4.1 Classifying Triangles

Two Ways to Classify Triangles: 1) Sides
2) Angles

By Angle Measure

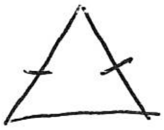
1) Acute Triangle – All angles less than 90°
2) Obtuse Triangle – one angle more than 90°
3) Right Triangle – one angle 90°

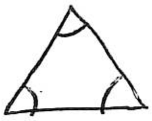
Equiangular – all angles are congruent
Equilateral

By Side Lengths

1) Scalene – all sides are different
2) Isosceles at least two sides are congruent
3) Equilateral – all sides are congruent

60°





Example

Find the measures of the sides of triangle RST. Classify the triangle by sides. R(-1, -3), S(4, 4) and T(8, -1).

$$d = \sqrt{(x-x)^2 + (y-y)^2}$$

$$RS = \sqrt{(-1-4)^2 + (-3-4)^2}$$

$$-5^2 + -7^2$$

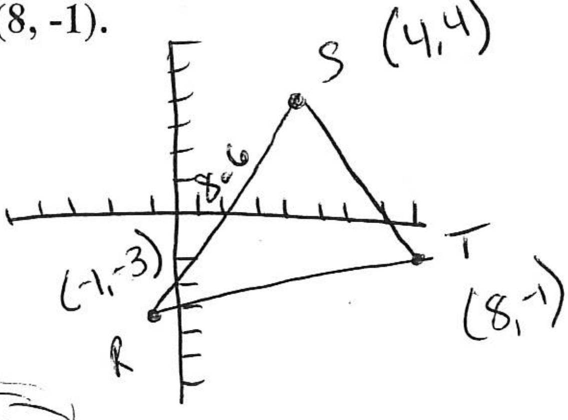
$$\sqrt{25 + 49} = \sqrt{74} \approx 8.6$$

$$ST = \sqrt{(4-8)^2 + (4+1)^2}$$

$$-4^2 + 5^2 \quad 16 + 25 = \sqrt{41} \approx 6.$$

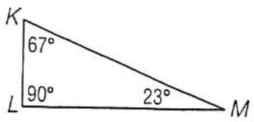
$$RT = 9.2$$

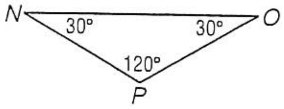
Scalene

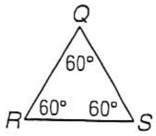


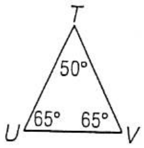
Examples

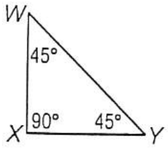
Classify each triangle as *acute*, *equiangular*, *obtuse*, or *right*.

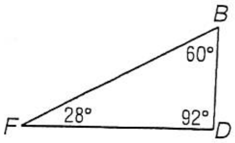
1. 

2. 

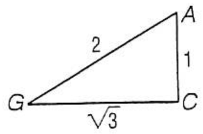
3. 

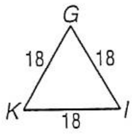
4. 

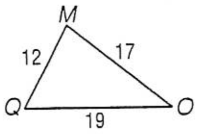
5. 

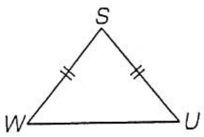
6. 

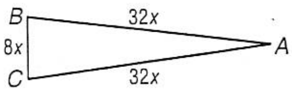
Classify each triangle as *equilateral*, *isosceles*, or *scalene*.

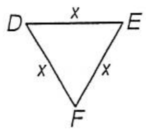
1. 

2. 

3. 

4. 

5. 

6. 

7. Find the measure of each side of equilateral $\triangle RST$ with $RS = 2x + 2$, $ST = 3x$, and $TR = 5x - 4$.

2 6 6

6 ~~$2x+2+3x+5x-4=180$~~

$2x+2 = 3x$

$2 = 1x$

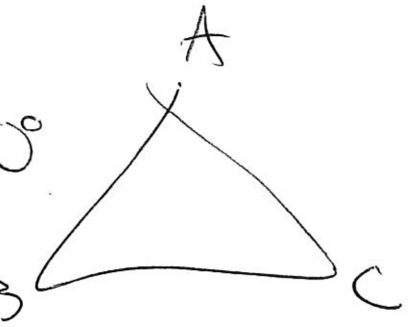
$x = 2$

Straight Line = 180°

4.2 Angles of Triangles

- Angle Sum Theorem – The sum of the measures of the angles of a triangle is 180°

$\angle A + \angle B + \angle C = 180°$

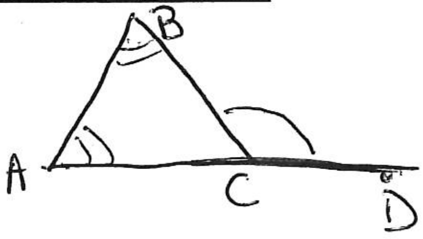


- Exterior Angle – an angle formed by one side of a triangle and the extension of another side

- Interior Angles – angle on the inside of the triangle that is not adjacent to the exterior angle

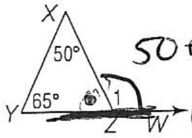
- Exterior Angle Theorem –

$\angle BCD = \angle A + \angle B$



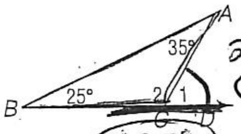
Examples

Find the measure of each numbered angle.

1. 

50 + 65

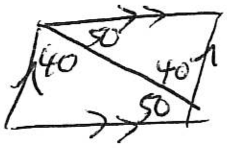
115°

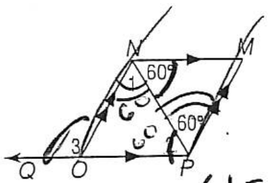
2. 

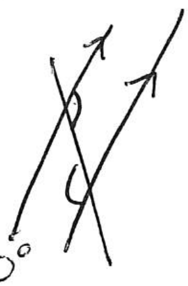
25 + 35

60°

120°



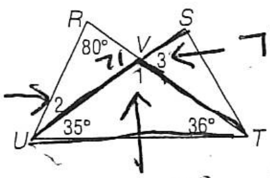
3. 



$\angle 1 = 60°$

$\angle 2 = 60°$

$\angle 3 = 120°$

4. 

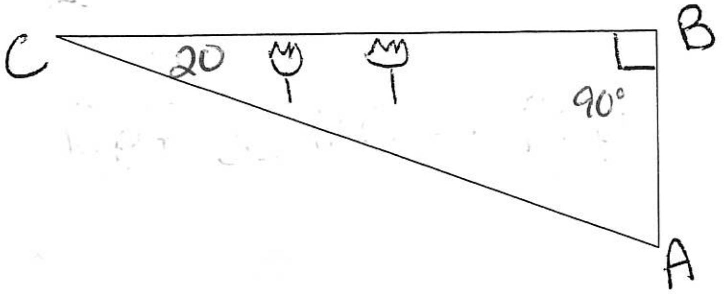
71°

29°

109°

Example

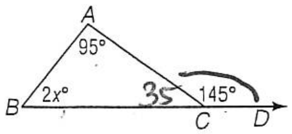
The flower bed shown is the shape of a right triangle. Find the measure of angle A if the measure of angle C is 20.



$180-20-90=70°$

Find x.

5.

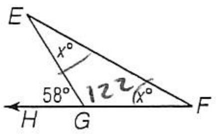


$2x+95=145$

$2x=50$

$x=25$

6.



$58=2x$

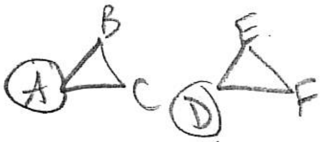
$2x+122=180$

$2x=58$

$x=29$

4.3 Congruent Triangles

$\overline{AB} \cong \overline{CD}$



- CPCTC (Corresponding Parts of Congruent Triangles are Congruent)

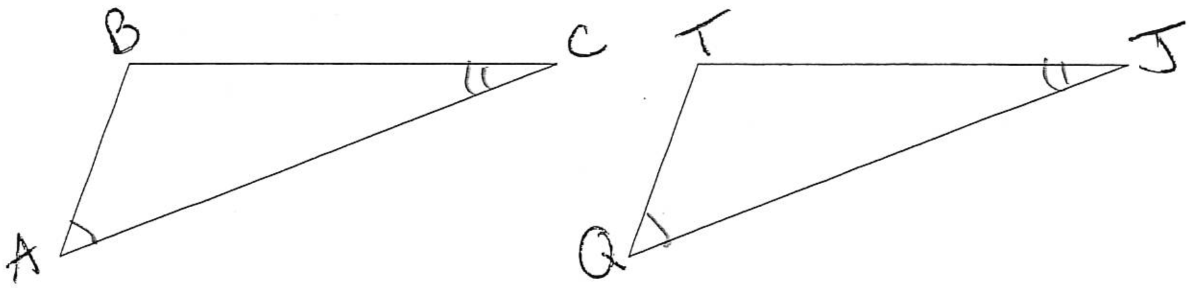
$\triangle ABC \cong \triangle DEF$

$\triangle ABC \cong \triangle FED$

- Congruent Triangles are Sides lengths that correspond are equal. & angles =

> Congruent Polygons – have corresponding sides that are congruent and angles.

Example



$\triangle ABC \cong \triangle QTJ$. List the congruent corresponding parts. Angles & Sides

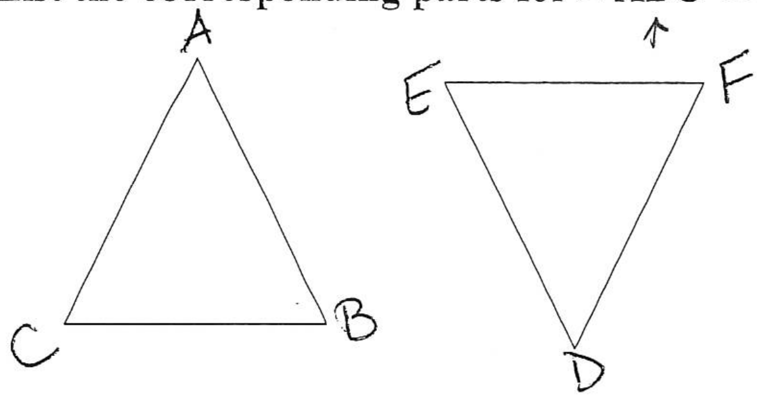
$\angle A \cong \angle Q$ $\overline{AB} \cong \overline{QT}$

$\angle C \cong \angle J$ $\overline{BC} \cong \overline{TJ}$

$\angle B \cong \angle T$ $\overline{AC} \cong \overline{QJ}$

Example

List the corresponding parts for $\triangle ABC \cong \triangle DEF$



$\angle B \cong \angle E$

$\angle C \cong \angle F$

$\angle A \cong \angle D$

Congruence Transformations/Isometries

Translation → slide

Rotation → turn

Reflection → flip

Example

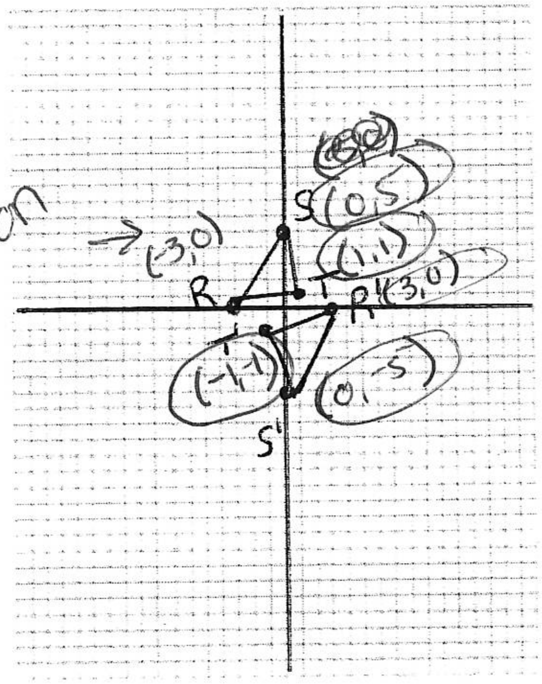
Verify that triangle RST is congruent to triangle R'S'T' and name the congruence transformation.

$d = \sqrt{(x-x)^2 + (y-y)^2}$

Yes

Reflection

origin



RST

$RS = \sqrt{(-3-0)^2 + (0-5)^2}$

$-3^2 + -5^2$

$9 + 25 = \sqrt{34} = 5.8$

$RT = \sqrt{(-3-1)^2 + (0-1)^2}$

$-4^2 + -1^2$

$16 + 1 = \sqrt{17} = 4.1$

$ST = \sqrt{(1-0)^2 + (5-1)^2}$

$1^2 + 4^2$

$1 + 16 = \sqrt{17} = 4.1$

R'S'T'

$R'S' = \sqrt{(3-0)^2 + (0+5)^2}$

$3^2 + 5^2$

$9 + 25 = \sqrt{34} = 5.8$

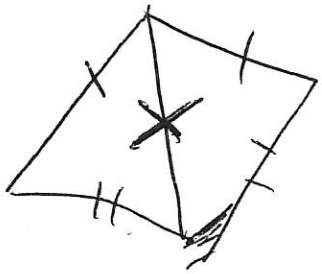
$R'T' = \sqrt{(3+1)^2 + (0+1)^2}$

$4^2 + 1^2 \quad \sqrt{17} = 4.1$

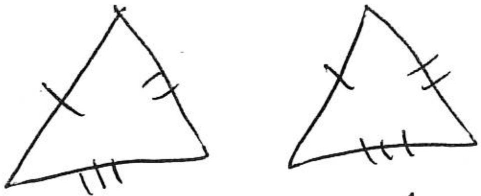
$S'T' = 4.1$

Triangle Congruence

There are different theorems that we can use to prove triangles are congruent.

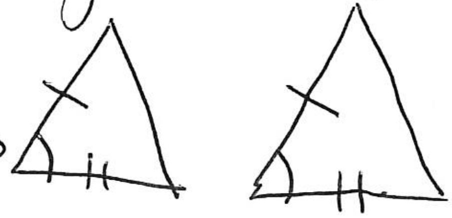


1. Side-Side-Side (SSS)



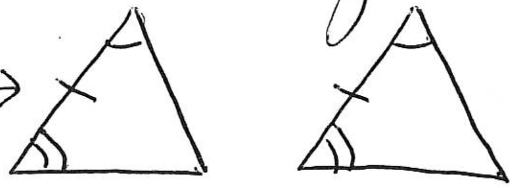
2. Side-Angle-Side (SAS)

Included Angles →

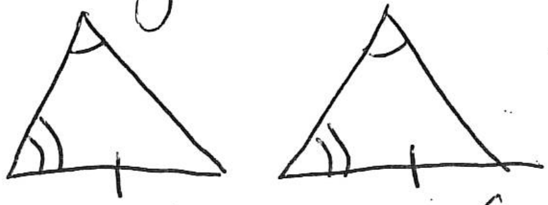


3. Angle-Side-Angle (ASA)

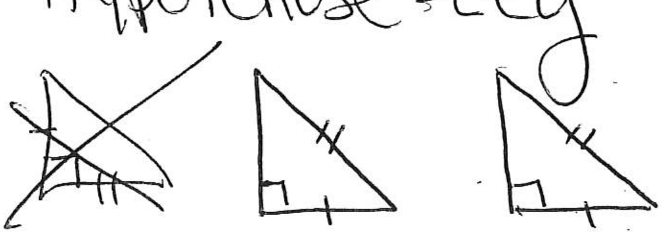
Included Side →



4. Angle-Angle-Side (AAS)



5. Hypotenuse-Leg (HL)



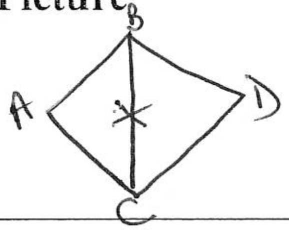
only works with right triangles

- A = A Reflexive
- A = B, B = A Symmetric
- A = B & B = C so A = C Trans/Sub.

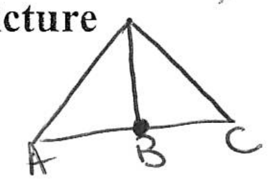
<u>**Triangle Proofs – Day 1**</u>

Now that we are proving triangles congruent, we need to add a couple new rules and/or look for the old ones in different ways. Questions to ask yourself:

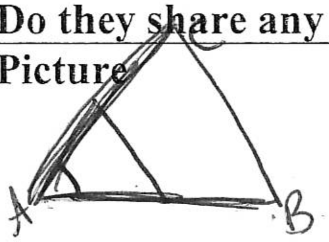
1. Do they share a side?

Picture	**What we Know**	**Justification**
	$\overline{BC} \cong \overline{BC}$	Reflexive

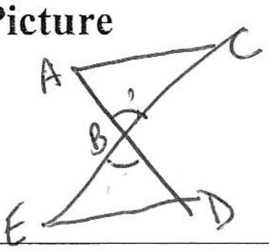
2. Does it say bisect, median, or midpoint?

Picture	**What we Know**	**Justification**
 B is midpt of AC	$\overline{AB} \cong \overline{BC}$	Def of midpt./median/or bisect

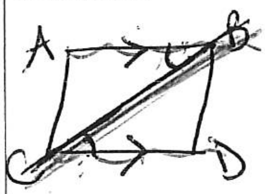
3. Do they share any angles?

Picture	**What we Know**	**Justification**
	$\angle CAB \cong \angle CAB$	Reflexive

4. Are there any vertical angles?

Picture	**What we Know**	**Justification**
	$\angle ABC \cong \angle DBE$	Vertical Angles

5. **Are there any parallel lines?**

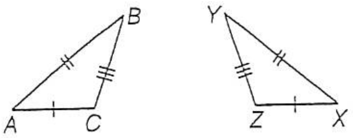
Picture	What we Know	Justification
	$\angle ABC \cong \angle BCD$	Alt. Int. Angles

When proving triangles congruent you will always end with either SSS, SAS, ASA, AAS, or HL.

Examples

Write a two-column proof.

1.

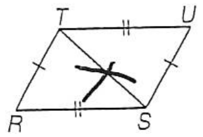


Given: $\overline{AB} \cong \overline{XY}, \overline{AC} \cong \overline{XZ}, \overline{BC} \cong \overline{YZ}$

Prove: $\triangle ABC \cong \triangle XYZ$

Statements	Reasons
1) $\overline{AB} \cong \overline{XY}$ $\overline{AC} \cong \overline{XZ}$ $BC \cong \overline{YZ}$	Given
2) $\triangle ABC \cong \triangle XYZ$	SSS

2.



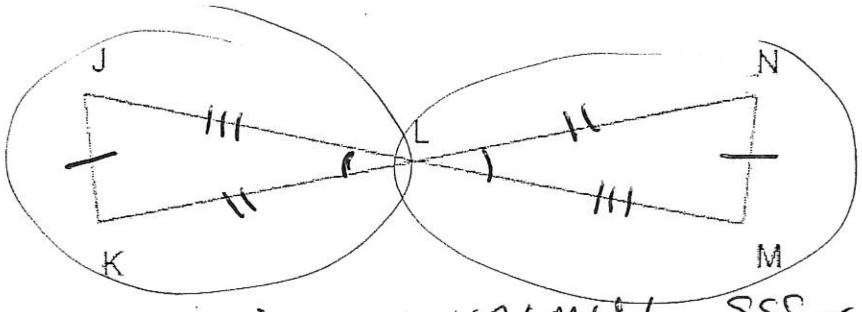
Given: $\overline{RS} \cong \overline{UT}, \overline{RT} \cong \overline{US}$

Prove: $\triangle RST \cong \triangle UTS$

Statements	Reasons
1) $\overline{RS} \cong \overline{UT}$ $\overline{RT} \cong \overline{US}$	Given
2) $\overline{ST} \cong \overline{ST}$	Reflexive
3) $\triangle RST \cong \triangle UTS$	SSS

3. Given: JM bisects KN, KN bisects JM, JK ≅ NM

Prove: △JLK ≅ △MLN

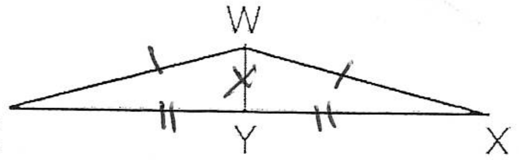


Statement	Reason
1) JM bisects KN, KN bisects JM, JK ≅ NM	Given
2) $\angle JLK \cong \angle NLM$	Vertical Angles
3) $\overline{KL} \cong \overline{LN}$	Def of bisect
4) $\overline{} \cong \overline{LM}$	Def. of bisect.
5) $\triangle JLK \cong \triangle MLN$	SSS or SAS

STOP

➢ **When trying to prove sides or angle congruent you will use** CPCTC

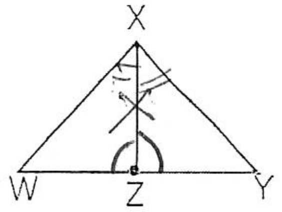
1) Given: $\overline{WZ} \cong \overline{WX}$, $\overline{WY}$ bisects $\overline{ZX}$
Prove: $\triangle WYZ \cong \triangle WYX$



	Given
2) $\overline{WY} \cong \overline{WY}$	Reflexive
3) $\overline{ZY} \cong \overline{YX}$	Def. Bisect
4) $\triangle WYZ \cong \triangle WYX$	SSS

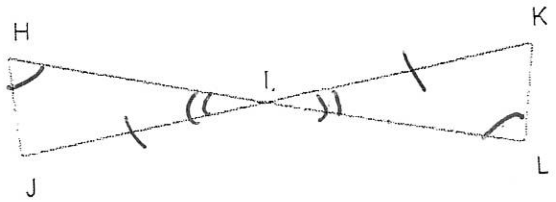
SAS
HL
SSS
ASA

2) Given XZ bisects $\angle WXY$ $\angle WZX \cong \angle XZY$
Prove $\angle XWZ \cong \angle XYZ$



	Given
2) $\overline{XZ} \cong \overline{XZ}$	Reflexive
3) $\angle WXZ \cong \angle YXZ$	Def of bisects
4) $\triangle WXZ \cong \triangle YXZ$	ASA
5) $\angle XWZ \cong \angle XYZ$	CPCTC ✓

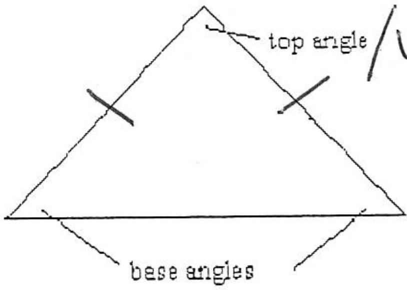
3) Given : HL bisects JK,
$\angle H \cong \angle L$
Prove: $\triangle JHI \cong \triangle LKI$



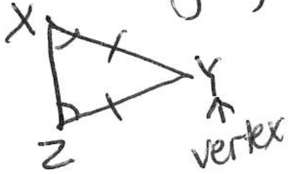
1)	Given
2) $\overline{JI} \cong \overline{IK}$	Def of bisect
3) $\angle HIJ \cong \angle KIL$	Vertical Angles
4) $\triangle JHI \cong \triangle LKI$	AAS

4.6 Isosceles Triangles

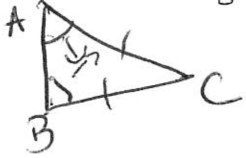
Remember: Isosceles triangles have two congruent sides.



- Isosceles Triangle Theorem – If you have isosceles triangle, then base angles are congruent



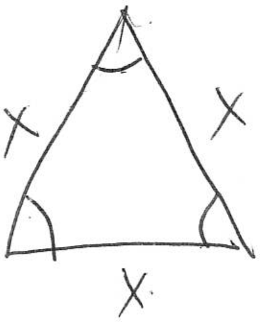
- Converse of Isosceles Triangle Theorem - If two angles of a triangle are congruent, then the sides opposite those angles are congruent.



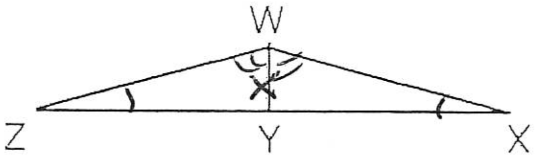
$AC \cong BC$

- A triangle is equilateral if and only if it is equiangular

$180/3 = 60°$



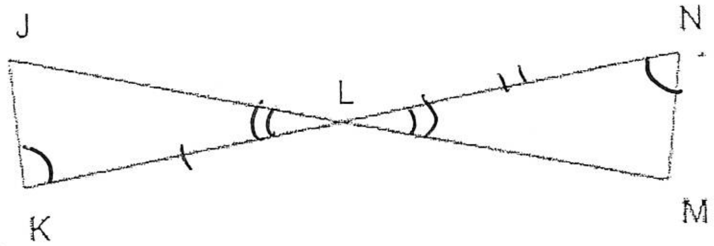
4) Given: WY bisects $\angle ZWX$
$\angle Z \cong \angle X$
Prove: $\triangle WYZ \cong \triangle WYX$



1)	Given
2) $\overline{WY} \cong \overline{WY}$	Reflexive
3) $\angle ZWY \cong \angle XWY$	Def of bisects
4) $\triangle WYZ \cong \triangle WYX$	AAS

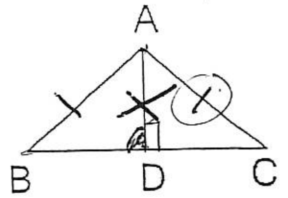
Stop

5) Given: JM bisects KN.
$\angle K \cong \angle N$
Prove: $\triangle JLK \cong \triangle MLN$



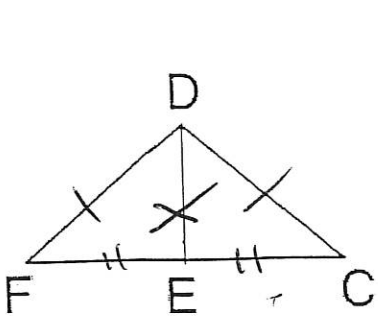
	Given
2) $\overline{KL} \cong \overline{LN}$	Def. bisects
3) $\angle JLK \cong \angle NLM$	Vertical Angle
4) $\triangle JLK \cong \triangle MLN$	ASA

6) Given: $\overline{AD} \perp \overline{BC}$, $\overline{BA} \cong \overline{AC}$
Prove: $\triangle ABD \cong \triangle ACD$



1)	Given
2) $\overline{AD} \cong \overline{AD}$	Reflexive
3) $\angle ADB$ & $\angle ADC = 90°$	Def of $\perp$
$\triangle ABD \cong \triangle ACD$	HL

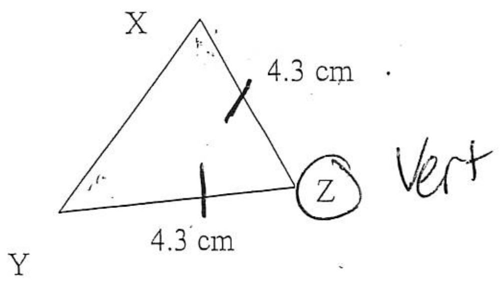
7) Given: DE is a perpendicular bisector of FC.
DF $\cong$ DC
Prove: $\triangle DEF \cong \triangle DEC$



1)	Given
2) $\overline{DE} \cong \overline{DE}$	Reflexive
3) $\overline{FE} \cong \overline{EC}$	Def. bisects
$\triangle DEF \cong \triangle DEC$	SSS

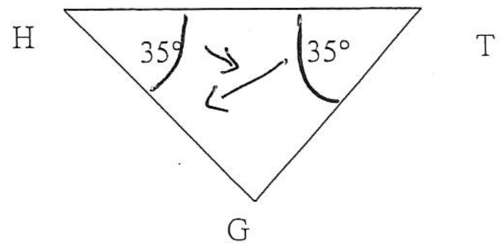
1. Which two angles in the figure to the right are congruent?

$\angle X$ & $\angle Y$



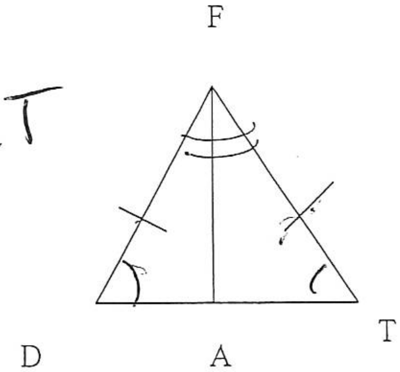
2. Which two sides are congruent in the figure to the right?

$\overline{HG} \cong \overline{TG}$



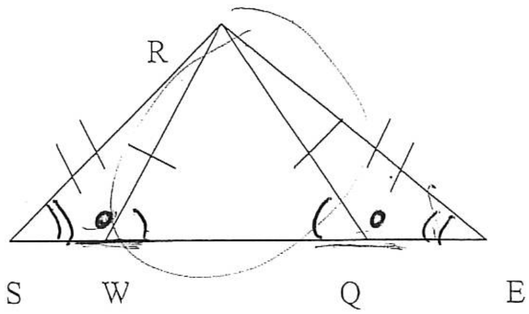
3. Which two angles are congruent in the figure to the right?

$\angle D \cong \angle T$



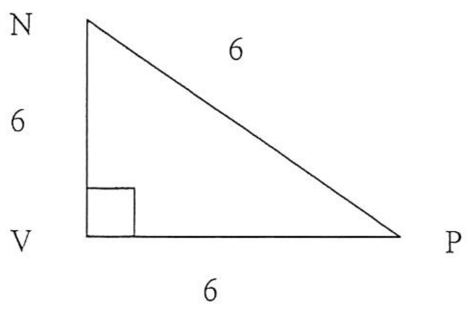
4. Which angles must be congruent in the figure to the right?

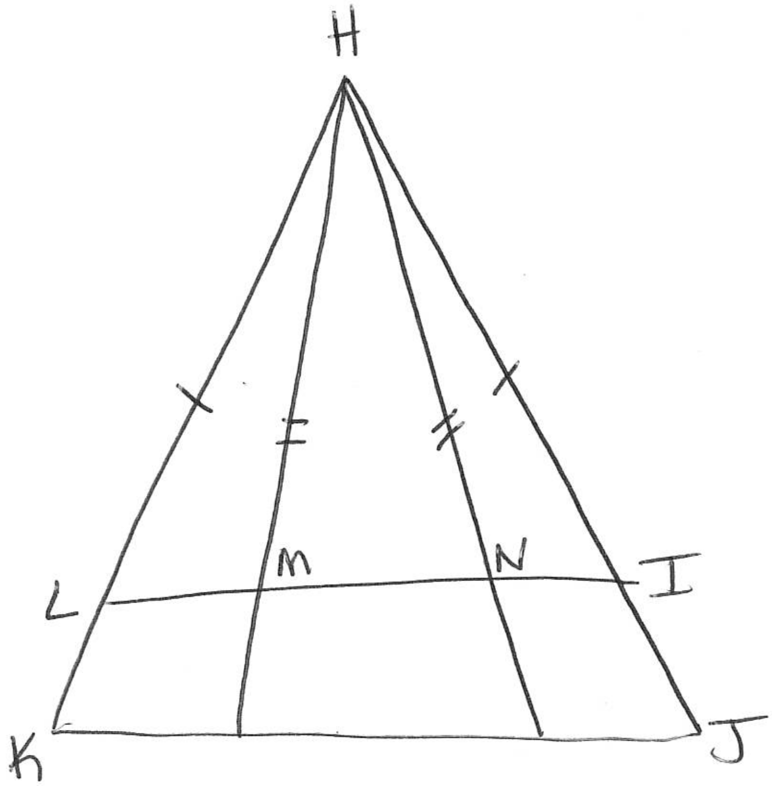
$\angle S$ & $\angle E$
$\angle W$ & $\angle Q$



5. What is wrong with the diagram to the right?

$\cancel{6^2+6^2=6^2}$





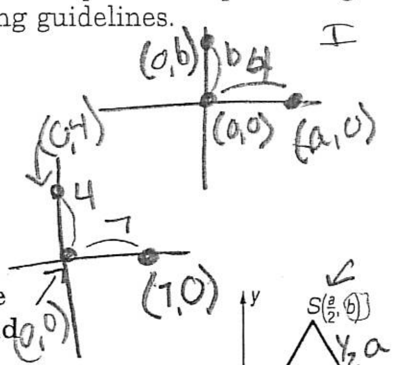
NAME _______________ DATE __________ PERIOD ____

Study Guide and Intervention

Triangles and Coordinate Proof

Position and Label Triangles A coordinate proof uses points, distances, and slopes to prove geometric properties. The first step in writing a coordinate proof is to place a figure on the coordinate plane and label the vertices. Use the following guidelines.

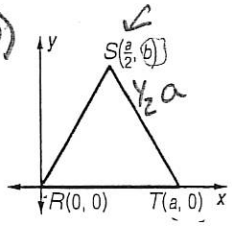
1. Use the origin as a vertex or center of the figure.
2. Place at least one side of the polygon on an axis.
3. Keep the figure in the first quadrant if possible.
4. Use coordinates that make the computations as simple as possible.



Example **Position an equilateral triangle on the coordinate plane so that its sides are a units long and one side is on the positive x-axis.**

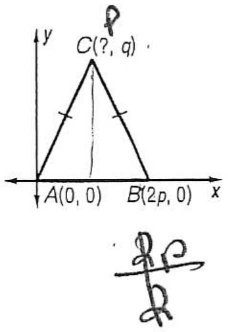
Start with $R(0, 0)$. If RT is a, then another vertex is $T(a, 0)$.

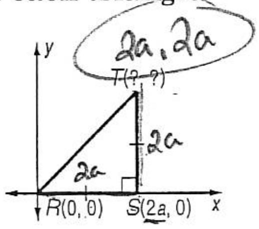
For vertex S, the x-coordinate is $\frac{a}{2}$. Use b for the y-coordinate, so the vertex is $S\left(\frac{a}{2}, b\right)$.

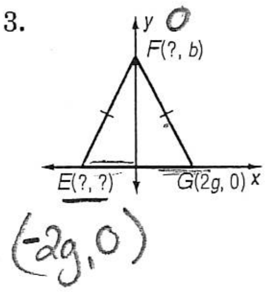


Exercises

Find the missing coordinates of each triangle.

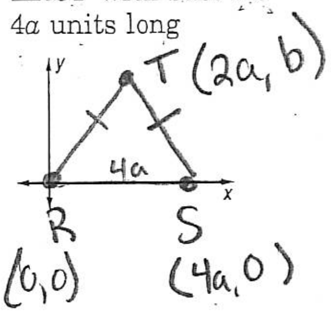
1.


2.


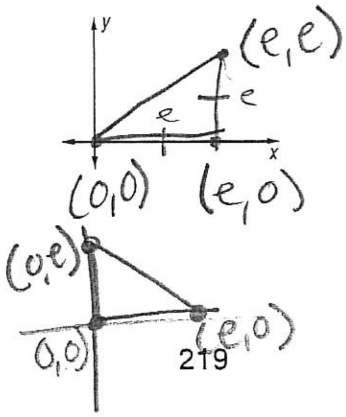
3.


Position and label each triangle on the coordinate plane.

4. isosceles triangle $\triangle RST$ with base $\overline{RS}$ $4a$ units long



5. isosceles right $\triangle DEF$ with legs e units long



6. equilateral triangle $\triangle EQI$ with vertex $Q(0, a)$ and sides $2b$ units long