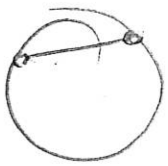
Key

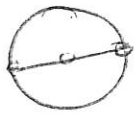
10.1 Circles and Circumference

Circles are named using the center of the circle

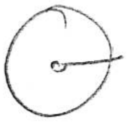
Chord – any segment with endpoints that are on the circle



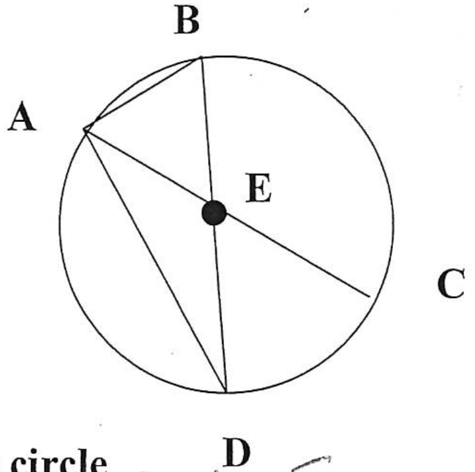
Diameter – a chord that passes through the center



Radius – any segment with endpoints that are the center and a point on the circle



Example



a. **Name the circle** Circle E

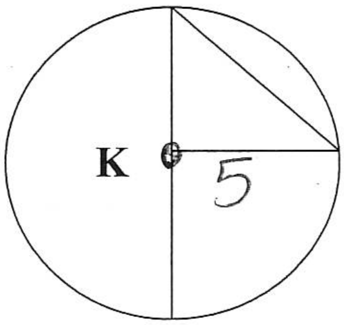
b. **Name a radius of the circle** $\overline{EC}$ $\overline{EA}$ $\overline{ED}$ $\overline{BE}$

c. **Name a chord of the circle** $\overline{AB}$ $\overline{AD}$ $\overline{AC}$ $\overline{BD}$

d. **Name a diameter of the circle** $\overline{AD}$ $\overline{AC}$

Circumference – the distance around a circle $2\pi r$ or $d\pi$

Example
Find the exact circumference of circle K.



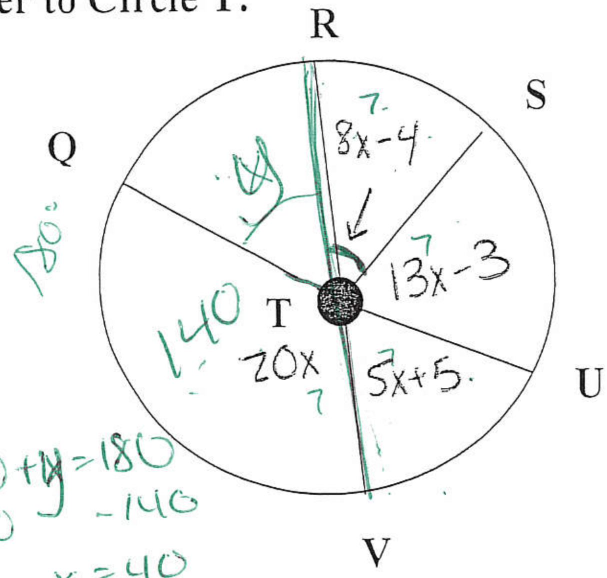
$5 \cdot 2\pi = 10\pi$

10.2 Angles and Arcs

- **Central Angle** – angle with center as the vertex.

360°

Example
Refer to Circle T.



$8x-4+13x-3+5x+5=180$

$26x-2=180$

$26x=182$

$x=7$

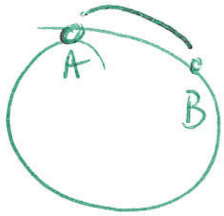
$140+x=180$

$-140 \quad -140$

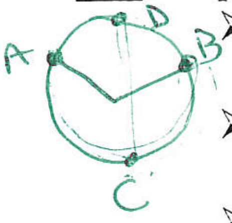
$x=40$

a) find $m\angle RTS$ 52°

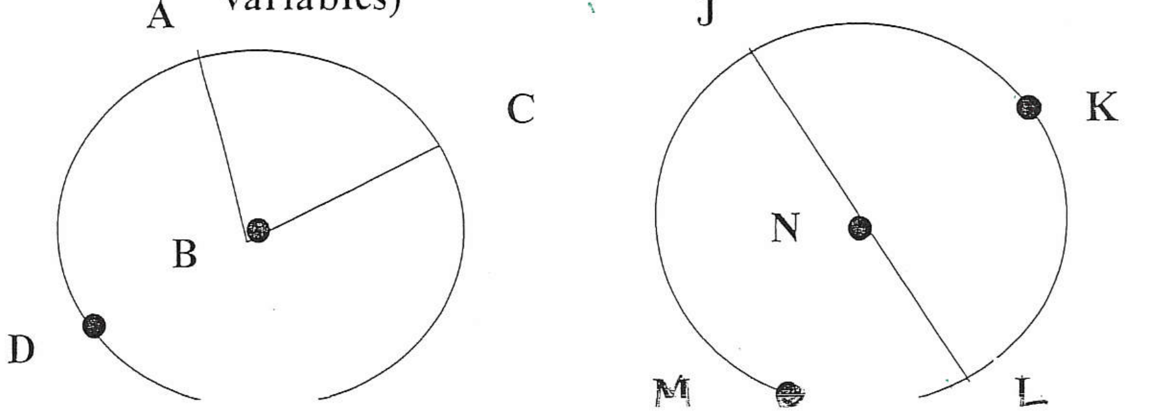
b) find $m\angle QTR$ 40°

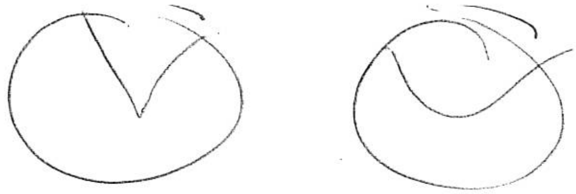


- **Arc** – a part of a circle defined by two endpoints $\overset{\frown}{AB}$



  - **Minor Arc** – same measure as central angle, less than 180° (denoted with two variables) $\overset{\frown}{DB}$ $\overset{\frown}{AD}$ $\overset{\frown}{AB}$
  - **Major Arc** – is 360 minus the minor arc, greater than 180° (denoted with three variables)
  - **Semicircle** – 180° (denoted with three variables)

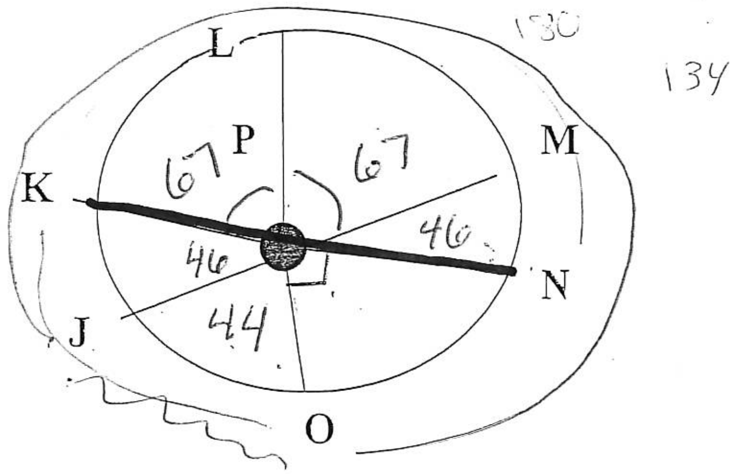




- **Theorem** – In the same or in congruent circles, two arcs are congruent iff their corresponding central angles congruent
  ≅

- **Arc Addition Postulate** – the measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

Example In circle P, $m\angle NPM = 46$, $\overline{PL}$ bisects $\angle KPM$ and $\overline{OP} \perp \overline{KN}$.



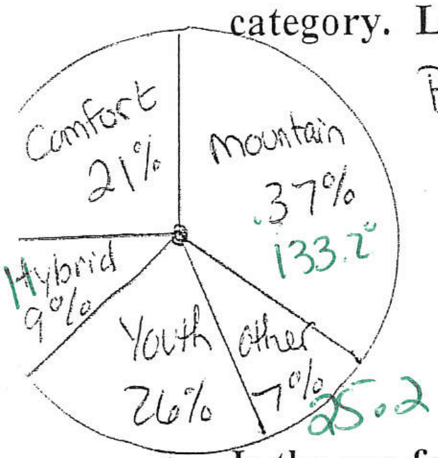
Find the measure of:

a) arc OK 90°
b) arc LM 67
c) arc JKO 316

Day 2

Example

Find the measurement of the central angle representing each category. List them from least to greatest.



Bicycles Bought In 2001
(by type)

Other 25.2°
Hybrid 32.4
Comfort 75.6
Youth 93.6
Mount. 133.2°

Is the arc for the wedge named YOUTH congruent to the arc for the combines wedges named OTHER and COMFORT?

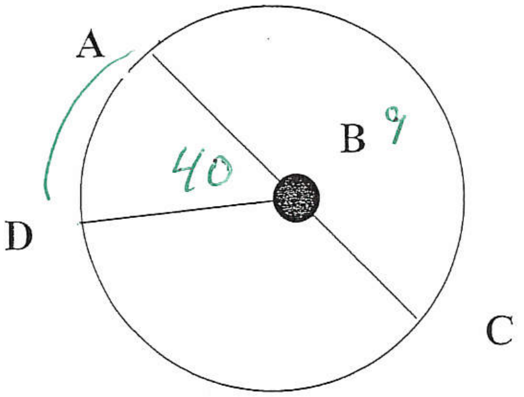
No $25.2 + 75.6 \neq 93.6$

Arc Length

L = $\frac{\text{Degree of Arc}}{360}$ X Circumference

Example

Given that AC = 9 and the measure of angle ABD = 40. Find the length of arc AD.



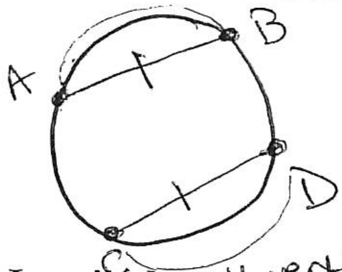
$\frac{40}{360} \cdot 9\pi$

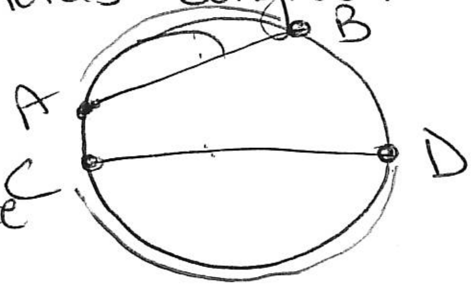
3.14

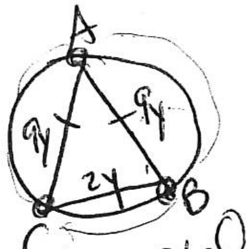
$C = 9\pi$

10.3 Arcs and Chords

- Arc – minor 2, major 3 part of circle
- Chord – segment w/ endpts on circle
- Theorem – in a circle two minor arc are congruent if and only if the chords congruent





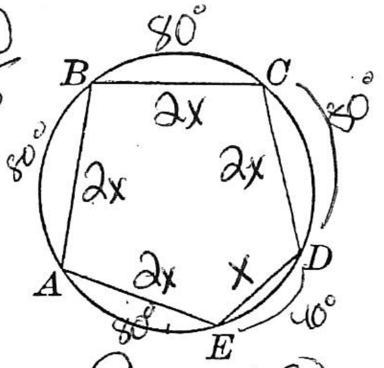


$20y = 360$

$\frac{20y}{20} = \frac{360}{20}$

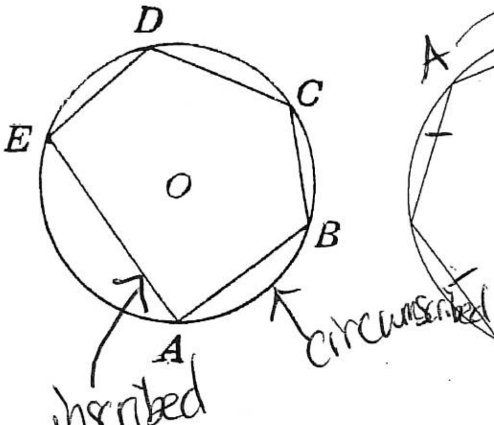
$y = 18$

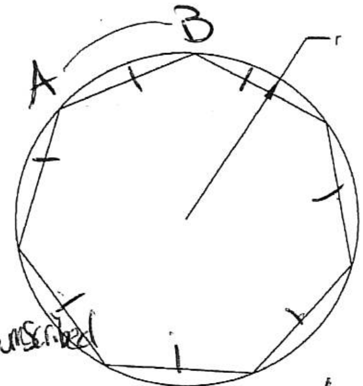
- Inscribed – all verticies are circle
- Circumscribed



$9x = 360$

$x = 40$





Find $m\widehat{AB}$

$360/7 = 51.4°$

Examples

Each <u>regular</u> polygon is inscribed in a circle. Determine the measure of each arc that corresponds to the side of the polygon.

1) octagon

$360/8 = 45°$

2) 15-gon

$360/15 = 24°$

3) 17-gon

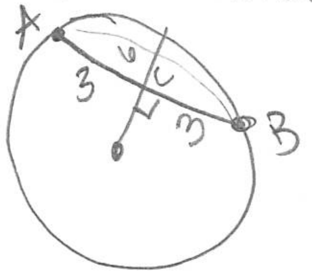
$360/17 = 21.17°$

4) nonagon

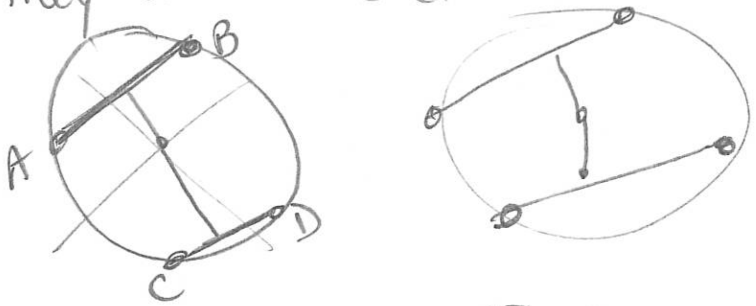
$360/9 = 40°$

10.3 Day 2

- Theorem – In a circle, if a diameter (or radius) is perpendicular to a chord, then chords will be bisected
  ½

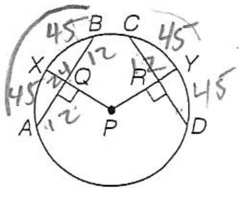


- Theorem – In a circle or in congruent circles, two chords are congruent iff they are same dist from center



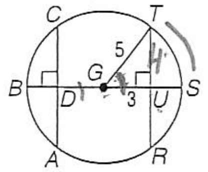
Examples

In ⊙P, $CD = 24$ and $m\overset{\frown}{CY} = 45$. Find each measure.



1. AQ 12
2. RC 12
3. QB 12
4. AB 24
5. $m\overset{\frown}{DY}$ 45°
6. $m\overset{\frown}{AB}$ 90°
7. $m\overset{\frown}{AX}$ 45°
8. $m\overset{\frown}{XB}$ 45°
9. $m\overset{\frown}{CD}$ 90°

In ⊙G, $DG = GU$ and $AC = RT$. Find each measure.



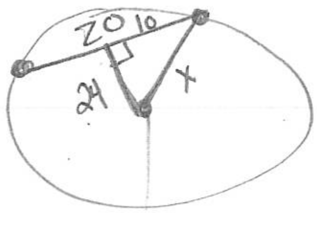
$3^2 + X^2 = 5^2$

10. TU 4
11. TR 8
12. $m\overset{\frown}{TS}$ 53.1
13. CD 4
14. GD 3
15. $m\overset{\frown}{AB}$ 53.1

SOH
CAH
TOA

16. A chord of a circle 20 inches long is 24 inches from the center of a circle. Find the length of the radius.

$X = 26$



$10^2 + 24^2 = x^2$

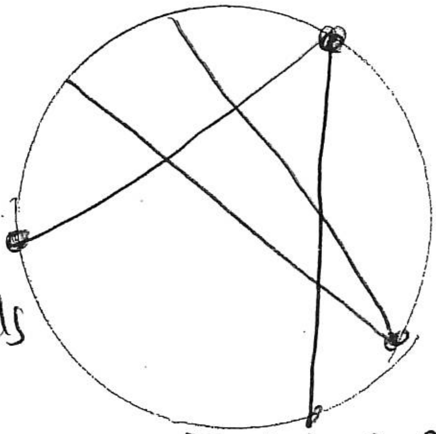
$\cos^{-1}$ $\cos^{-1}$

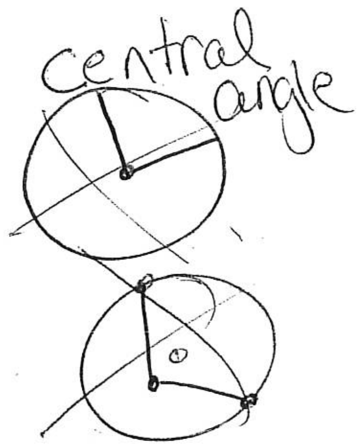
$\cos X = \frac{3}{5}$

53.1°

10.4 Inscribed Angles

- Inscribed Angle –
  - vertex on circle
  - sides chords

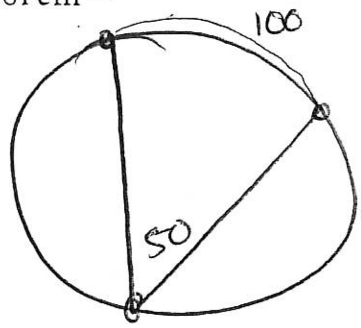




↖ intercepted arc

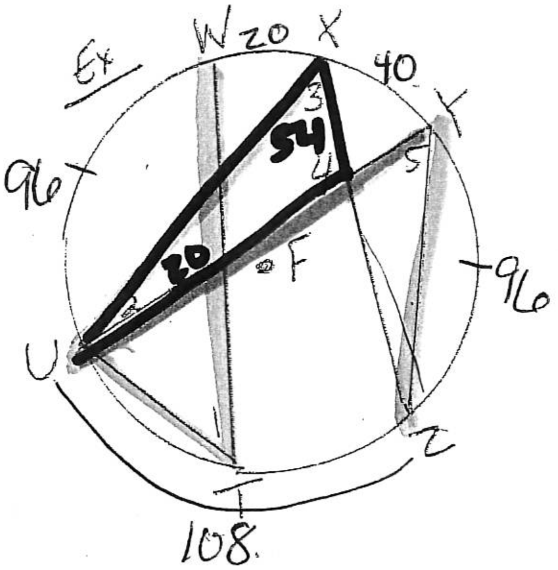
- Inscribed Angle Theorem –

Inscribed Angle = ½ arc



Example

Find the measure of the numbered angles given the following:



$m\,\widehat{WX} = 20$

$m\,\widehat{XY} = 40$

$m\,\widehat{UZ} = 108$

$m\,\widehat{UW} = m\,\widehat{YZ}$

$\angle 1 = \frac{96}{2} = 48°$

$\angle 2 = \frac{40}{2} = 20°$

$\angle 3 = \frac{108}{2} = 54°$

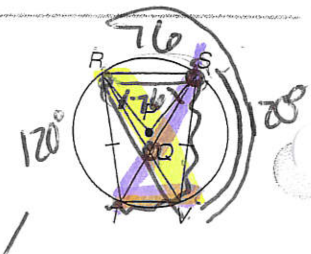
$* \angle 4 \quad 106°$

$\angle 5 = \frac{108}{2} = 54°$

$360 - 20 - 40 - 108 = \frac{192}{2}$

- Theorem – If two inscribed angles of a circle intercept congruent arcs of the same arc, then the angles are congruent.

Exercises



Use $\odot P$ for Exercises 1–10. In $\odot P$, $\overline{RS} \parallel \overline{TV}$ and $\overline{RT} \cong \overline{SV}$.

1. Name the intercepted arc for $\angle RTS$. $\widehat{RS}$

2. Name an inscribed angle that intercepts $\widehat{SV}$. $\angle SRV$ $\angle STV$

In $\odot P$, $m\widehat{SV} = 120$ and $m\angle RPS = 76$. Find each measure.

3. $m\angle PRS$ 52

4. $m\widehat{RSV}$ 196

5. $m\widehat{RT}$ 120°

6. $m\angle RVT$ 60°

7. $m\angle QRS$ 60°

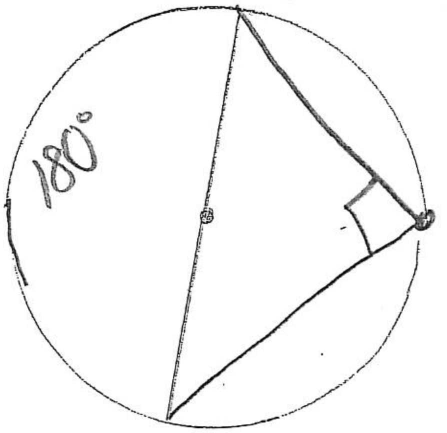
8. $m\angle STV$ 60°

9. $m\widehat{TV}$ 44°

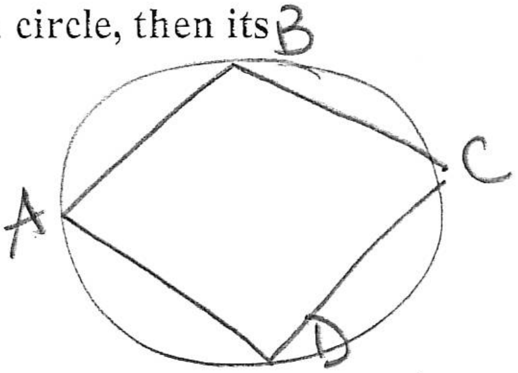
10. $m\angle SVT$ 98°

10.4 Inscribed Angles (Day 2)

- Theorem – if an inscribed angle intercepts a semicircle, the angle is a $90°$ angle.



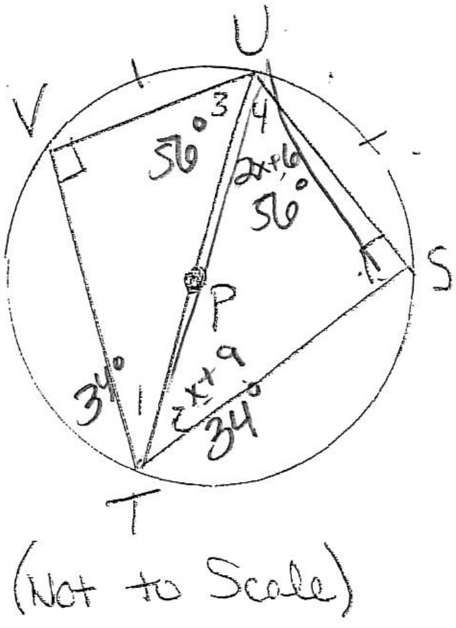
- Theorem – if a quadrilateral is inscribed in a circle, then its opposite angles are Supplementary
  (+ 180°)



$\angle A + \angle C = 180$

$\angle B + \angle D = 180$

Examples



Triangle TVU and TSU are inscribed in $\odot P$, $\overset{\frown}{VU} \cong \overset{\frown}{SU}$. Find the numbered angles if $m\angle 2 = x+9$ and $m\angle 4 = 2x+6$

$2x+6+x+9+90 = 180$

$3x+15+90 = 180$

$3x+105 = 180$

$3x = 75$

$x = 25$

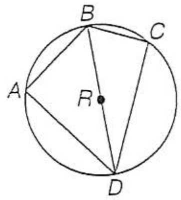
NAME ______________________________ DATE ____________ PERIOD _____

10-4 Study Guide and Intervention *(continued)*

Inscribed Angles

Angles of Inscribed Polygons

An **inscribed polygon** is one whose sides are chords of a circle and whose vertices are points on the circle. Inscribed polygons have several properties.



- If an angle of an inscribed polygon intercepts a semicircle, the angle is a right angle.
- If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

If $\widehat{BCD}$ is a semicircle, then $m\angle BCD = 90$.

For inscribed quadrilateral $ABCD$, $m\angle A + m\angle C = 180$ and $m\angle ABC + m\angle ADC = 180$.

Example In $\odot R$ above, $BC = 3$ and $BD = 5$. Find each measure.

a. $m\angle C$

$\angle C$ intercepts a semicircle. Therefore $\angle C$ is a right angle and $m\angle C = 90$.

b. CD

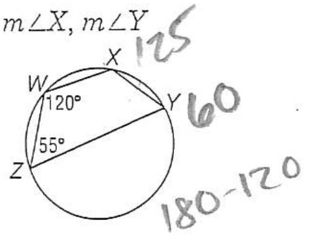
$\triangle BCD$ is a right triangle, so use the Pythagorean Theorem to find CD.

$$(CD)^2 + (BC)^2 = (BD)^2$$
$$(CD)^2 + 3^2 = 5^2$$
$$(CD)^2 = 25 - 9$$
$$(CD)^2 = 16$$
$$CD = 4$$

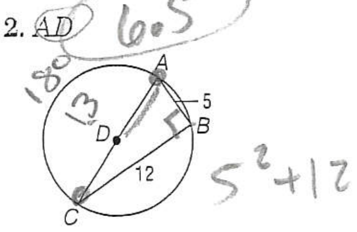
Exercises

Find the measure of each angle or segment for each figure.

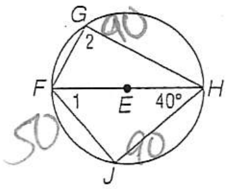
1. $m\angle X$, $m\angle Y$



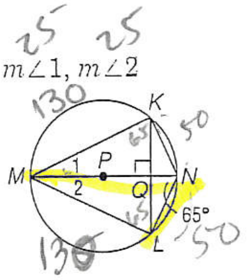
2. AD 60.5



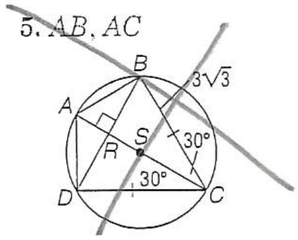
3. $m\angle 1$, $m\angle 2$



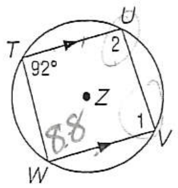
4. $m\angle 1$, $m\angle 2$ 25 25



5. ~~AB, AC~~



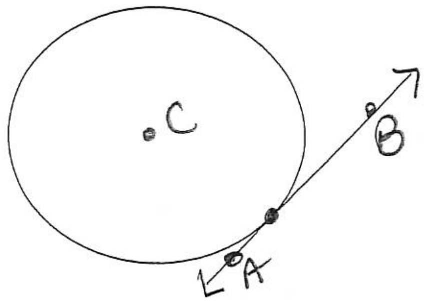
6. $m\angle 1$, $m\angle 2$

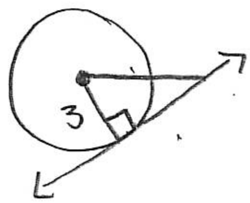


10.5 Tangents

A line tangent to a circle intersects the circle in EXACTLY one point.

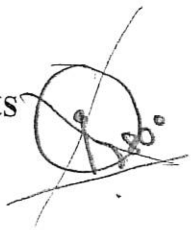
$\overleftrightarrow{AB}$ tangent to $\odot C$



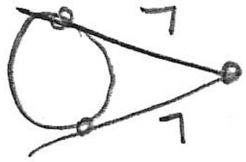


- Theorem – If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency.

- Theorem – If a line is perpendicular to a radius of a circle at its endpoint on the circle, then the line is tangent to the circle.

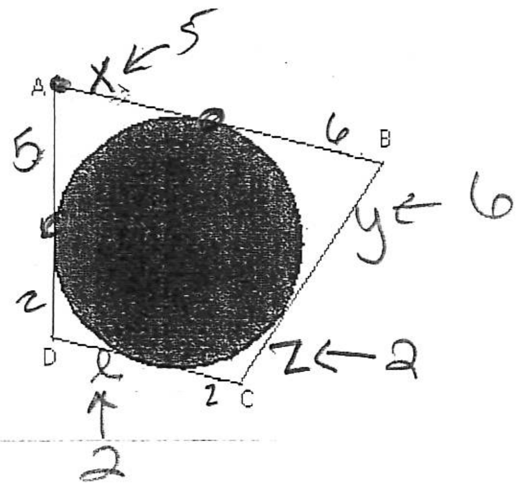


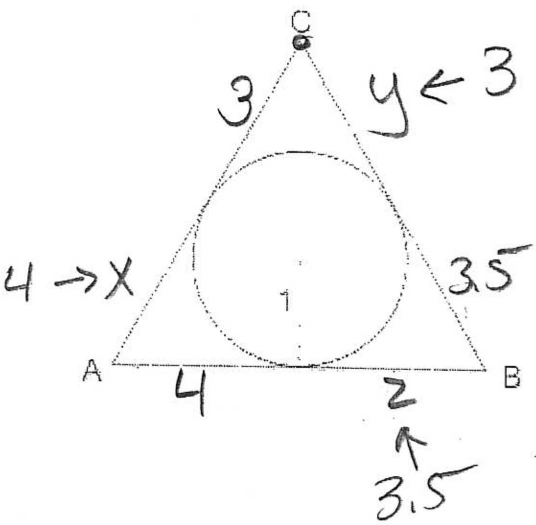
- Theorem – If two segments from the same exterior point are tangent to a circle, then they are congruent



- Circumscribed Polygons - polygons where every side is tangent to circle

Fig: 1





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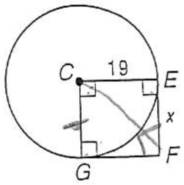
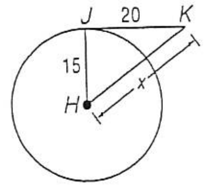
10-5 Study Guide and Intervention *(continued)*

Exercises

Find x. Assume that segments that appear to be tangent are tangent.

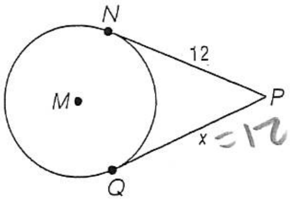
1.


$x = 19$

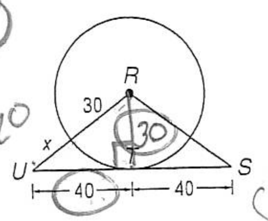
2.


$15^2 + 20^2 = x^2$

$x = 25$

3.


$x = 12$

4.


20

$30^2 + 40^2 = (30+x)^2$

$2500 = (30+x)(30+x)$

$= 900 + 30x + 30x + x^2$

$2500 = 900 + 60x + x^2$

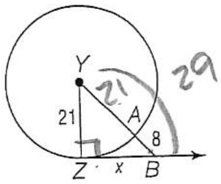
$1600 = 60x + x^2$

$x^2 + 60x - 1600 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

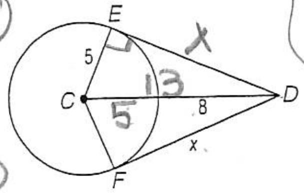
$\frac{-60 \pm \sqrt{10{,}000}}{2}$

$\frac{-60 \pm 100}{2}$

5.


$21^2 + x^2 = 29^2$

$x = 20$

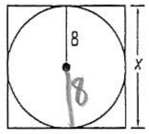
6.


$5^2 + x^2 = 13^2$

$25 + x^2 = 169$

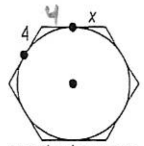
$x = 12$

Find x. Assume that segments that appear to be tangent are tangent.

1.


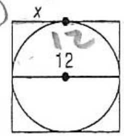
square

16

2.


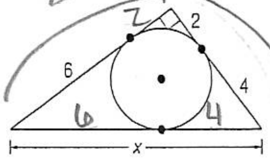
regular hexagon

4

3.


square

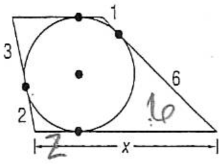
$x = 6$

4.


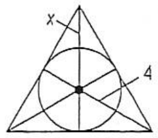
8

6

10

5.


8

6.


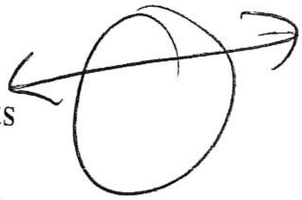
equilateral triangle

4

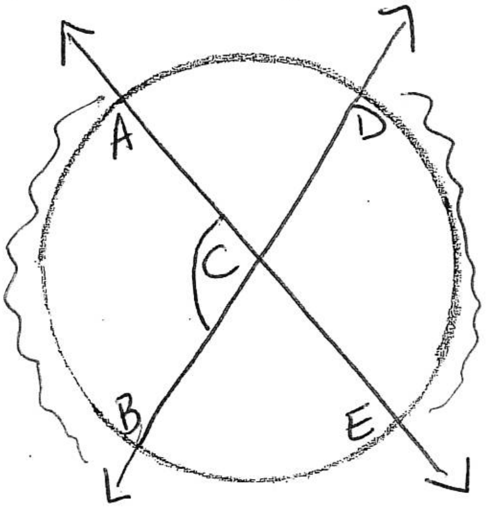
Lesson 10-5

10.6 Secants, Tangents, and Angle Measures

Secant – a line that intersects a circle in EXACTLY 2 points

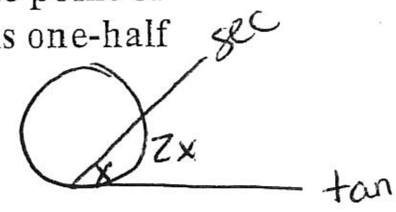


- Theorem – if two secants intersect in the interior of a circle, then the measure of an angle formed is ½ sum of the measure of the arc intercepted by the angle and its vertical angle



$\angle ACB = \frac{1}{2}(\overset{\frown}{AB} + \overset{\frown}{DE})$

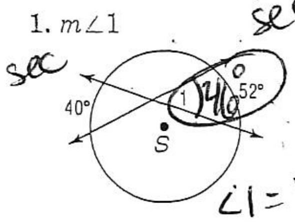
- Theorem – if a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one-half the measure of its intersected arc



Exercises

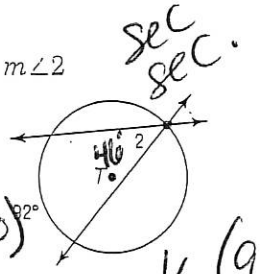
Find each measure.

1. $m\angle 1$



$\angle 1 = \frac{1}{2}(52+40)$

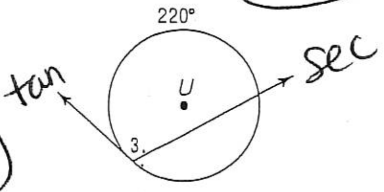
2. $m\angle 2$



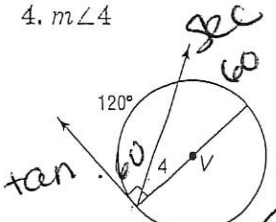
$\frac{1}{2}(92+0)$

3. $m\angle 3$

110°

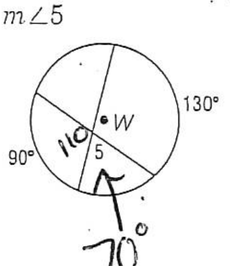


4. $m\angle 4$



30°

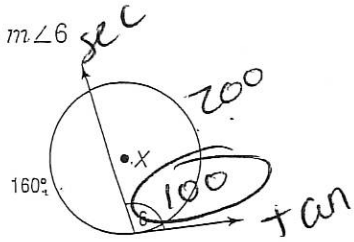
5. $m\angle 5$



sec
sec

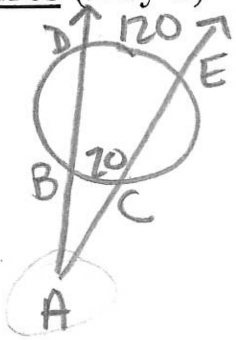
70°

6. $m\angle 6$



10.6 Secants, Tangents, and Angle Measures (Day 2)

Two Secants



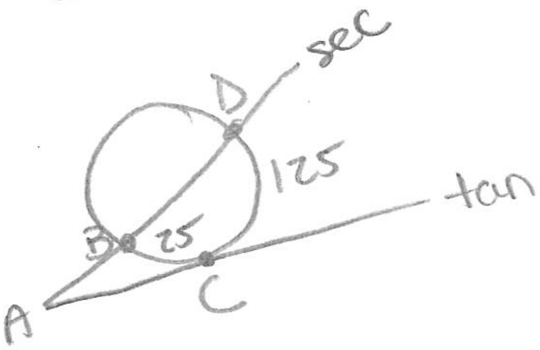
$$m\angle A = \frac{1}{2}(mDE - mBC)$$

$\frac{1}{2}(120 - 20)$

$\frac{1}{2}\ 100$

50°

Secant-Tangent



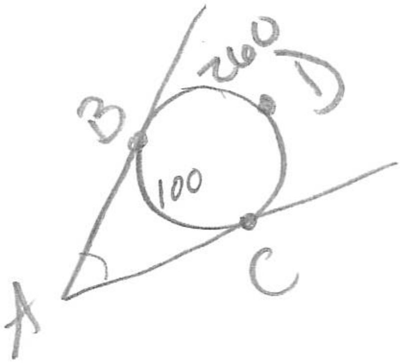
$$m\angle A = \frac{1}{2}(mDC - mBC)$$

$125 - 25$

$\frac{1}{2}\ 100$

50°

Two Tangents



$$m\angle A = \frac{1}{2}(mBDC - mBC)$$

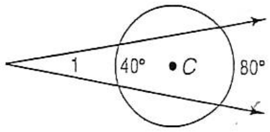
$260 - 100$

$\frac{1}{2} \cdot 160$

80°

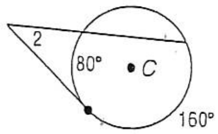
Find each measure.

1. $m\angle 1$



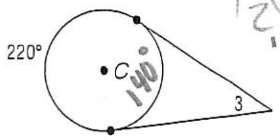
20°

2. $m\angle 2$



40°

3. $m\angle 3$

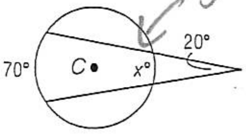


$\frac{1}{2}(220 - 140)$

$\frac{1}{2} \cdot 80$

40

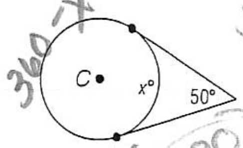
4. x



30°

$70 - x = 40$

5. x



$50 = \frac{1}{2}(360 - x - x)$

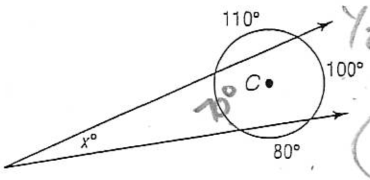
$50 = \frac{1}{2}(360 - 2x)$

$50 = 180 - x$

$-130 = -x$

$x = 130$

6. x



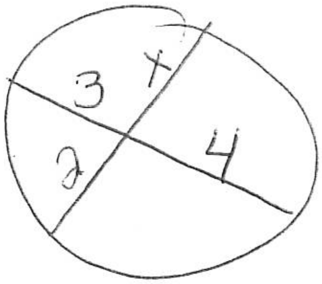
$\frac{1}{2}(100 - 70)$

15°

10.7 Special Segments in Circles

Segments that Intersect Inside a Circle:

Theorem – If two chords intersect in a circle, then the product of the measures of the segments are equal.



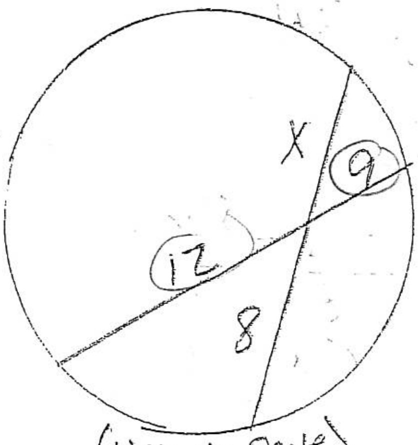
$3 \cdot 4 = 2x$

$\frac{12}{2} = \frac{2x}{2}$

$x = 6$

Example

Find x.



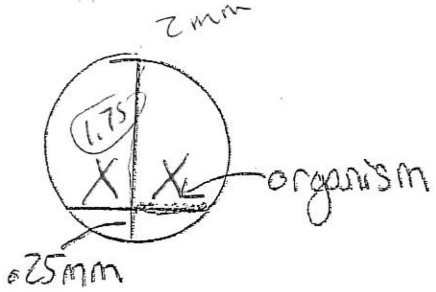
(Not to Scale)

$\frac{12 \cdot 9}{8} = \frac{8x}{8}$

$x = 13.5$

Example

Biologists often examine organisms under microscopes. The circle represents the field of view under the microscope with a diameter of 2 mm. Determine the length of the organism if it is located .25 mm from the bottom of the field of view. Round to the nearest hundredth.



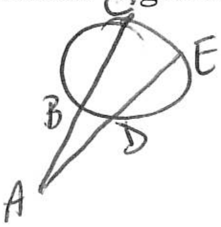
$1.75 \cdot .25 = x \cdot x$

$\sqrt{.4375} = \sqrt{x^2}$

$x = .66mm$

10.7 Segments Intersecting Outside a Circle

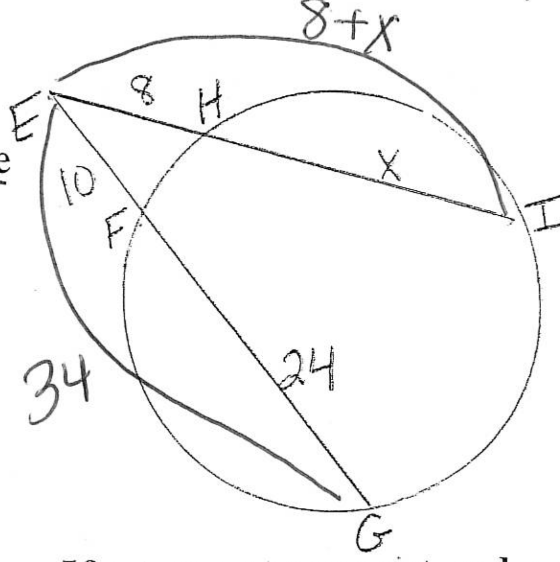
Theorem – If two secants are drawn from an exterior point, the product of the exterior part and the whole segment for each secant equals one another.



$AB \cdot AC = AD \cdot AE$

outside · whole = outside · whole

Example



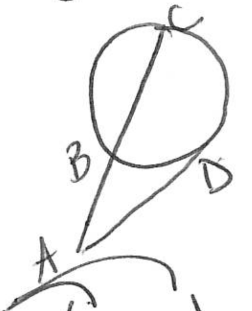
$10 \cdot 34 = 8 \cdot (8+x)$

$340 = 64 + 8x$

$\frac{276}{8} = \frac{8x}{8}$

$x = 34.5$

Theorem – If a tangent segment and a secant segment are drawn to a circle from an exterior point, then the square of the measure of the tangent is equal to the product of the secant segment and its external secant segment.

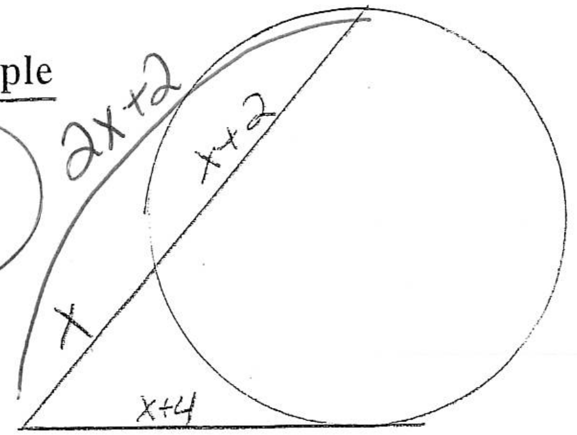


$AB \cdot AC = AD \cdot AD$

AD^2

Example

8



$x \cdot (2x+2) = (x+4)(x+4)$

$2x^2 + 2x = x^2 + 4x + 4x + 16$

$2x^2 + 2x = x^2 + 8x + 16$

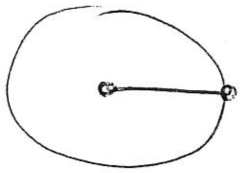
$-x^2 \quad -8x \quad -x^2 \quad -8x$

$x^2 - 6x = 16 \qquad x^2 - 6x - 16 = 0$

10.8 Equations of Circles

An equation for a circle with center at (h, k) and a radius of r units is $(x-h)^2+(y-k)^2=r^2$

* Center radius



Example
Write an equation for each circle:

a) center at (3, -3), d=12 h k $r=6$

$(x-3)^2+(y--3)^2=6^2$

$\boxed{(x-3)^2+(y+3)^2=36}$

b) center at (-12, -1), r = 8

$(x--12)^2+(y--1)^2=8^2$

$\boxed{(x+12)^2+(y+1)^2=64}$

Ex center (-3,9) radius endpt (0,4)

$(x--3)^2+(y-9)^2=(5.8)^2$

$\sqrt{(-3)^2+(5)^2}$

$9+25=\sqrt{34}$

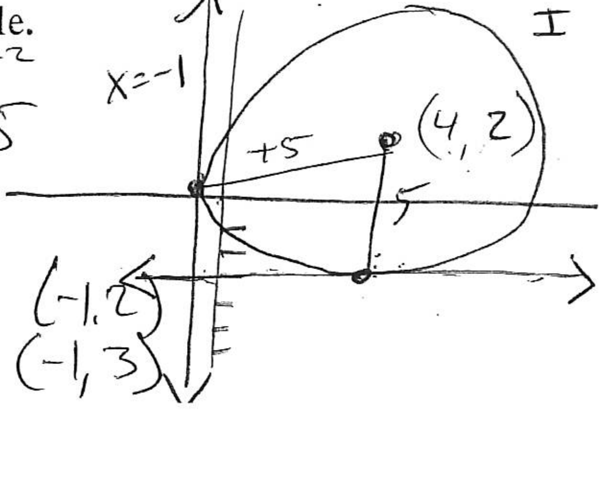
5.8

Example
A circle with a diameter of 10 has its center in the first quadrant. The lines y = -3 and x = -1 are tangent to the circle. Write an equation of the circle.

$r=5$

$(x-4)^2+(y-2)^2=25$ 5^2

(2, -3)
(4, -3)



NAME ______________________________ DATE ____________ PERIOD _____

10-8 Study Guide and Intervention *(continued)*

Equations of Circles

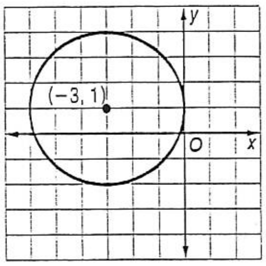
Graph Circles If you are given an equation of a circle, you can find information to help you graph the circle.

Example Graph $(x + 3)^2 + (y - 1)^2 = 9$.

Use the parts of the equation to find (h, k) and r.

$(x - h)^2 + (y - k)^2 = r^2$

$(x - h)^2 = (x + 3)^2$	$(y - k)^2 = (y - 1)^2$	$r^2 = 9$
$x - h = x + 3$	$y - k = y - 1$	$r = 3$
$-h = 3$	$-k = -1$	
$h = -3$	$k = 1$	

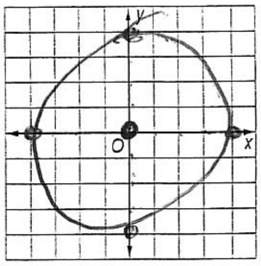


The center is at $(-3, 1)$ and the radius is 3. Graph the center. Use a compass set at a radius of 3 grid squares to draw the circle.

Exercises

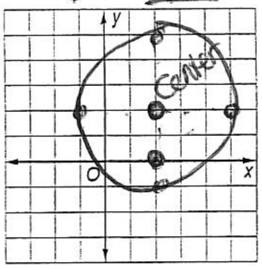
Graph each equation.

1. $x^2 + y^2 = 16$ $\sqrt{16} = 4$



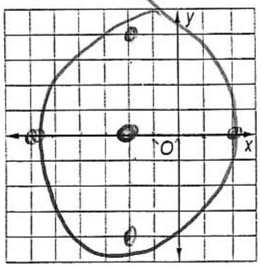
2. $(x - 2)^2 + (y - 1)^2 = 9$ (2, 1) $\sqrt{9} = 3$

x − h



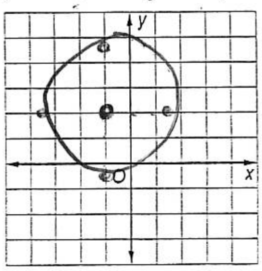
3. $(x + 2)^2 + y^2 = 16$ (−2, 0)

$\sqrt{16} = 4$



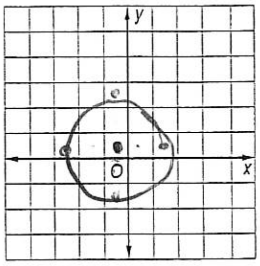
4. $(x + 1)^2 + (y - 2)^2 = 6.25$ (−1, 2)

2.5



5. $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{1}{4}\right)^2 = 4$ (−½, ¼)

2



6. $x^2 + (y - 1)^2 = 9$ (0, 1)