Key

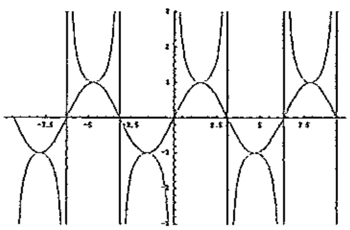
5.1 Fundamental Identities

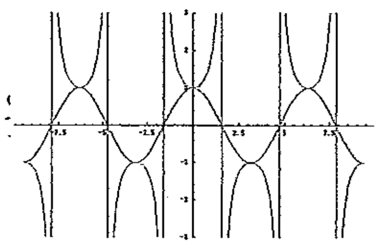
Basic Trigonometric Identities

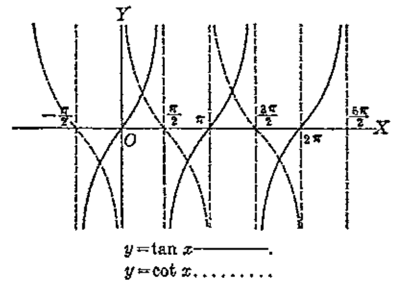
Reciprocal Identites

$\csc\theta = \dfrac{1}{\sin\theta}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad \cos\theta = \dfrac{1}{\sec\theta}$ $\qquad \tan\theta = \dfrac{1}{\cot\theta}$







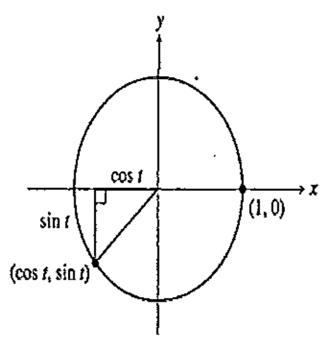
- Domain of Validity – #s for which both sides of eqn. are defined.

Pythagorean Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$



$$\frac{\cos^2\theta + \sin^2\theta = 1}{\cos^2\theta}$$

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta = 1}{\sin^2\theta}$$

$$\boxed{\cot^2\theta + 1 = \csc^2\theta}$$

#1-4 Geom.

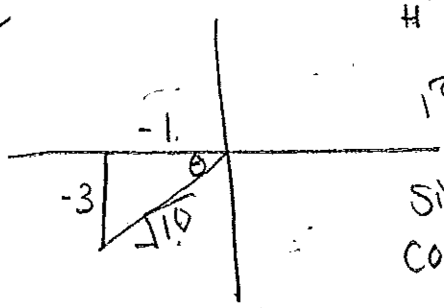
Ex

Example

$\frac{O}{A}$ both neg

Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = 3$ and $\cos\theta < 0$.

$\frac{3}{1}$ $\quad \frac{A}{H} \leftarrow$ neg



$1^2 + 3^2$

$\sin\theta = -3/\sqrt{10}$

$\cos\theta = -1/\sqrt{10}$

Algebraic $(\tan\theta)^2$

$1 + \tan^2\theta = \sec^2\theta$

$1 + (3)^2 = \sec^2\theta$

$10 = \sec^2\theta$

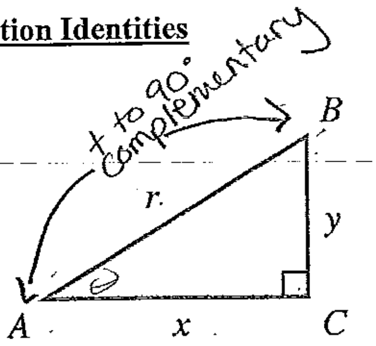
$\pm\sqrt{10} = \sec\theta$

$-\sqrt{10}$ b/c $\sec\theta < 0$

$\cos\theta = -1/\sqrt{10}$

$\sin\theta = 3/\sqrt{10}$

Cofunction Identities



Angle A: $\boxed{\sin A = \frac{y}{r}}$ $\quad \tan A = \frac{y}{x} \quad \sec A = \frac{r}{x}$

$\cos A = \frac{x}{r} \quad \cot A = \frac{x}{y} \quad \csc A = \frac{r}{y}$

Angle B: $\sin B = \frac{x}{r} \quad \tan B = \frac{x}{y} \quad \sec B = \frac{r}{y}$

$\boxed{\cos B = \frac{y}{r}}$ $\quad \cot B = \frac{y}{x} \quad \csc B = \frac{r}{x}$

sin A & cos B?

tan A & cot B? same ratios

sec A & csc B?

90° ≠

sin 30 = cos 60

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta \cdot \frac{x}{r}$ $\qquad \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$

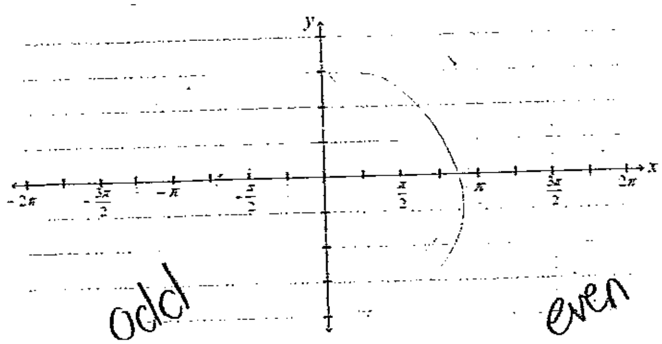
$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$ $\qquad \cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

$\sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$ $\qquad \csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$

So $\sin\left(\frac{\pi}{2}-60\right)=\cos 60$

$\sec\left(\frac{\pi}{2}-10\right)=\csc 80$

Odd-Even Identities



odd $\qquad$ even $\qquad$ odd

$\sin(-x)=-\sin x \qquad \cos(-x)=\cos x \qquad \tan(-x)=-\tan x$

$\csc(-x)=-\csc x \qquad \sec(-x)=\sec x \qquad \cot(-x)=-\cot x$

Example

#5-8 If $\cos\theta = .34$, find $\sin\left(\theta-\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}-\theta\right)$

$\sin\left(\theta-\frac{\pi}{2}\right) = -\sin\left(\pi/2-\theta\right) = -\cos\theta$

$= \boxed{-.34}$

Ex If $\tan(\pi/2-\theta) = -5.32$ find $\cot\theta$ $\quad -5.32$

#9-16

Example

Solve algebraically

$$\overset{x^3}{\cos^3 x} + \overset{x\,y^2}{\cos x \sin^2 x} = \overset{x}{\cos x}\overset{(x^2+y^2)}{(\cos^2 x + \sin^2 x)}$$

$$\cos x \cdot 1$$

$$\boxed{=\cos x}$$

~~Ex~~ #12 $\cot u \sin u$

$$\frac{\cos u}{\cancel{\sin u}} \cdot \frac{\cancel{\sin u}}{1}$$

$$\boxed{\cos u}$$

Stop

Recall

① $x^2+6x+5=0$

② $x^2-4=0$

③ $x^2+2x+1=0$

Example

Simplify the expression $(sec^2x + csc^2x) - (tan^2x + cot^2x)$.

$$(1+\tan^2 x + \cot^2 x + 1) - (\tan^2 x + \cot^2 x)$$

$$2 + \cancel{\tan^2 x} + \cancel{\cot^2 x} - \cancel{\tan^2 x} - \cancel{\cot^2 x}$$

2

Example

Simplify the expression $\frac{\cos x}{\cos x} \cdot \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x}$

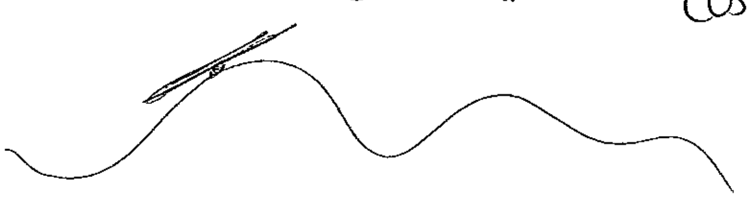
$$\frac{\cos x \sec x}{\cos x \sin x} - \frac{\sin^2 x}{\cos x \sin x} = \frac{\cancel{\cos x} \sec x - \sin^2 x}{\cos x \sin x}$$

$\cos x \cdot \frac{1}{\cos x}$

$$= \frac{1 - \sin^2 x}{\cos x \sin x}$$

$$= \frac{\cos^2 x}{\cos x \sin x} = \frac{\cos x}{\sin x}$$

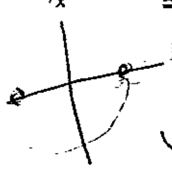
$$\boxed{= \cot x}$$



#51-56

Example

Find all values of x in the interval $[0, 2\pi)$ that solve $\sin^3 x/\cos x = \tan x$.



$$\frac{\sin^3 x}{\cos x} = \overset{\tan x}{\frac{\sin x}{\cos x}}$$

$$\sin^3 x = \sin x$$

$$\sin^3 x - \sin x = 0$$

$$\sin x(\sin^2 x - 1) = 0$$

want zero on one side

$x^2+2x+3=0$

$x(x^2-1)=0$

$\sin x = 0$ or $-\cos^2 x = 0$

$\sin x = 0$

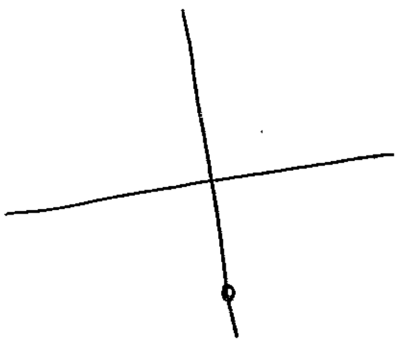
0 & π

Example

$2x^2 + x - 1$

Find all solutions to the trigonometric equation $2sin^2x + \sin x = 1$

$2\sin^2 x + \sin x - 1 = 0$



$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\qquad\qquad\qquad\qquad\quad -1 \quad -1$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$

$\frac{3\pi}{2} + 2\pi n$

$\frac{\pi}{6} + 2\pi n$

$\frac{5\pi}{6} + 2\pi n$

Example

Solve $\cos x = .37$ using a calculator.

$1.1918 + 2\pi n$

5.2 Proving Trigonometric Identities

We are now going to use proofs with Trigonometric Identities!

Recall:

Prove the algebraic identity:

$$\frac{1}{x} - \frac{1}{2} = \frac{2-x}{2x}$$

$$\frac{2}{2}\left(\frac{1}{x}\right) - \left(\frac{1}{2}\right)\frac{x}{x}$$

$$\frac{2}{2x} - \frac{x}{2x}$$

$$= \frac{2-x}{2x}$$

Example

Prove the identity: $\sin x - \csc x = -\cos x \cot x$.

$$\sin x - \csc x$$

$$= \sin x - 1/\sin x$$

$$= \frac{\sin^2 x - 1}{\sin x}$$

$$= \frac{-\cos^2 x}{\sin x} \quad \text{Pyth. Ident.}$$

$$= \left(\frac{\cos x}{\sin x}\right) \cdot -\cos x$$

$$= -\cos x \cot x$$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

Example

Prove the identity: $\frac{\sin t}{1+\cos t} = \frac{1-\cos t}{\sin t}$.

$$\frac{\sin t}{1+\cos t} \cdot \frac{1-\cos t}{1-\cos t}$$

$$= \frac{\sin t\,(1-\cos t)}{1-\cos^2 t}$$

$$= \frac{\sin t\,(1-\cos t)}{\sin^2 t}$$

$$= \frac{1-\cos t}{\sin t}$$

Example

Prove the identity: $\frac{\tan^2 x}{1+\sec x} = \frac{\sin^2 x}{\cos x + \cos^2 x}$.

$$\frac{\tan^2 x}{1+\sec x}$$

$$= \frac{\sec^2 x - 1}{1+\sec x}$$

$$= \frac{(\sec x + 1)(\sec x - 1)}{1+\sec x}$$

$$= \sec x - 1$$

Now Start RHS

$$\frac{\sin^2 x}{\cos x + \cos^2 x}$$

$$= \frac{1-\cos^2 x}{\cos x\,(1+\cos x)}$$

$$= \frac{(1-\cos x)(1+\cos x)}{\cos x\,(1+\cos x)}$$

$$= \frac{1-\cos x}{\cos x}$$

$= \frac{1}{\cos x} - 1$ $\sec x - 1$

Identities in Calculus

1. $\cos^3 x = (1-\sin^2 x)(\cos x)$
2. $\sec^4 x = (1+\tan^2 x)(\sec^2 x)$
3. $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$
4. $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$
5. $\sin^5 x = (1-2\cos^2 x + \cos^4 x)(\sin x)$
6. $\sin^2 x \cos^5 x = (\sin^2 - 2\sin^4 x + \sin^6 x)(\cos x)$

$\frac{3\pi}{2}$

5.3 Sum and Difference Identities

$-1 \quad 0 \quad 1$

1. Let $u = \pi$ and $v = \pi/2$. Find sin(u + v). Find sin(u) + sin(v). = 1

   Are they equal? No

   $1 \quad 2$

2. Let $u = 0$ and $v = 2\pi$. Find cos(u + v). Find cos(u) + cos(v).

   Are they equal? No

Cosine of a Sum or Difference

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

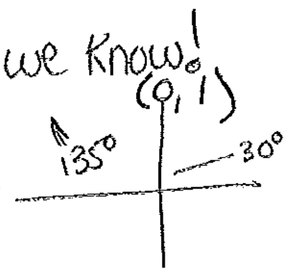
Khan Academy Video

(Note the sign switch in either case.)

Example

#1-10

Find the exact value of cos 105° without using a calculator. Break into Values we Know!



$\cos(135 - 30)$

$= \cos 135 \cos 30 + \sin 135 \sin 30$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{-\sqrt{6} + \sqrt{2}}{4}$

Example

#40

Prove the identities a) $\cos\left(\left(\frac{\pi}{2}\right) - x\right) = \sin x$ and b) $\sin\left(\left(\frac{\pi}{2}\right) - x\right) = \cos x$

a)
$\cos\frac{\pi}{2} \cos x + \sin\frac{\pi}{2} \sin x$

$0 \cdot \cos x + 1 \cdot \sin x$

$= \sin x$

b) $\sin\frac{\pi}{2} \cos x - \cos\frac{\pi}{2} \sin x$

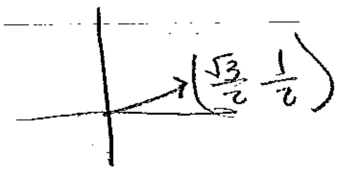
$1 \cdot \cos x - 0 \cdot \sin x$

$\cos x$

Sine of a Sum or Difference

$$\sin(u \pm v) = \sin u \cos v \pm \sin v \cos u$$

(Note that the sign does *not* switch in either case.)



#11-22 **Example**

Write each of the following expressions as the sine or cosine of an angle.

(a) sin 25° cos 5° + cos 25° sin 5° → a) $\sin(25+5) = \sin 30$ Diff ratios so Sum of Sine or Diff

(b) $\cos\frac{\pi}{4}\cos\frac{\pi}{5} + \sin\frac{\pi}{4}\sin\frac{\pi}{5}$ → Same so cos

(c) sin 3*x* sin 5*x* − cos 3*x* cos 5*x*

b) $\cos\left(\frac{4\pi}{4} - \frac{4\pi}{5}\right)$

$\cos \frac{5\pi}{20} - \frac{4\pi}{20}$

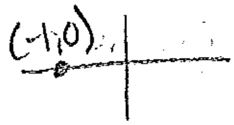
$\cos \frac{\pi}{20}$

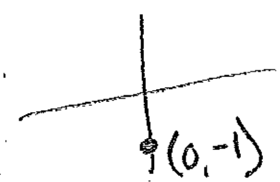
c) Same so cos

$\cos(3x+5x)$

$\cos 8x$

Example





#40 Prove the reduction formulas.

(a) $\sin(\pi - x) = \sin x$

(b) $\cos\left(x - \frac{3\pi}{2}\right) = -\sin x$

a) $\sin(\pi - x)$

$\sin\pi\cos x - \cos\pi\sin x$

$0 \cdot \cos x - -1 \sin x$

$= 1 \sin x$

b) $\cos x \cos\frac{3\pi}{2} + \sin x \sin\frac{3\pi}{2}$

$\cos x \cdot 0 + \sin x \cdot -1$

$= -\sin x$

Tangent of a Difference or Sum

$$\tan(u \pm v) = \frac{\sin(u \pm v)}{\cos(u \pm v)} = \frac{\sin u \cos v \pm \sin v \cos u}{\cos u \cos v \mp \sin u \sin v}$$

or

$$\boxed{\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}}$$

Example

#44-45

Express f(x)= 2 sin x + 5 cos x as a sinusoid in the form f(x)= a sin (bx + c)

OR Graph

Sum of Sin Formula $a \sin(bx + c)$

$= a(\sin bx \cos c + \cos bx \sin c)$

$2\sin x + 5\cos x = (a\cos c)\sin bx + (a\sin c)\cos bx$

$b = 1 \quad a\cos c = 2 \quad a\sin c = 5$

Turn into Trig Ident. so easier

$(a\cos c)^2 + (a\sin c)^2 = 2^2 + 5^2$

$a^2\cos^2 c + a^2\sin^2 c$

$a^2(\cos^2 c + \sin^2 c) = 29$

$a^2 \cdot 1 = 29$

$a^2 = 29$

$a = \pm\sqrt{29}$

so $\cos c = \frac{2}{\sqrt{29}}$

$\sin c = \frac{5}{\sqrt{29}}$

must be pos arcsin + arccos

$\boxed{5.39\sin(x + 1.2)}$

So $a\sin(bx + c)$

$\sqrt{29}\sin\left(x + \cos^{-1}\frac{2}{\sqrt{29}}\right)$

or

$\sqrt{29}\sin\left(x + \sin^{-1}\frac{5}{\sqrt{29}}\right)$

5.4 Multiple-Angle Identities

$\sin 2u = 2\sin u\cos u$

$$\cos 2u = \begin{cases} \cos^2 u - \sin^2 u \\ 2\cos^2 u - 1 \\ 1 - 2\sin^2 u \end{cases}$$

$$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$$

$$\sin^2 u = \frac{1-\cos 2u}{2}$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\tan^2 u = \frac{1-\cos 2u}{1+\cos 2u}$$

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos u}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos u}{2}}$$

$$\tan\frac{u}{2} = \begin{cases} \pm\sqrt{\frac{1-\cos u}{1+\cos u}} \\ \frac{1-\cos u}{\sin u} \\ \frac{\sin u}{1+\cos u} \end{cases}$$

Example

Prove the identity: $\cos 2u = \cos^2 u - \sin^2 u$.

$\cos(u+u) = \cos u \cos u - \sin u \sin u$

$\cos^2 u - \sin^2 u$

-Find exact value

$\cancel{\cos 120°}$

Example

Prove the identity $\cos 6x = 2\cos^2 3x - 1$.

$\cos 2(3x) = 2\cos^2 3x - 1$ Substitution

Example

Rewrite $\cos^4 x$ in terms of trigonometric functions with no power greater than 1.

$(\cos^2 x)^2 = \left(\frac{1+\cos 2x}{2}\right)^2$ By Power Red Ident.

$\frac{1+2\cos 2x+\cos^2 2x}{4}$ FOIL

$\frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\cos^2 2x$

$\frac{1}{4}\left(\frac{1+\cos 4x}{2}\right)$

$\frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{8}+\frac{1}{8}\cos 4x$

$=\frac{1}{8}(3+4\cos 2x+\cos 4x)$

#31-70

Example

Use half-angle identities to find an exact value without a calculator $\sin(5\pi/12)$

$\sin\frac{5/6\pi}{2}$ ← 75°

$\pm\sqrt{\frac{1-(-\sqrt{3}/2)}{2}}$ $=\pm\sqrt{\frac{1-\cos(5/6\pi)}{2}}$

$\pm\sqrt{\frac{1}{2}\left(1+\frac{\sqrt{3}}{2}\right)}$

$\pm\sqrt{\frac{1--\frac{\sqrt{3}}{2}}{2}}$ $\left(1+\frac{\sqrt{3}}{2}\right)2$ $\sqrt{\frac{2+\sqrt{3}}{2}}$ $\frac{1}{2}\sqrt{2+\sqrt{3}}$

$2\cos x = 0$ $\quad 2\sin x + 1 = 0$ Recall factoring $(x-1)(x+1) = 0$

$\sin x - 1 = 0 \quad \sin x + 1 = 0$

Example

Solve algebraically in the interval $[0, 2\pi)$: $\cos 2x = \sin x$.

#5-10

$\cos 2x = \sin x$

Ident. $\{$ $\cos^2 x - \sin^2 x = \sin x$

$1 - \sin^2 x - \sin^2 x = \sin x$

$1 - 2\sin^2 x - \sin x = 0$

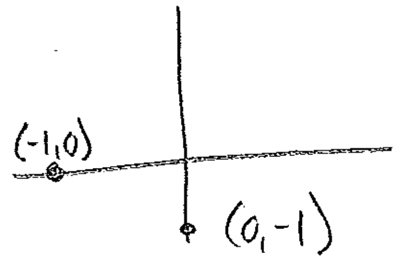
So easier to factor $-1x$ $\quad (-2\sin^2 x - \sin x + 1)$

$2\sin^2 x + \sin x - 1$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$\pi/6$ & $\frac{5\pi}{6}$ $\quad \frac{3\pi}{6} = \frac{\pi}{2}$



Example

Solve $\cos^2 x = (\sin(x/2))^2$.

$\cos^2 x - (\sin(x/2))^2$ (3 intersections)

$\cos^2 x = \frac{1-\cos x}{2}$

$2\cos^2 x = 1 - \cos x$

$2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1)$

$\cos x = \frac{1}{2} \quad \cos x = -1$

$\pi/3, \frac{5\pi}{3} \quad \pi$

Solve Alg. #6

$\sin 2x = \sin x$

$\sin 2x - \sin x = 0$

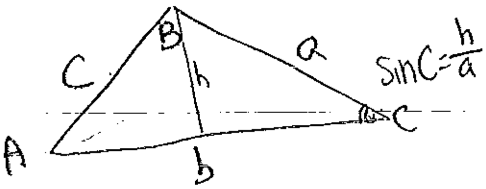
$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{2}$

$0, \pi, \pi/3, \frac{5\pi}{3}$

5.5 The Law of Sines



base height

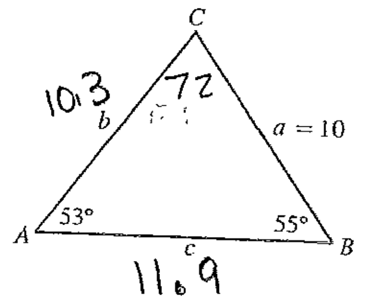
$$\text{Area} = \left(\tfrac{1}{2} ba\sin C = \tfrac{1}{2} bc\sin A = \tfrac{1}{2} ac\sin B\right)2$$

$$\frac{ba\sin C = bc\sin A = ac\sin B}{abc}$$

$$\boxed{\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{Law of Sines}}$$

Example

Solve ΔABC given that $\angle A = 53°$, $\angle B = 55°$, and $a = 10$.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 53}{10} = \frac{\sin 55}{b}$$

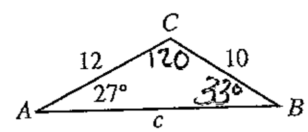
$$b\sin 53 = 10\sin 55 \qquad b = 10.3$$

$$\frac{\sin 72}{c} = \frac{\sin 53}{10}$$

$$c = 11.9$$

Example

Solve ΔABC given that $a = 10$, $b = 12$, and $\angle A = 27°$.



$$\frac{\sin 27}{10} = \frac{\sin B}{12}$$

$$12\sin 27 = 10\sin B$$

$$\frac{12\sin 27}{10} = \sin B$$

$$33°$$

$$\frac{\sin 27}{10} = \frac{\sin 120}{c}$$

$$10\sin 120 = c\sin 27$$

$$c = 19.1$$

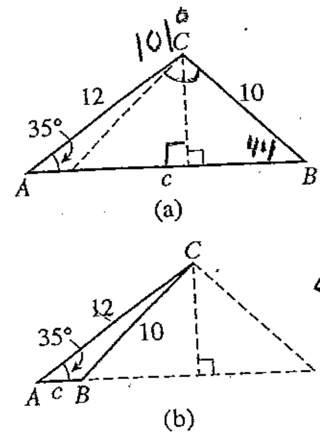
\# of Triangles

AAS, ASA = 1 Δ

SSA = 1, 0, 2 b/c calc only knows acute angles

Example

Solve ΔABC given that $a = 10$, $b = 12$, and $\angle A = 35°$.



Both Possible

$$\frac{\sin 35}{10} = \frac{\sin B}{12}$$

$$\frac{12 \sin 35}{10} = \frac{\cancel{10} \sin B}{\cancel{10}}$$

$B = 43.5$

$\approx 44°$

$C = 101$

$A = 35$

$$\frac{\sin 101}{c} = \frac{\sin 35}{10}$$

$10 \sin 101 = \sin 35 \cdot c$

$c = 17.1$

$a = 10$

$b = 12$

Angle B = 180 − 44

b/c obtuse

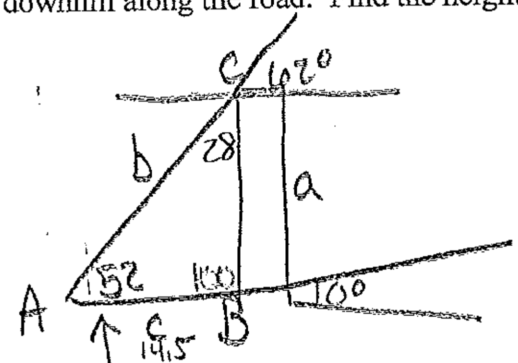
Option B

$A = 35$	$a = 10$
$B = 136°$	$b = 12$
$C = 9°$	$c = 2.7$

$$\frac{\sin 9}{c} = \frac{\sin 35}{10}$$

Example

A road slopes 10 degrees above the horizontal, and a vertical telephone pole stands beside the road. The angle of elevation of the Sun is 62 degrees, and the pole casts a 14.5 foot shadow downhill along the road. Find the height of the telephone pole.



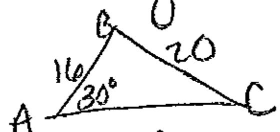
Elevation − Slope of Rd

$$\frac{\sin 52}{a} = \frac{\sin 28}{14.5}$$

$14.5 \sin 52 = a \sin 28$

24.3 ft

Ex How many Δs

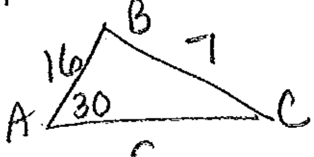
(1) 

$$\frac{\sin 30}{20} = \frac{\sin C}{16}$$ $\sin C = .4$ pos in 1 & 4 but

$C = 24°$

$156 + 30 > 180$

so 1 Δ

(2) 

$$\frac{\sin 30}{7} = \frac{\sin C}{16}$$ $\sin C = 1.1$ $\boxed{\text{Not poss}}$

(3) 

$$\frac{\sin 30}{10} = \frac{\sin B}{16}$$ $\sin B = .8$

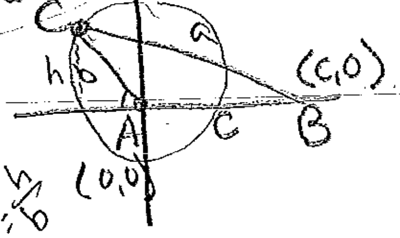
$B = 53.13$ or $B = 127$ b/c 127 ...

$\boxed{2 \Delta s}$

Factor stretched or shrink

$(b\cos A, b\sin A)$

5.6 The Law of Cosines



$\sin\theta = \frac{h}{b}$

Dist Formula

$a = \sqrt{(b\cos A - c)^2 + (b\sin A - 0)^2}$

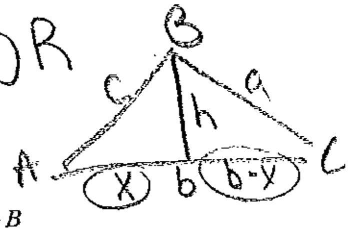
$a^2 = b^2\cos^2 A - 2bc\cos A + c^2 + b^2\sin^2 A$

$a^2 = b^2(\cos^2 A + \sin^2 A) - 2bc\cos A + c^2$

$a^2 = b^2 - 2bc\cos A + c^2$

$a^2 = b^2 + c^2 - 2bc\cos A$

OR



$\cos A = \frac{x}{c}$

$x^2 + h^2 = c^2$

$h^2 + (b-x)^2 = a^2$

So $h^2 = c^2 - x^2$

$c^2 - x^2 + (b-x)^2 = a^2$

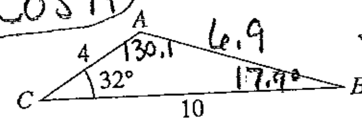
$c^2 - \cancel{x^2} + b^2 - bx + \cancel{x^2} = a^2$

$c^2 + b^2 - bx = a^2$

$c^2 + b^2 - b \cdot \cos A\, c = a^2$

Example

Solve ΔABC given that $a = 10$, $b = 4$, and $C = 32°$.



$c^2 = 4^2 + 10^2 - 2 \cdot 4 \cdot 10 \cos 32$

$c^2 = 48.15$

$c = 6.9$

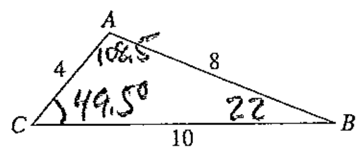
Now can use LoS or LoC

$\frac{\sin 32}{6.9} = \frac{\sin B}{4}$

$B = 17.9$

Example

Solve ΔABC given that $a = 10$, $b = 4$, and $c = 8$.



$8^2 = 4^2 + 10^2 - 2 \cdot 4 \cdot 10 \cos C$

$64 = 116 - 80\cos C$

$-52 = -80\cos C$

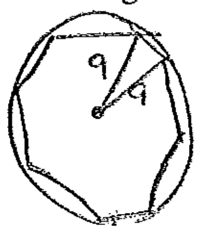
$.65 = \cos C$

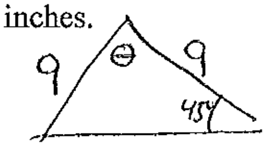
$C = 49.5$

$\frac{\sin 49.5}{8} = \frac{\sin B}{4}$

Recall:

Find the area of a regular octagon inscribed inside a circle of radius 9 inches.





$\frac{1}{2} bc \sin A$

$\frac{1}{2} \cdot 9 \cdot 9 \sin 45$

$= 40.5 \cdot \frac{\sqrt{2}}{2}$

$\times$ 8 Triangles

$\approx 229 \text{ in}^2$

Heron's Formula

Let a, b, and c be the sides of ∇ABC, and let s denote the **semiperimeter** $(a+b+c)/2$. Then the area of ∇ABC is given by

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Example

Find the area of a triangle with sides 13, 17, and 20.

$S = (13+17+20)/2 = 25$

$\text{Area} = \sqrt{25(25-13)(25-17)(25-20)}$

$= \sqrt{25 \cdot 12 \cdot 8 \cdot 5}$

$\sqrt{50} \approx \cancel{u^2}$

Apply: Video