7.1 Solving Systems

Recall you have learned 4 methods to solve systems of equations they are

1) Graphing
2) Substitution
3) Elimination.
4) matrix

Method 1
Solve the System

Graphing

$y = x^2$
$y - 9 = 0$
$+9 \quad +9$
$y = 9$

x y
(-3, 9)
(3, 9)

2nd Trace Intersect

Method 2
Solve the System

Subst.

$2x + 2y = 100$
$xy = 300$

$\frac{2y}{2} = \frac{100 - 2x}{2}$
$y = 50 - 1x$

$x(50 - x) = 300$
$50x - x^2 = 300$
$-x^2 + 50x - 300 = 0$

(6.972, 43.028) (43.027, 6.973)

Method 3
Solve the system

$2x + y = 10$
$-2(x - 2y = -5)$
$-2x + 4y = 10$
$2x + y = 10$
$5y = 20$
$y = 4$

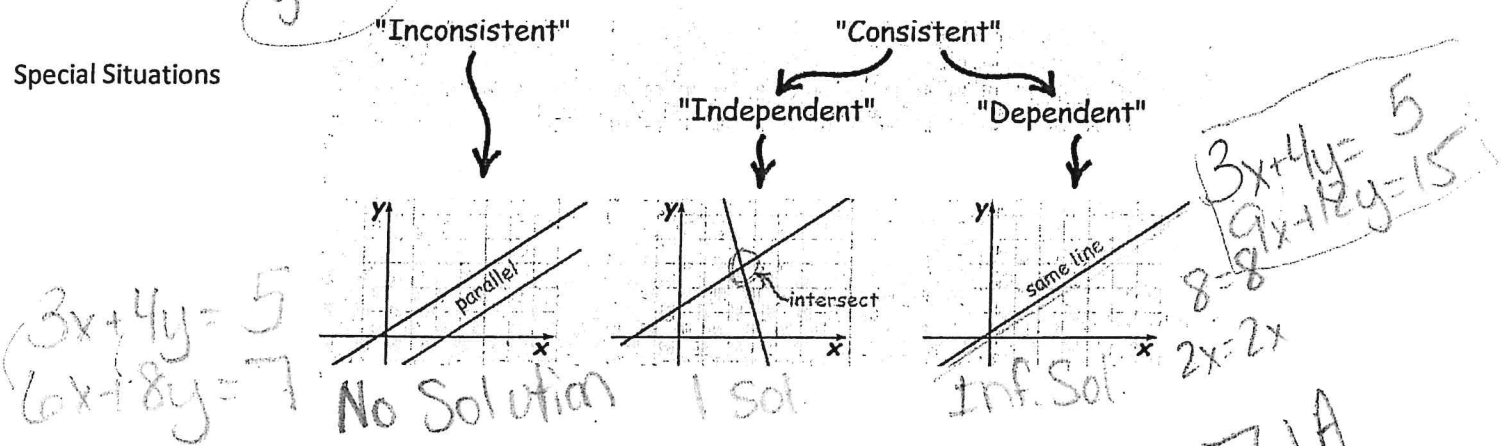
$2x + 4 = 10$
$2x = 6$
$x = 3$

(3, 4)

$-2(x - 3y = -2)$
$2x - 6y = 4$

$-2x + 6y = 4$
$2x - 6y = 4$
$0 = 8$
No Solution!

Special Situations



$3x + 4y = 5$
$6x + 8y = 7$
No Solution

1 Sol.

Inf. Sol.

$3x + 4y = 5$
$9x + 12y = 15$
$8 = 8$
$2x = 2x$

7.1A

7.1 Day 2 Apply Systems

- Break Even Point – where start to make profit
- Equilibrium Price – where supply & demand meet

Tips: line up the units to make it easier to sort out

Example

CHEMISTRY How many liters of 15% acid and 33% acid should be mixed to make 40 liters of 21% acid solution?

% $.15x + .33y = .21(40)$

Amt $-.15(x + y = 40)$

$-.15x - .15y = -6$

$.18y = 2.4$

$\frac{.18y}{.18} = \frac{2.4}{.18}$

$y = 13.3$

$x + 13\frac{1}{3} = 40$

$.15x + .33(40 - x) = .21$

$13\frac{1}{3}$ of 33%

$26\frac{2}{3}$ of 15%

Example

SHOPPING Two stores are having a sale on T-shirts that normally sell for \$20. Store S is advertising an s percent discount, and Store T is advertising a t dollar discount. Rose spends \$63 for three T-shirts from Store S and one from Store T. Manny spends \$140 on five T-shirts from Store S and four from Store T. Find the discount at each store.

$3S + T = 63) -4$

$5S + 4T = 140$

$-12S - 4T = -252$

$\frac{-7S}{-7} = \frac{-112}{-7}$

$S = 16$

Costs \$

$3(16) + T = 63$

$48 + T = 63$

$-48 \quad -48$

$T = 15$

Store S $\frac{4}{20} = 20\%$ disc.

Store T $20 - 15 = \$5$ discount

7.2 Matrix Algebra

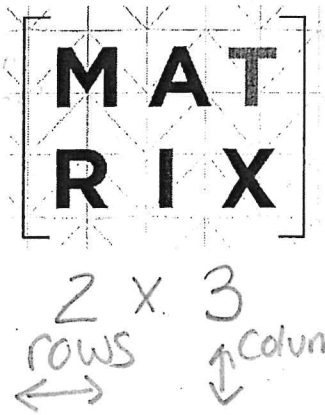
$A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$ 2×2

$B = \begin{bmatrix} 1 & 8 \\ 0 & -2 \end{bmatrix}$ 2×2 (2 rows, 2 columns)

Sum of a Matrix A + B $\begin{bmatrix} 3 & 12 \\ 3 & 3 \end{bmatrix}$

Difference of a Matrix A – B $\begin{bmatrix} 1 & -4 \\ 3 & 7 \end{bmatrix}$

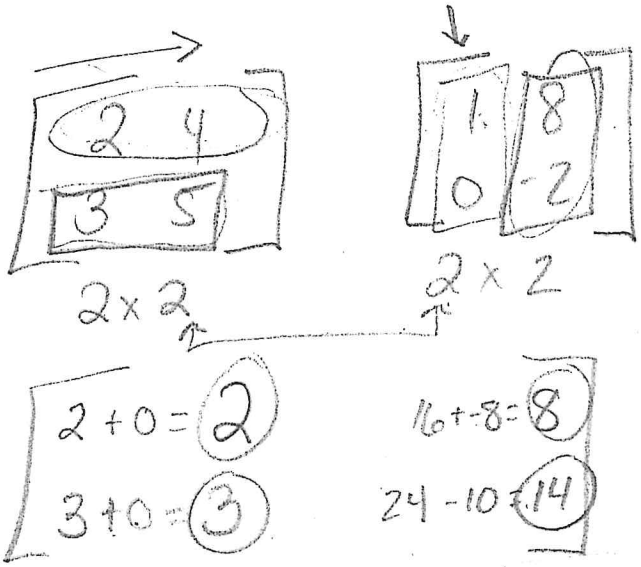
Scalar 3A $\begin{bmatrix} 6 & 12 \\ 9 & 15 \end{bmatrix}$



2 × 3

rows ↔ columns ↕

Matrix Multiplication AB



To solve a system you use the determinant of a matrix to solve the system.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example

Solve $\begin{matrix} 2x + 3y = 10 \\ 3x - y = 4 \end{matrix}$

$$\begin{bmatrix} 2 & 3 \\ 3 & -1 \end{bmatrix}^{-1}\begin{bmatrix} 10 \\ 4 \end{bmatrix}$$

$ad - bc$

(2, 2)

Solve $\begin{matrix} 2x - y = 10 \\ x - z = -1 \\ y + z = -9 \end{matrix}$

$$\begin{matrix} x & y & z \\ \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} & \begin{bmatrix} 10 \\ -1 \\ -9 \end{bmatrix} \\ 3 \times 3 & 3 \times 1 \end{matrix}$$

$x = 0$
$y = -10$
$z = 1$

Example

At the Pittsburgh zoo, children ride a train for 25 cents, adults pay $1, and senior citizens 75 cents. On a given day, 1400 passangers paid a total of $740 for the rides. There were 250 more children riders than all other riders. Find the number of children, adult, and senior riders.

C a S

$.25x + 1y + .75z = 740$

$x + y + z = 1400$

~~$x = 250 + y + z$~~

$x - y - z = 250$

$$\begin{bmatrix} .25 & 1 & .75 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}^{-1}\begin{bmatrix} 740 \\ 1400 \\ 250 \end{bmatrix}$$

$c = 825$
$a = 410$
$s = 165$

7.3 Multivariate Linear Systems (AKA back substitution)

(You learned how to solve these with matricies previously, bu tyou can also do them by hand!)

$x = 2$, $y = -1$, $z = 3$

#1-2

$x - 2y + z = 7$

$y - 2z = -7$

$z = 3$

$y - 2(3) = -7$

$y - 6 = -7$

$y = -1$

$x - 2(-1) + 3 = 7$

$x + 2 + 3 = 7$

$x + 5 = 7$

$x = 2$

Tips for completing Gaussian Elimination:

1) Interchange two equations if needed
2) Multiply (or divide) one of the equations by any nonzero real number
3) Add a multple of one equation to any other equation in the system.

Example

$-3(x - 2y + z = 7)$

$3x - 5y + z = 14$

$2x - 2y - z = 3$

$-3x + 6y - 3z = -21$

$3x - 5y + z = 14$

$y - 2z = -7$ Replace middle eqn

$-2(x - 2y + z = 7) \rightarrow -2x + 4y - 2z = -14$

$y - 2z = -7$

$2x - 2y - z = 3 \rightarrow 2x - 2y - z = 3$

$2y - 3z = -11$ Replace 3rd eqn.

$x - 2y + z = 7$

$-2(y - 2z = -7)$

$2y - 3z = -11$

$-2y + 4z = 14$

$2y - 3z = -11$

$z = 3$ Replace 3rd.

$x - 2y + z = 7$

$y - 2z = -7$

$x - 2(-1) + 3 = 7$

$x + 2 + 3 = 7$

$y - 2(3) = -7$

$x = 2$

$y = -1$

$z = 3$

Example

$-3(x + 2y - z = 3)$

$3x + 7y - 3z = 12$

$-2x - 4y + 3z = -5$

$-3x - 6y + 3z = -9$

$3x + 7y - 3z = 12$

$y = 3$ Replace 2nd eqn.

$(-2, 3, 1)$

$2(x + 2y - z = 3)$

$y = 3$

$-2x - 4y + 3z = -5$

$2x + 4y - 2z = 6$

$-2x - 4y + 3z = -5$

$z = 1$

$x + 2y - z = 3$

$y = 3$

$z = 1$

$x + 2(3) - 1 = 3$

$x + 5 = 3$

$x = -2$

Example

$-2(x - 2y + z = 8)$

$2x + y - 3z = -9$

$-3x + y + 3z = 5$

$-2x + 4y - 2z = -16$

$2x + y - 3z = -9$

$5y - 5z = -25$

Replace 2nd.

$3(x - 2y + z = 8)$

$5y - 5z = -25$

$-3x + y + 3z = 5$

$3x - 6y + 3z = 24$

$-3x + y + 3z = 5$

$-5y + 6z = 29$

$(2, -1, 4)$

$x - 2y + z = 8$

$5y - 5z = -25$

$-5y + 6z = 29$

Replace 3rd

$5y - 5z = -25$

$-5y + 6z = 29$

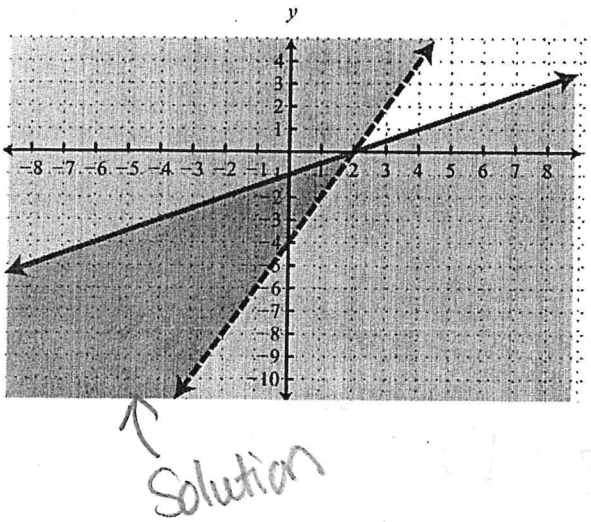
$z = 4$

$x - 2y + z = 8$

$5y - 5z = -25$

$z = 4$

7.5 Systems of Inequalities

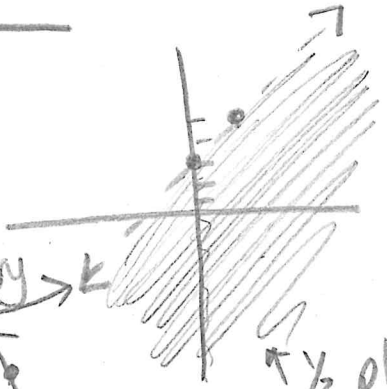


Boundary Lines

$<$ $>$ -----

$\leq$ $\geq$ ——

$y \leq 2x+3$



Example

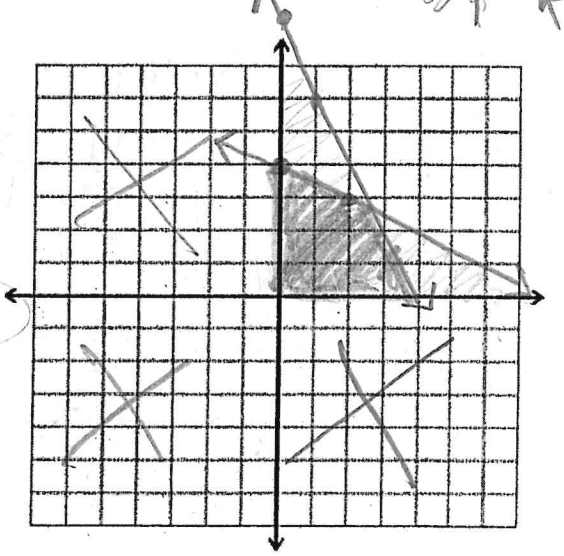
Solve the system of inequalities

$2x + y \leq 80$
$x + 2y \leq 80$
$x \geq 0$
$y \geq 0$

$y \leq 80-2x$

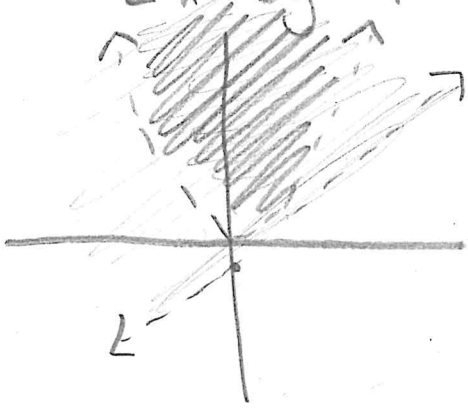
$\frac{2y}{2} \leq \frac{80-x}{2}$

$y \leq 40-\frac{1}{2}x$



Ex

$y \geq x^2$



$2x-3y<4$

$\frac{-3y}{-3} < \frac{4-2x}{-3}$

$y > -\frac{4}{3}+\frac{2}{3}x$

Ex

$y \leq \sin x$