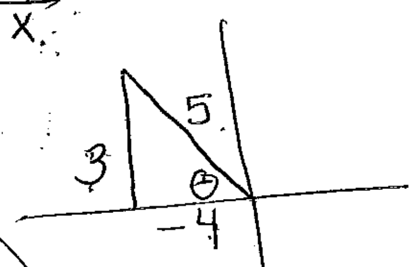
~~(c)~~ cot θ is undefined and sin θ is positive

a) $\cos \theta = -\frac{4}{5}$ $\quad \tan \theta < 0$ Neg.

$\frac{Adj}{Hyp} \quad \frac{x}{r}$ $\quad \frac{O}{A} \quad \frac{y}{x}$ ← pos

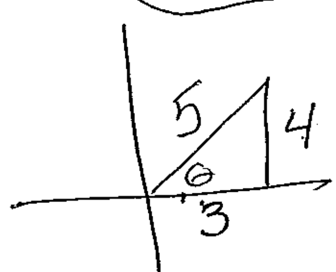
$\sin \theta = 3/5$

$\tan \theta = \frac{-3}{4}$



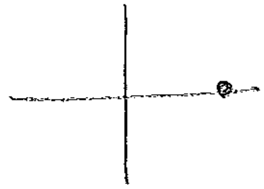
b) $\sec \theta = \frac{5}{3}$ $\quad \sin \theta > 0$

$\frac{r}{x}$ $\quad$ pos $\frac{y}{r}$



$\sin \theta = 4/5$

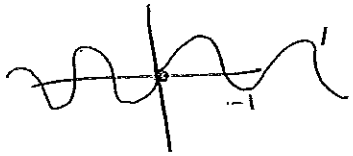
$\tan \theta = \frac{4}{3}$



4.4 Graphs of Sine and Cosine: Sinusoids

Radians

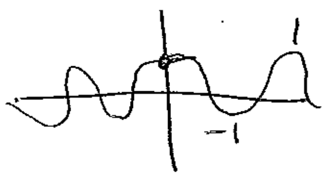
- The Sine Function



D: $\mathbb{R}$
Range: $[-1, 1]$
Continuous

odd
Bounded
No asymptotes
No end behavior

- The Cosine Function



D: $\mathbb{R}$
Range: $[-1, 1]$
Continuous

Even (y-axis)
Bounded
No asymptotes
No end behavior

- Sinusoid ~ Functions in form

$$f(x) = a \sin(bx + c) + d \qquad a \neq 0,\ b \neq 0$$

a ↑ vert stretch/shrink; b ↑ Hor; c ↑ Rt/Left; d ↑ up/down

- Amplitude of Sinusoid

= ½ height of wave
or height x-axis up if no shift

$|a|$

Example

Find the amplitude of each function and use the language of transformations to describe how the graphs are related.

#1-6

(a) $y_1 = \sin x$ (b) $y_2 = \frac{1}{3}\sin x$ (c) $y_3 = -2\sin x$

Amp 1 — $1/3$ — 2

Vert Shrink $1/3$

vert stretch 2
Reflect over x-axis

- Period of Sinusoid

= smallest value where function repeats

$$\frac{2\pi}{|b|}$$

Recall period of
Sine $= 2\pi$
Cos $= 2\pi$
Tan $= \pi$

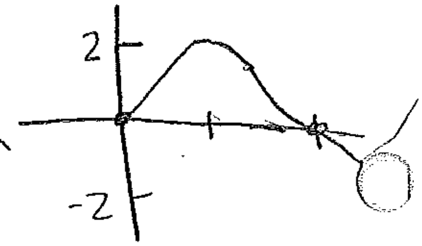
Factor $\frac{1}{|b|}$

Ex
$\sin \frac{x}{3}$
Hor stretch 3

#17-22 (17) $2\sin x$

Amp 2

Period 2π



Example

Find the period of each function and use the language of transformations to describe how the graphs are related.

#1-12

(a) $y_1 = \sin x$

$\frac{2\pi}{1}$

Period 2π

(b) $y_2 = -3\sin\left(\frac{x}{2}\right)$

$\frac{2\pi}{(1/2)} = 4\pi$

- Hor. strech 2.
- Vert. strech 3
- Reflect over x-axis

(c) $y_3 = 2\sin(-2x)$

$\frac{2\pi}{|-2|}$ $\frac{2\pi}{2} = \pi$

- Hor. shrink 1/2
- Vert. stretch 2
- Reflect y-axis

➢ Frequency of a Sinusoid – # of complete cycles completed by the wave in a unit interval (Recip. of Period) $\frac{|b|}{2\pi}$

Example

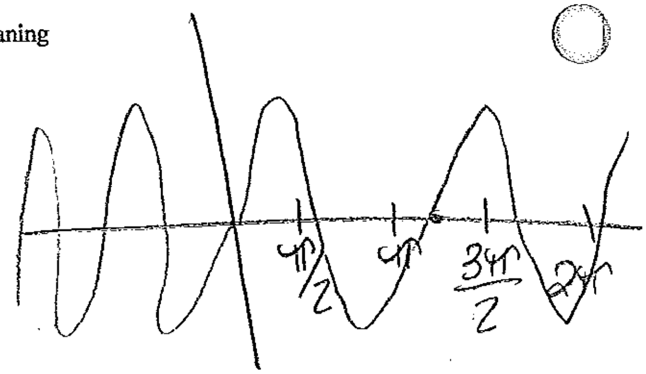
#13-16

Find the frequency of the function $f(x) = 3\sin(3x/2)$ and interpret its meaning graphically.

Sketch the graph in the window $[-2\pi, 2\pi]$ by $[-3, 3]$.

Period $\frac{2\pi}{|3/2|} = \frac{4\pi}{3}$

Freq. $\frac{3}{4\pi}$



Day 2

➢ **Phase Shift** – translation left or rt

$a\sin(bx+c)+d$

amp, Period/Freq., rt/left, up/down

Phase shift $-c$

Example

#42

a) Write the cosine function as a phase shift of the sine function. left

$\frac{\pi}{2}$ $\cos x = \sin\left(x + \frac{\pi}{2}\right)$

b) Write the sine function as a phase shift of the cosine function.

$\frac{\pi}{2}$ $\sin x = \cos\left(x - \frac{\pi}{2}\right)$

$a \sin(bx+c)+d$

Example

Construct a sinusoid with period $\Pi/5$ and amplitude 6 that goes through (2,0)

$a = 6$

$b = \dfrac{2\pi}{\frac{\pi}{5}} = 10$

$6\sin(10x)$ through (0,0)

Rt 2 $\quad -2$

$6\sin 10(x-2)$

$\boxed{= 6\sin(10x-20)}$

The graphs of $y = a\sin(b(x-h))+k$ and $y = a\cos(b(x-h))+k$ (where $a \neq 0$ and $b \neq 0$) have the following characteristics:

amplitude $= |a|$;

period $= \dfrac{2\pi}{|b|}$;

frequency $= \dfrac{|b|}{2\pi}$.

Example

Construct a sinusoid $y = f(x)$ that rises from a minimum value of $y = 2$ at $x = 0$ to a maximum of $y = 6$ at $x = 6$ (Figure 4.10).

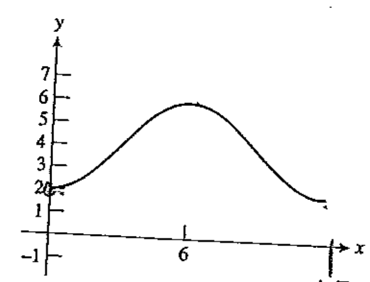


Figure 4.10 A sinusoid with specifications.

57-60

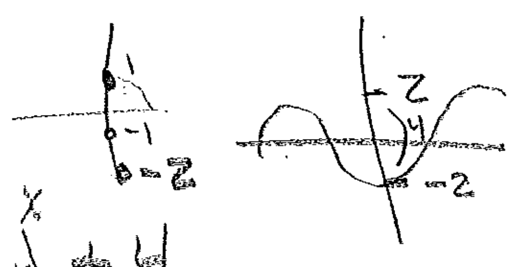
- Amp $\dfrac{6-2}{2} = 2$ a
- Period $\dfrac{2\pi}{|b|} = 12$

$\dfrac{2\pi}{12} = |b| = \dfrac{\pi}{6}$ b

- Cosine max $\quad y=1 \quad x=0$

Neg amp $\quad -2\cos\frac{\pi}{6}x$

$-2\cos(\frac{\pi}{6}x) + 4$



$y = a\sin(bx+c)+d$

High 7:12 $^{.2}$ 11 ft Max

Low 1:24 $^{.004}$ 7 ft Min

Example

#75 On Labor Day the high tide in Southern California occurs at 7:12 AM. At that time you measure the water at the end of the Santa Monica Pier to be 11 ft deep. At 1:24 PM it is low tide and you measure the water to be only 7 ft deep. Assume the depth of the water is a sinusoidal function of time with a period of ½ lunar day, which is about 12 hr 24 min.

Find eqn

a) At what time on that Labor Day does the first low tide occur?

1 am

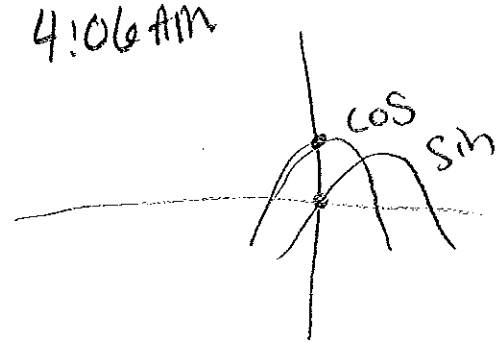
b) What was the approximate depth of the water at 4:00 AM and at 9:00 PM?

21

8.9   10.5

c) What is the first time on that Labor Day that the water is 9 ft deep?

4:06 Am



- Amp. - Ht. of wave $\frac{11-7}{2} = 2$

- Period time By Repeats

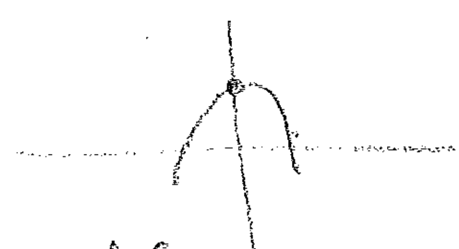
12 hr. 24 min

$\frac{24}{60}$

12.4

$$\frac{2\pi}{b} = 12.4$$

$$\frac{2\pi}{12.4} = \boxed{\frac{\pi}{6.2}}$$



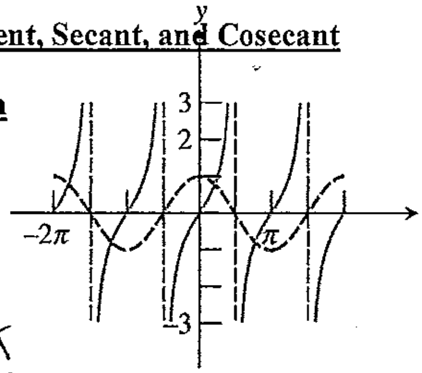
- Vert. Shift $\frac{11+7}{2} = 9$

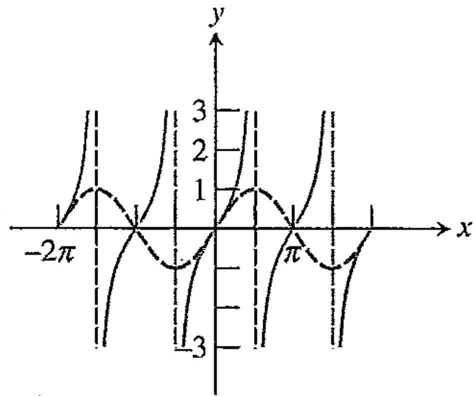
- Max must $x = 7{:}12$ 7.2 hrs after mid.

Radians $D(t) = 2\cos\left(\frac{\pi}{6.2}(t - 7.2)\right) + 9$

4.5 Graphs of Tangent, Cotangent, Secant, and Cosecant

➢ **Tangent Function**





Domain: $x \neq$ odd mult $\pi/2$

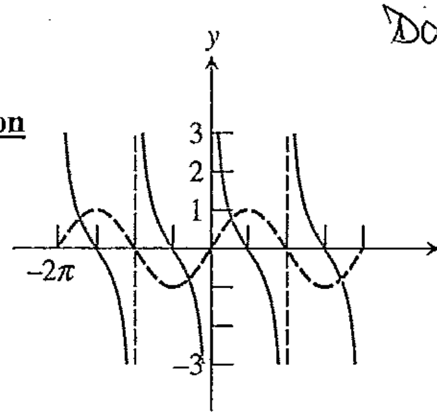
$n\pi + \pi/2$

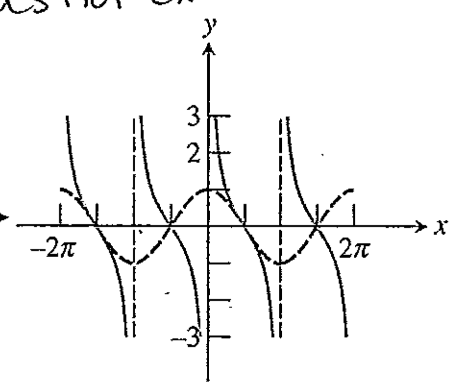
$\tan = \frac{\sin}{\cos}$

DNE

Does not exist

➢ **Cotangent Function**





$\cot = \frac{\cos}{\sin}$

Use Hor Factor to get new period & asymptotes.

Example

#6-28 Describe the graph of $y = -\tan 3x$ in terms of a basic trigonometric function. Locate the vertical asymptotes and graph four periods of the function.

$y = \tan x$

Norm. asym. odd $\frac{\pi}{2}$

Normal period π

- over x-axis, Hor. shrink 1/3
- $\frac{\pi}{2} \cdot \frac{1}{3} = \frac{\pi}{6}$ Asymptotes odd mult.
- Period $\pi \cdot 1/3 = \frac{\pi}{3}$

Example

Describe the graph of $f(x) = 2 \cot (x/2) - 1$ in terms of a basic trigonometric function. Locate the vertical asymptotes and graph four periods of the function.

cot x

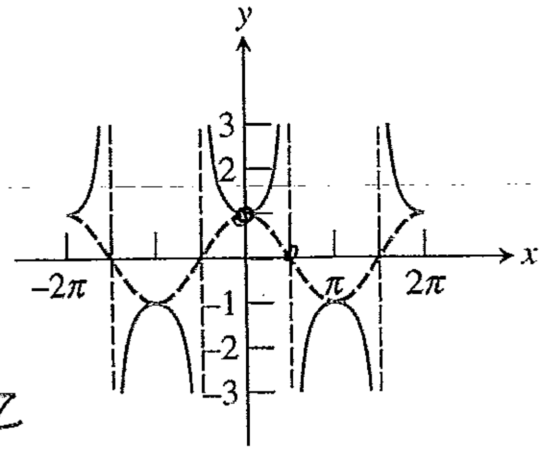
Normal Asymptotes $n\pi$

Period π

- Vert Stretch 2
- Hor. stretch 2
- Down 1

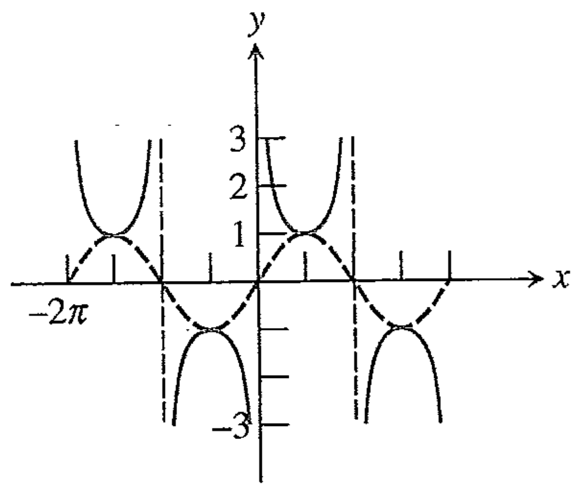
- Asym. $\pi \cdot 2 = 2\pi$
- Period $\pi \cdot 2 = 2\pi$

➢ **Secant Function** $\frac{1}{\cos x}$

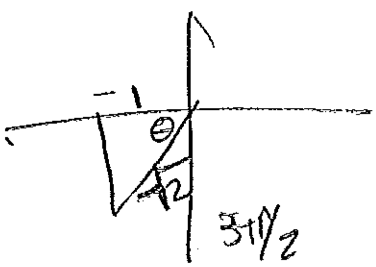


- No x-int
- Even
- Asymptotes odd mult $\pi/2$

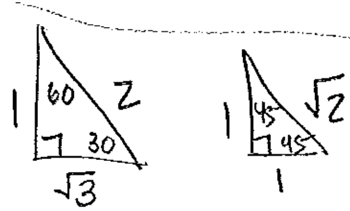
➢ **Cosecant Function** $\frac{1}{\sin x}$



- Odd
- Asymptotes πn



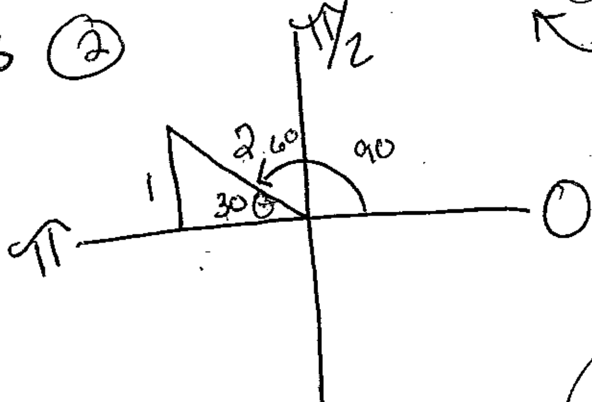
225° or $\frac{5\pi}{4}$



Example

Find the value of x between $\pi/2$ and π that solves $\csc x = 2$.

#30-38 2



Recip. $\sin \frac{O}{H}$

$\csc \frac{H}{O} \quad \frac{2}{1}$

150° or $\frac{5\pi}{6}$

$\cdot \frac{\pi}{180}$

#40 $\tan x = .3$

$0 \le x \le 2\pi$

.291 radians or 16.699°

$+\pi$ 3.433 rad

196.699°

#32 $\sec x = -\sqrt{2}$

$\pi \le x \le 3\pi/2$

#

Summary: Basic Trigonometric Functions

Function	Period	Domain	Range
$\sin x$	2π	All reals	$[-1, 1]$
$\cos x$	2π	All reals	$[-1, 1]$
$\tan x$	π	$x \neq \pi/2 + n\pi$	All reals
$\cot x$	π	$x \neq n\pi$	All reals
$\sec x$	2π	$x \neq \pi/2 + n\pi$	$(-\infty, -1] \cup [1, \infty)$
$\csc x$	2π	$x \neq n\pi$	$(-\infty, -1] \cup [1, \infty)$

Function	Asymptotes	Zeros	Even/Odd
$\sin x$	None	$n\pi$	Odd
$\cos x$	None	$\pi/2 + n\pi$	Even
$\tan x$	$x = \pi/2 + n\pi$	$n\pi$	Odd
$\cot x$	$x = n\pi$	$\pi/2 + n\pi$	Odd
$\sec x$	$x = \pi/2 + n\pi$	None	Even
$\csc x$			

8 Find $x^2 = \csc x$

$y = x^2$

$y = \csc x$ ← Radians

$\frac{1}{\sin x}$

Note: $\sin^2 x$ & $(\sin x)^2$ same thing

4.6 Graphs of Composite Trigonometric Functions

Recall:

Polynomial Functions $y=x^2$ $y=3x+1$

Exponential Functions $y=2^x$ $y=5\cdot\frac{1}{3}^x$

Logarithmic Functions $y=\log x$

Rational Functions $y=\frac{1}{x}$ $y=\frac{1}{x^2}-3$

Trigonometric Functions $y=\sin x$ $y=\cos x$ } only ones periodic

Example

Graph each of the following functions for $-2\Pi \le x \le 2\Pi$, adjusting the vertical window as needed. Which of the functions appear to be periodic?

#1-8

a) $y = sinx + x^2$ No
b) $y = x^2 \sin x$ No
c) $y = (\sin x)^2$ Yes
d) $y = \sin(x)^2$ Yes

* When compose may or may not be periodic
If it is: $\sin(x+2\pi) = \sin x$

Example

Verify algebraically that $f(x) = (\cos x)^2$ is periodic and determine its period graphically.

9-12

show $f(x+2\pi) = f(x)$
We know $\cos(x+2\pi) = \cos x$
$= (\cos(x+2\pi))^2$
$= (\cos x)^2$ Subst.
$= f(x)$
$\therefore$ Periodic
Period π

Example

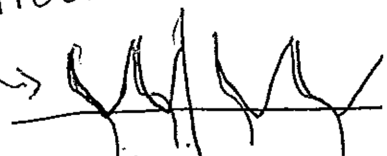
Find the domain, range, and period of each of the following functions. Sketch a graph showing four periods.

13-18

(a) $f(x) = |\cos x|$ (b) $g(x) = |\cot x|$

Domain $\mathbb{R}$ $(-\infty,\infty)$
Rng: $[0,1]$ Normal $[-1,+1]$
Period: π

Domain: $x \neq n\pi$
Rng: $[0,\infty)$ Normal $\mathbb{R}$
Period: π



Example

$y=1$
$y=-1$

The graph of f(x) = 0.5x + sin x oscillates between two parallel lines. What are the equations of the two lines?

#19-22

$y = .5x - 1$
$y = .5x + 1$

Sums That Are Sinusoid Functions

If $y_1 = a_1 \sin(b(x - h_1))$ and $y_2 = a_2 \cos(b(x - h_2))$, then

$$y_1 + y_2 = a_1 \sin(b(x - h_1)) + a_2 \cos(b(x - h_2))$$

is a sinusoid with period $2\pi/|b|$.

Bs must be same

Example

Determine whether f(x) is a sinusoid.

#23-28

a) F(x) = sin x – 3 cos x Yes
b) F(x) = 2 cos Πx + sin Πx Yes
c) F(x) = 3 sin 2x – 5 cos x No

Diff periods

Example

Let $f(x) = 3 \sin x + 4 \cos x$. Since both $\sin x$ and $\cos x$ have period 2π, f is periodic and is a sinusoid.

#29-34

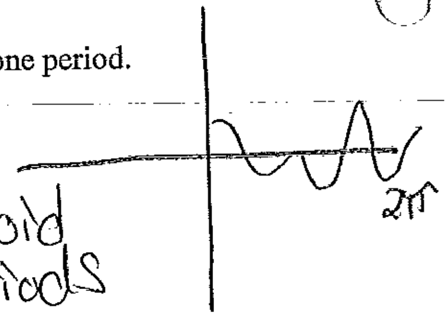
(a) Find the period of f. 2π
(b) Estimate the amplitude and phase shift graphically (to the nearest hundredth).
(c) Give a sinusoid in the form $a \sin(b(x - h))$ that approximates $f(x)$.

Graph amp. 5
phase shift $-.93$
$y = 5\sin(x + .93)$

Example

2π 2π

Show that $f(x) = \sin 2x + \cos 3x$ is periodic but not a sinusoid. Graph one period.



5-38

$2\pi \cdot \frac{1}{2} = \pi$ $2\pi \cdot \frac{1}{3}$ $\frac{2\pi}{3}$

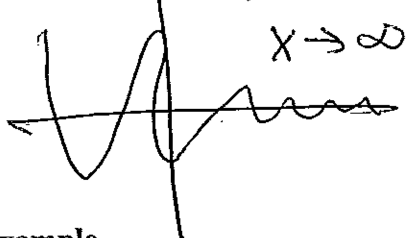
Not sinusoid
diff. periods

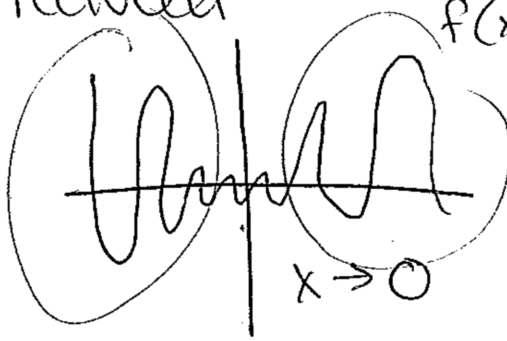
Know $\sin(x+2\pi) = \sin x$
$\cos(x+2\pi) = \cos x$

$f(x+2\pi)$
$= \sin 2(x+2\pi) + \cos 3(x+2\pi)$
$\sin(2x + \cancel{4\pi}) + \cos(3x + \cancel{6\pi})$
$\sin 2x + \cos 3x$
$f(x)$

- Damped Oscillation

amp is reduced





Example

Tell whether the function exhibits damped oscillation. If so, identify the damping factor and tell whether the damping occurs as $x \to 0$ or as $x \to \infty$.

43-48

A) $f(x) = e^{-x} \sin 3x$

$x \to \infty$ e^{-x}

B) $f(x) = x^3 \sin 5x$

$x \to 0$ x^3

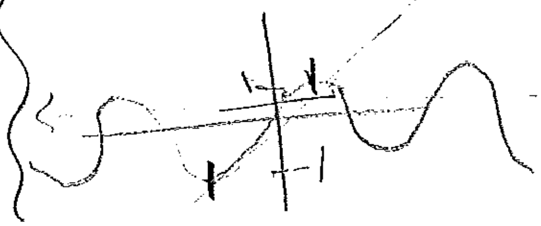
Reciprocal $^{-1}$

$\sin x = 1/\csc x$

$1/\sin x = \csc x$

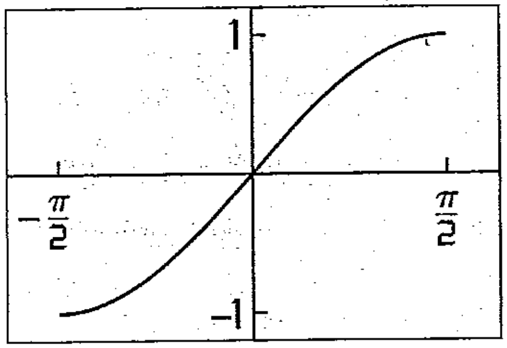
$(\sin x)^{-1}$

arcsin x $\sin^{-1} x$



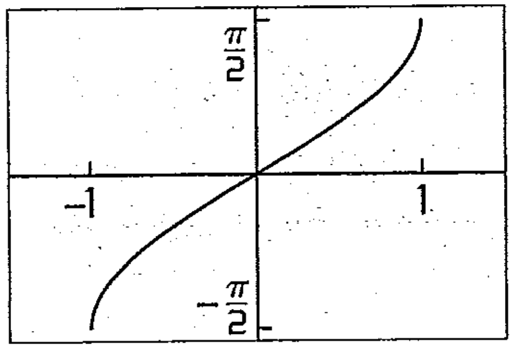
4.7 Inverse Trigonometric Functions

➤ **Inverse Sine Function**



[–2, 2] by [–1.2, 1.2]

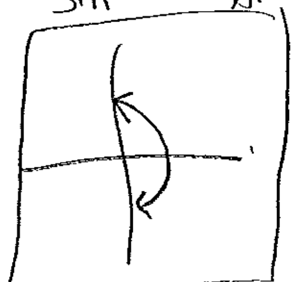
(a)



[–1.5, 1.5] by [–1.7, 1.7]

(b)

$\sin^{-1}$



Domain $[-1, 1]$

Rng $[-\pi/2, \pi/2]$

Recall: sin is y-value

– Recall reflection over y=x

– Domain + Rng flipped

(cos, sin)

Example

#1-22

Find the exact value of each expression without a calculator.

(a) $\sin^{-1}\left(-\frac{1}{2}\right)$ angle $-30° = -\frac{\pi}{6}$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\pi/3$ 60° (c) $\sin^{-1}\left(-\frac{\pi}{2}\right)$ $-\pi/2$ DNE

$= -1.57$ $\quad -1 \ldots < -1$

↑ No not in domain

(d) $\cancel{\sin^{-1}}\left(\cancel{\sin}\left(\frac{\pi}{5}\right)\right)$ (e) $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

$\pi/5$ $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\pi/4$

45°

✓ In domain so cancels

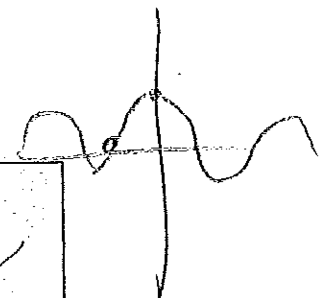
Example

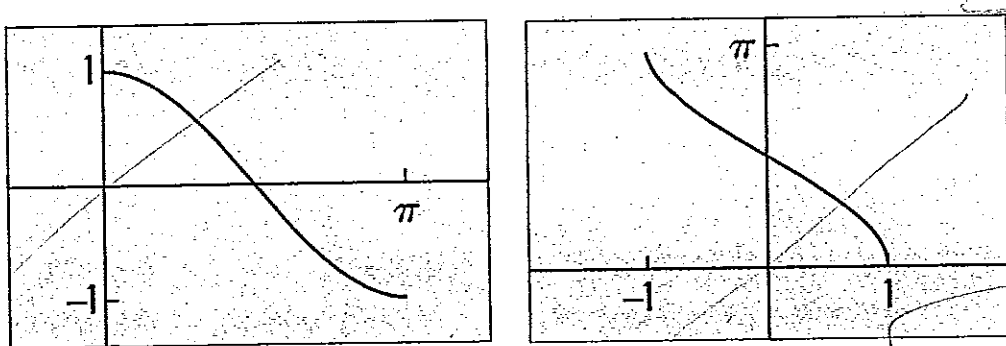
Use a calculator in radian mode to evaluate these inverse sine values:

(a) $\sin^{-1}(-0.73)$ $-.818$

(b) $\sin^{-1}(\sin(3.45\pi))$ -1.414

- **Inverse Cosine Function**

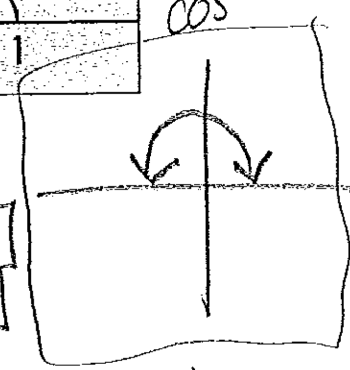




[−1, 4] by [−1.4, 1.4]
(a)

[−2, 2] by [−1, 3.5]
(b)

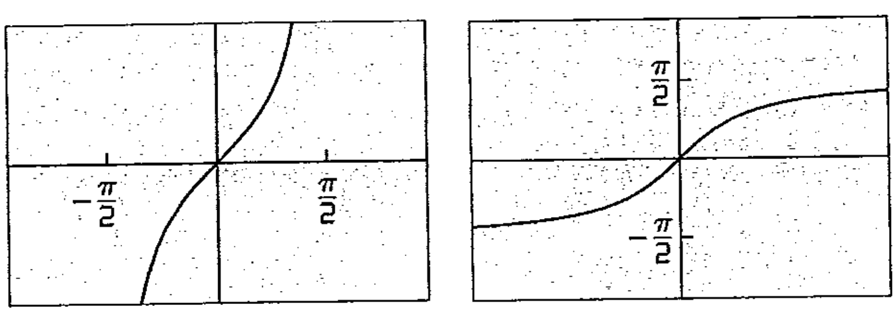
$\cos^{-1}$



Domain $[-1,1]$
Rng $[0,\pi]$

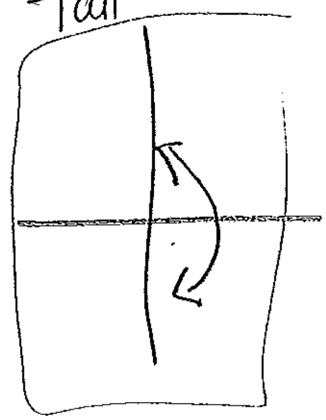
- **Inverse Tangent Function**

$\tan^{-1}$



[−3, 3] by [−2, 2]
(a)

[−4, 4] by [−2.8, 2.8]
(b)



Domain $\mathbb{R}$
Rng $(-\pi/2, \pi/2)$

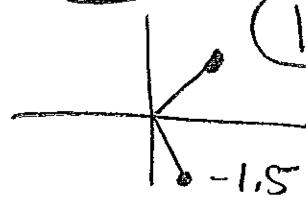
Example

Find the exact value of each expression without a calculator. ← Recall Domain $-1, 1$

(a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\pi/6$ $30°$

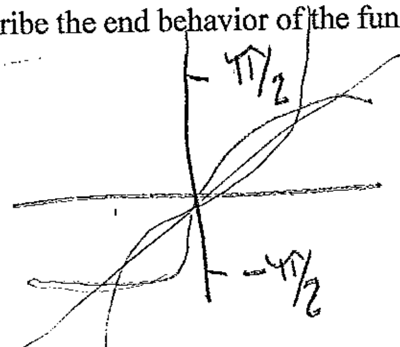
(b) $\tan^{-1} 1$ $\frac{\pi}{4}$

(c) $\cancel{\cos^{-1}}(\cancel{\cos}(-1.5))$ 1.5 1.5



Example

Describe the end behavior of the function $f(x) = tan^{-1}x$



$\lim_{x \to -\infty} f(x) = -\pi/2$

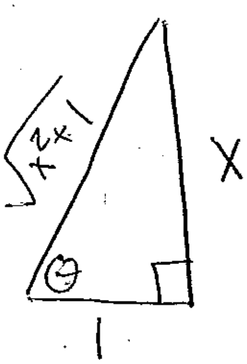
$\lim_{x \to \infty} f(x) = \pi/2$

Day 2 – Composing Functions – Combining Knowledge

① $\cos(\tan^{-1} 1)$ $\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$

② $\sin^{-1}(\cos \pi/3)$ $1/2$ $\pi/6$

Exploration #1 pg. 383



1) $\tan\theta = \frac{x}{1} = x$

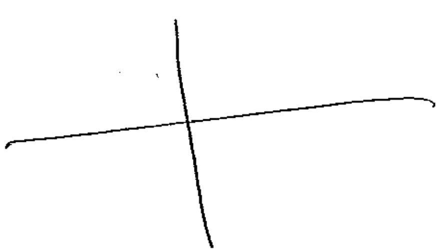
2) $\tan^{-1} x = \theta$

3) hypotenuse $= \sqrt{x^2+1}$

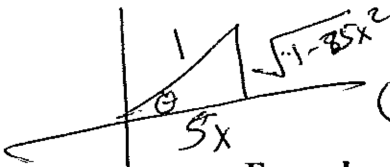
4) $\sin(\tan^{-1}(x))$ $\sin\theta = \frac{x}{\sqrt{x^2+1}}$

5) $\sec(\tan^{-1}(x))$ $\sec\theta = \frac{\sqrt{x^2+1}}{1} = \sqrt{x^2+1}$

6) If $x < 0$, are #4+5 valid?



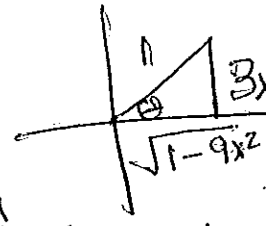
Yes,



$\cos 5x$

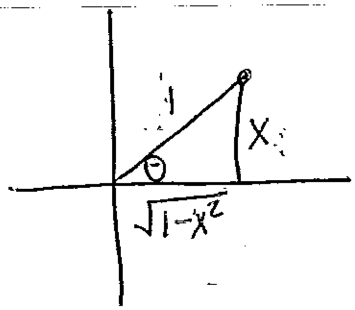
$\arcsin 3x$

$\sin \frac{3x}{1}$



$\sin^{-1} x/1$

Example
Compose each of the six basic trig functions with arcsin x and reduce the composite function to an algebraic expression involving no trig functions.



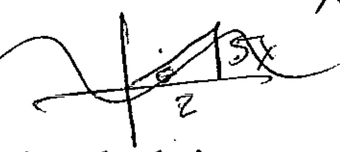
$\sin(\sin^{-1} x) = x$ $\qquad$ $\csc(\sin^{-1} x) = 1/x$

$\cos(\sin^{-1} x) = \sqrt{1-x^2}$ $\qquad$ $\sec(\sin^{-1} x) = 1/\sqrt{1-x^2}$

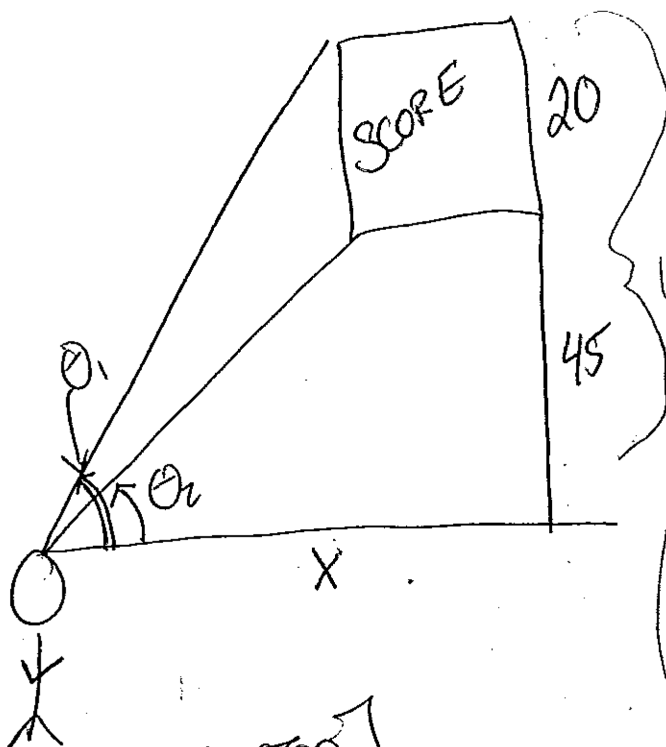
$\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1-x^2}}$ $\qquad$ $\cot(\sin^{-1} x) = \dfrac{\sqrt{1-x^2}}{x}$

Example – Real World

$\tan^{-1} \frac{5x}{2}$



The bottom of a 20-foot replay screen at Dodger Stadium is 45 feet above the playing field. As you move away from the wall, the angle formed by the screen at your eye changes. There is a distance from the wall at which the angle is the greatest. What is that distance?



65

$\tan \theta_1 = \frac{65}{x}$ $\qquad$ $\theta_1 = \tan^{-1}\frac{65}{x}$

$\tan \theta_2 = \frac{45}{x}$ $\qquad$ $\theta_2 = \tan^{-1}\frac{45}{x}$

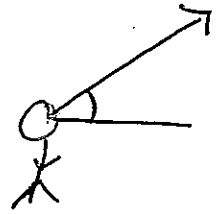
$$\boxed{\tan^{-1}\frac{65}{x} - \tan^{-1}\frac{45}{x}}$$

[0,500]
[0,20]

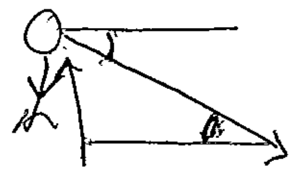
54ft

4.8 Solving Problems with Trig

- **Angle of Elevation** – horizontal up



- **Angle of Depression** –horizontal down

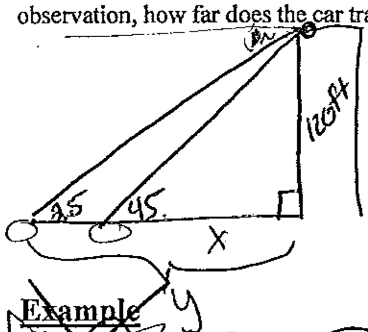


not here

alt. int. same

Example

From the top of the 120-ft tall Tallman Hall a man observes a car moving toward the building. If the angle of depression of the car changes from 25° to 45° during the period of observation, how far does the car travel?



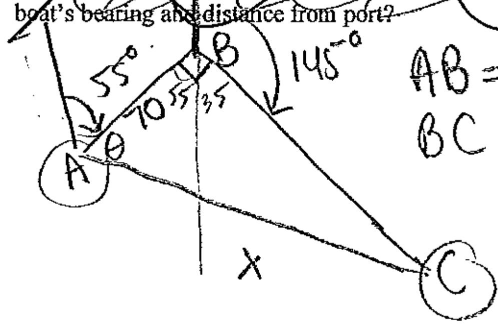
$\tan 45 = \frac{120}{x}$ $x = 120ft.$

$\tan 25 = \frac{120}{y}$ $y = 257.34$

$= 137.34 ft$

Example

A U.S. Coast Guard patrol boat leaves port and averages 35 knots (nautical mph) traveling for 2 hours on a course of 55° and then 3 hours on a course of 145°. What is the boat's bearing and distance from port?



Dist=rate x time

$AB = 35 \cdot 2 = 70$ miles

$BC = 35 \cdot 3 = 105$ miles

– Bearing is angle from North

$x^2 = 70^2 + 105^2$

$x = 126.2$ Dist.

– $\boxed{111.3° \quad 126 miles}$

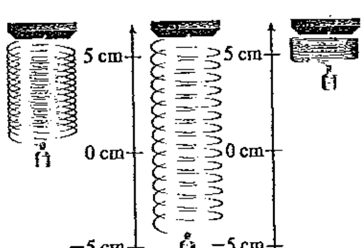
$\tan\theta = \frac{105}{70}$

$\theta = 56.3°$

Example

#35

A mass oscillating up and down on the bottom of a spring (assuming perfect elasticity and no friction or air resistance) can be modeled as harmonic motion. If the weight is displaced a maximum of 5 cm, find the modeling equation if it takes 3 seconds to complete one cycle. (See Figure 4.29.)



a sin bx   a cos bx

5

Period = 3

$\frac{\omega}{2\pi} = \frac{1}{3}$

$5 \sin \frac{2\pi}{3} x$

2π