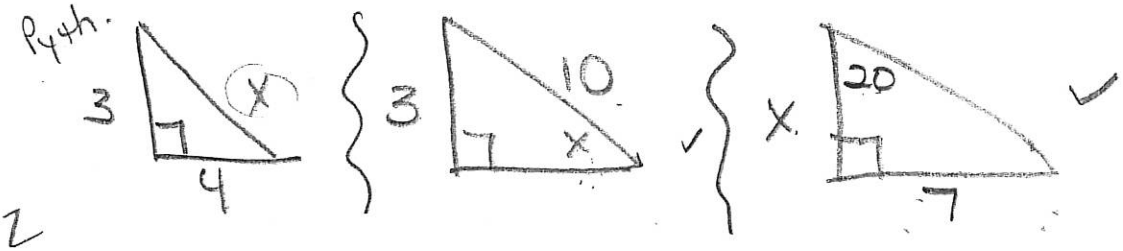
Pyth.

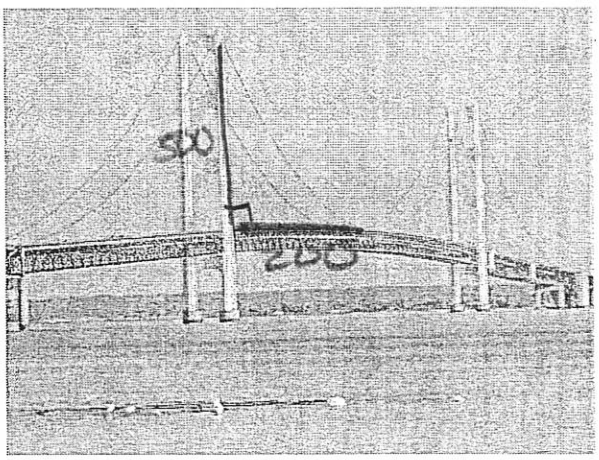


8.2

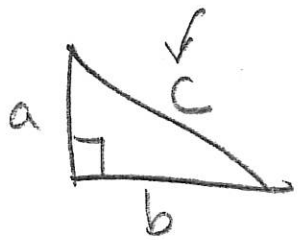
7.2 Pythagorean Theorem

How can we find the length of the cable on the bridge to the right? What bridge is this?

$500^2 + 200^2 = x^2$



Source: http://www.historicbridges.org/other/images.htm



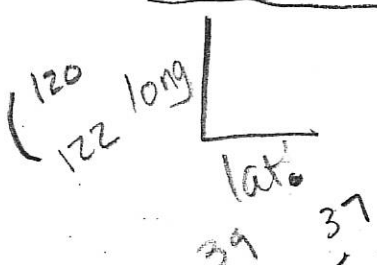
➢ Pythagorean Theorem – $a^2 + b^2 = c^2$

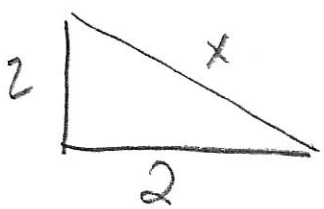
legs

hypotenuse
- opp. rt. angle
- longest side

Example

Carson City Nevada is located at about 120 degrees longitude and 39 degrees latitude. Use the lines of longitude and latitude to find the distance to the nearest tenth degree if you were to travel directly from NASA Ames (122 degrees longitude and 37 degrees latitude) to Carson City.





$a^2 + b^2 = c^2$

$2^2 + 2^2 = x^2$

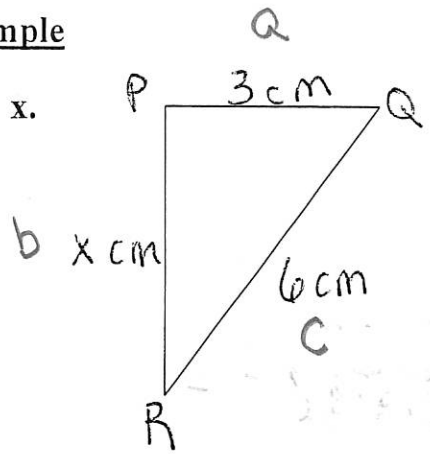
$4 + 4 = x^2$

$\sqrt{8} = \sqrt{x^2}$

$x = 2.82$

Example

Find x.



$3^2 + x^2 = 6^2$

$9 + x^2 = 36$

$-9 \quad -9$

$\sqrt{x^2} = \sqrt{27}$

$x = 5.196$

5.2

Does the converse of the Pythagorean Theorem work?

Yes

Example

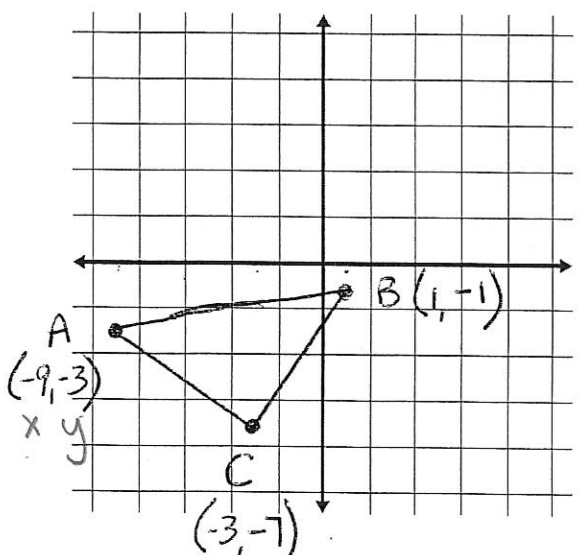
Verify that triangle ABC is a right triangle.

$d = \sqrt{(x-x)^2 + (y-y)^2}$

$AC = \sqrt{(-9+3)^2 + (-3+7)^2}$

$-6^2 + 4^2$

$36 + 16 = \sqrt{52}$



$BC = \sqrt{52} = 7.2$

$AB = \sqrt{104} = 10.2$

$\sqrt{52}^2 + \sqrt{52}^2 = \sqrt{104}^2$ x y

$52 + 52 = 104$

Yes

➢ Pythagorean Triple – three whole numbers that satisfy the Pythagorean theorem.

Examples

Determine whether each set of measures are the sides of a right triangle. Then state whether they form a Pythagorean Triple.

a) 9, 12, and 15

1 $a^2+b^2=c^2$ 2

$9^2+12^2=15^2$

Yes Yes

b) 21, 42, and 54

No No

c) $4\sqrt{3}$, 4 and 8

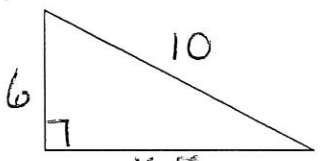
$(4\sqrt{3})^2+(4)^2=8^2$ No

$48+16=64$

Yes

Practice:

1) Find x.

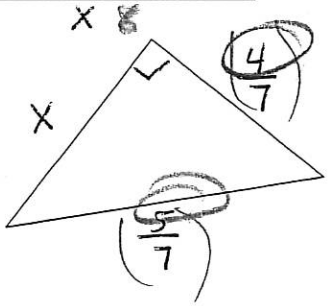


$6^2+x^2=10^2$ $x=8$

$36+x^2=100$

$x^2=64$

2) Find x.



3 4 5

$x^2+\left(\frac{4}{7}\right)^2=\left(\frac{5}{7}\right)^2$

$x^2+\frac{16}{49}=\frac{25}{49}$

$x^2=\frac{9}{49}$ $\frac{3}{7}$

3) Are the following a Pythagorean Triple? $\sqrt{40}$, 20, 21

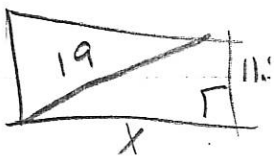
No

4) Computer monitors are usually measured along the diagonal of the screen. A 19-inch monitor has a diagonal that measures 19 inches. If the height of the screen is 11.5 inches, how wide is the screen?

$11.5^2+x^2=19^2$

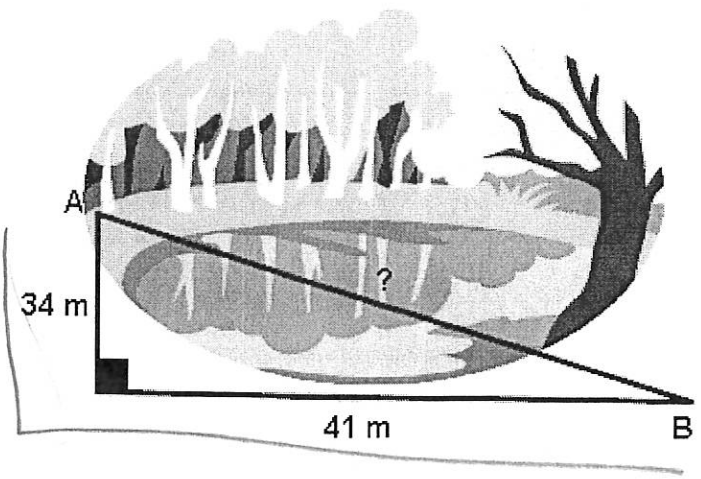
$132.25+x^2=361$

15.1 in



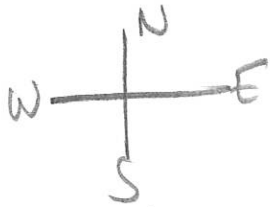
Pythagorean Theorem Day 2 - Applications

1.



$a^2+b^2=c^2$

leg leg hypotenuse



To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the *nearest meter*, how many meters would be saved if it were possible to walk through the pond?

$34^2+41^2=c^2$

$2837=\sqrt{c^2}$

$c=53.26$

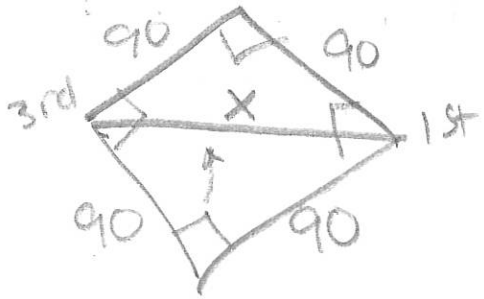
$34+41=75$ now

-53.26 pond

21.74

22 m

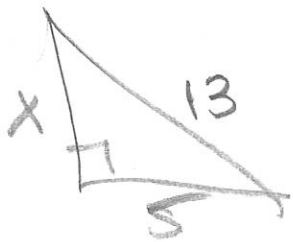
A baseball diamond is a square with sides of 90 feet. What is the shortest distance, to the *nearest tenth* of a foot, between first base and third base?



$90^2+90^2=x^2$

$x=127.3$ ft

Ms. Green tells you that a right triangle has a hypotenuse of 13 and a leg of 5. She asks you to find the other leg of the triangle without using paper and pencil. What is your answer?



$x=12$

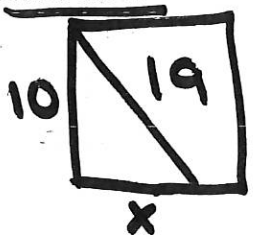
25 169

-25

$x^2+5^2=13^2$

$x^2+25=169$

In a computer catalog, a computer monitor is listed as being 19 inches. This distance is the diagonal distance across the screen. If the screen measures 10 inches in height, what is the actual width of the screen to the *nearest inch*?



16in

A suitcase measures 24 inches long and 18 inches high. What is the diagonal length of the suitcase to the *nearest tenth* of a foot?

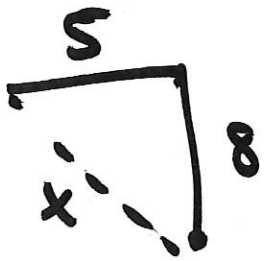
$\frac{30\text{in}}{12}$

2.5ft

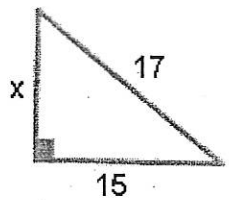
The older floppy diskettes measured 5 and 1/4 inches on each side. What was the diagonal length of the diskette to the *nearest tenth* of an inch?

7.4in

Two joggers run 8 miles north and then 5 miles west. What is the shortest distance, to the *nearest tenth* of a mile, they must travel to return to their starting point?



9.4m



x=8

Find x

4, 9, 16, 25 Quick Rev

$\sqrt{2}\cdot\sqrt{2} = 2$

$\sqrt{20} = \sqrt{4\cdot5}$ $2\sqrt{5}$

$\sqrt{63} = \sqrt{9\cdot7}$ $3\sqrt{7}$

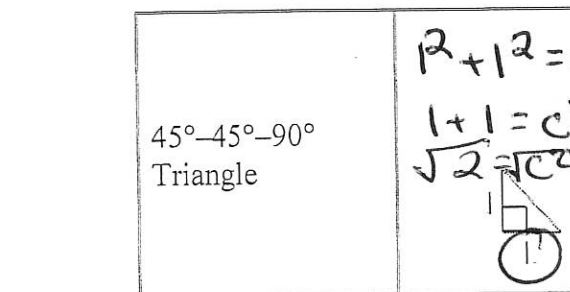
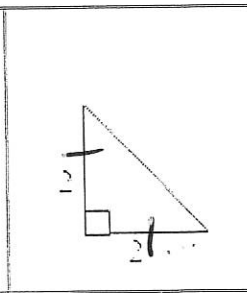
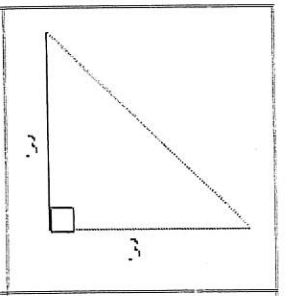
$\sqrt{24} = \sqrt{4\cdot6}$ $2\sqrt{6}$

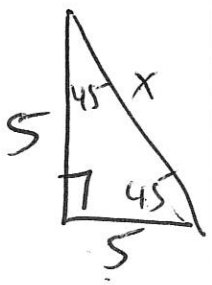
8.3

~~7.3~~ Special Right Triangles

45-45-90 Triangles

Three 45°-45°-90° triangles are shown below. Complete the table by using the Pythagorean Theorem to find the length of each triangle's hypotenuse. Write each length in simplest form.

45°–45°–90° Triangle			
Length of leg	1 unit	2 units	3 units
Length of hypotenuse	$\sqrt{2}$	$\sqrt{8} = \sqrt{4\cdot2}$ $2\sqrt{2}$	$\sqrt{18} = \sqrt{9\cdot2}$ $3\sqrt{2}$



$5\sqrt{2}$

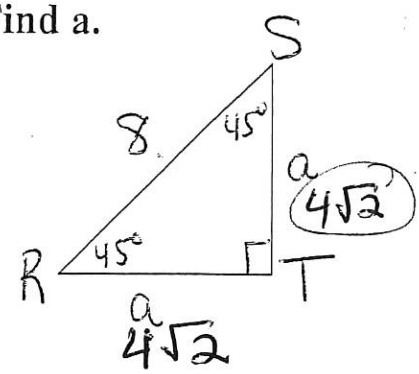
Theorem 7.6

In a 45°- 45°- 90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of the leg.

Start	End	Process
leg	hyp.	mult. $\sqrt{2}$
leg	leg	same
hyp.	leg	divide $\sqrt{2}$

Example

Find a.



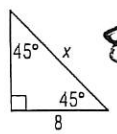
$$\frac{8}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{8\sqrt{2}}{2}$$

$$4\sqrt{2}$$

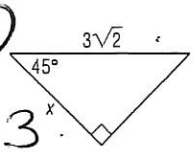
Examples

Find the missing side lengths below:

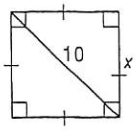
Find x.

1. 

$8\sqrt{2}$

2. 

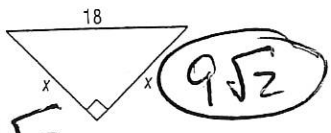
$$\frac{3\sqrt{2}}{\sqrt{2}}$$

3. 

$5\sqrt{2}$

$$\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2}$$

$5\sqrt{2}$

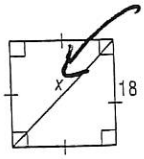
4. 

$9\sqrt{2}$

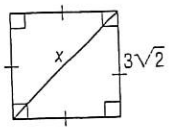
$9\sqrt{2}$

$$\frac{18}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{18\sqrt{2}}{2}$$

5. 

$18\sqrt{2}$

6. 

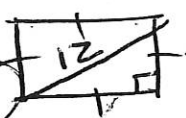
$3\sqrt{2} \cdot \sqrt{2}$

$3 \cdot 2$

6

7. Find the perimeter of a square with diagonal 12 centimeters.

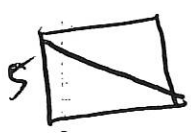
$24\sqrt{2}$ $24\sqrt{2}$



$$\frac{12}{\sqrt{2}}$$

$6\sqrt{2}$

$\times 4$



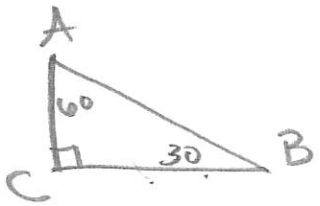
8. Find the diagonal of a square with perimeter 20 inches.

$5\sqrt{2}$

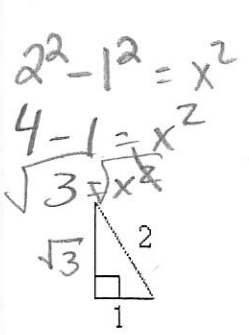
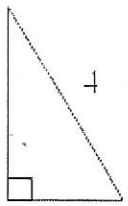
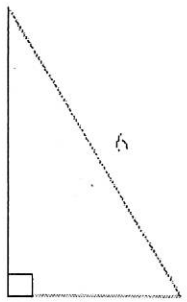
9. Find the diagonal of a square with perimeter 28 meters.

$7\sqrt{2}$

30-60-90 Triangles



Three 30°-60°-90° triangles are shown below. Complete the table by using the Pythagorean theorem to find the length of each triangle's hypotenuse. Write each length in simplest form.

Triangle			
Length of hypotenuse	2 units	4 units	6 units
Length of shorter leg	1 unit	2 units	3 units
Length of longer leg	$1\sqrt{3}$	$\sqrt{12} = \sqrt{4 \cdot 3}$ $2\sqrt{3}$	$\sqrt{27} = \sqrt{9 \cdot 3}$ $3\sqrt{3}$

X2

Theorem 7.7

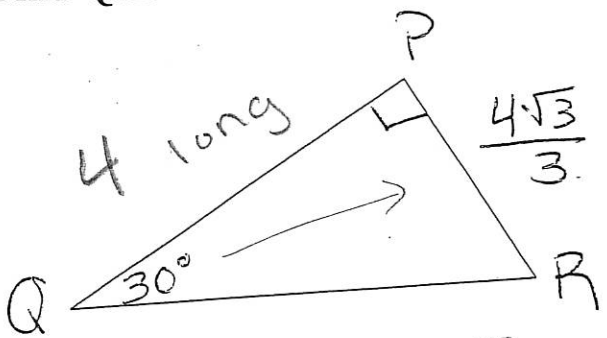
In a 30°- 60°- 90° triangle,

1) **the length of the hypotenuse is** X 2 **the length of the shorter leg,**
2) **the length of the longer leg is** $\sqrt{3}$ **times the length of the shorter leg.**

Start	End	Process
Short	long leg	mult $\sqrt{3}$
long leg	short leg	$\div \sqrt{3}$
Short	hyp.	X 2
hyp.	short	$\div$ 2

$\frac{4}{3} \cdot \frac{2}{1}$

Example
Find QR.



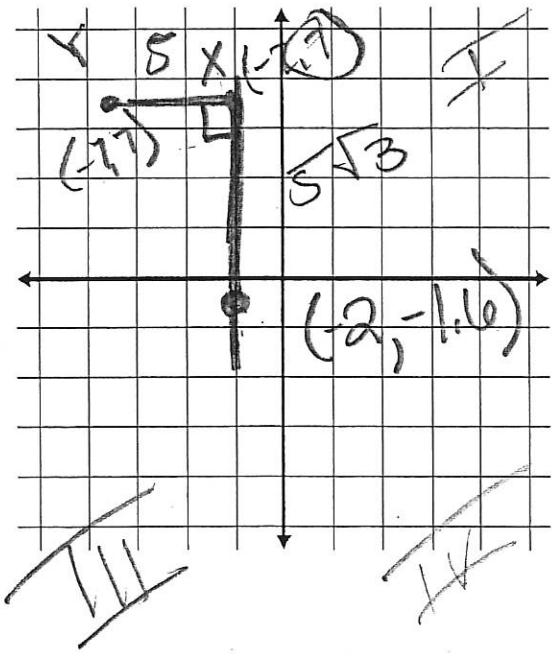
short $\frac{4\sqrt{3}}{3}$

$\frac{4\sqrt{3}}{3} \cdot \frac{2}{1} = \frac{8\sqrt{3}}{3}$

$\frac{4\sqrt{3}}{3} \cdot \frac{\sqrt{3}}{1} = \frac{4\sqrt{9}}{3}$

$\frac{12}{3}$

Example
Triangle WXY is a 30-60-90 triangle. The right angle is angle X. WX is the longer leg. Graph points W(-2, 7) and Y(-7, 7) and locate point W in quadrant III.



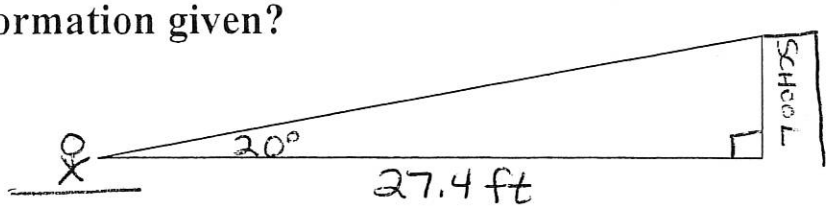
$5\sqrt{3} = 8.6$

$$\begin{array}{r} 7 \\ -8.6 \\ \hline -1.6 \end{array}$$

8.4

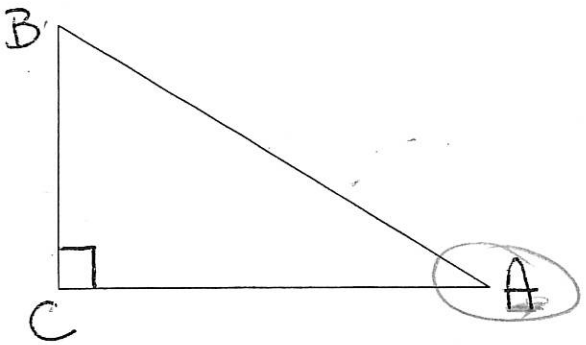
7.4 Trigonometry

Suppose you are sitting on the ground in the school parking lot and want to find the height of the building. How could you do that using the information given?



- **Trigonometry** – measurement using triangles
- **Trigonometric Ratio – ratio of the lengths of sides of a right triangle**

Three most common:



Sine of $\angle A$ = Sin

$$\frac{\text{measure of leg opposite } \angle A}{\text{measure of hypotenuse}} \quad \frac{BC}{BA}$$

Cosine of $\angle A$ = cos

$$\frac{\text{measure of leg adjacent } \angle A}{\text{measure of hypotenuse}} \quad \frac{CA}{BA}$$

Tangent of $\angle A$ = tan

$$\frac{\text{measure of leg opposite } \angle A}{\text{measure of leg adjacent to } \angle A} \quad \frac{BC}{CA}$$

SOH – CAH – TOA

S o h C a h T o a
O p p A d j O p p
H y p H y p A d j

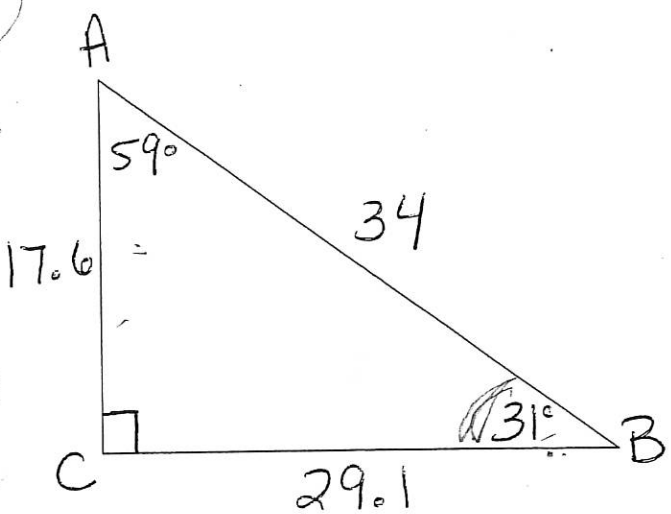
Set up Trig Ratios using the identified angles.

Angle A

Sin 59 = $\frac{29.1}{34}$

Cos 59 = $\frac{17.6}{34}$

Tan 59 = $\frac{29.1}{17.6}$



Angle B

Sin 31 = $\frac{17.6}{34}$

Cos 31 = $\frac{29.1}{34}$

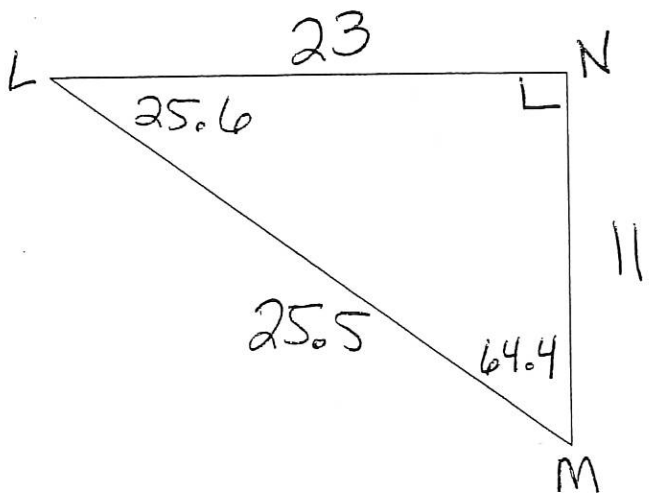
Tan 31 = $\frac{17.6}{29.1}$

Angle L

Sin 25.6 =

Cos 25.6 =

Tan 25.6 =



Angle M

Sin 64.4 =

Cos 64.4 =

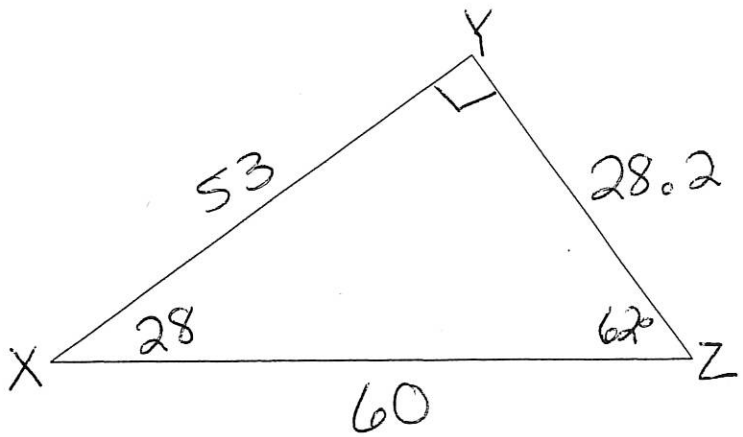
Tan 64.4 =

Angle X

Sin =

Cos =

Tan =



Angle Z

Sin =

Cos =

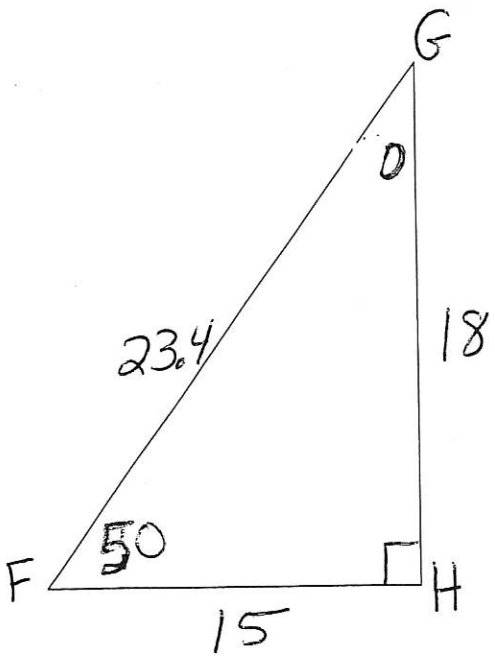
Tan =

Angle F

Sin =

Cos =

Tan =



Angle G

Sin =

Cos =

Tan =

O - opp. H - hyp.
A - adj.

8.4

~~7.4~~ Trigonometry – Day 2

SOH - CAH - TOA

We can use the trig ratios discussed yesterday to solve for missing lengths of sides in triangles.

Recall some basic equations:

Solve the following

1) $\frac{2}{x} = 23.6 \cdot x$

$2 = 23.6x$

$\frac{2}{23.6} = \frac{23.6x}{23.6}$

$x = .0847$

2) $\frac{x}{5} = 14 \cdot 5$

$x = 70$

3) $\frac{3}{x-1} = 5(x-1)$

$3 = 5x - 5$

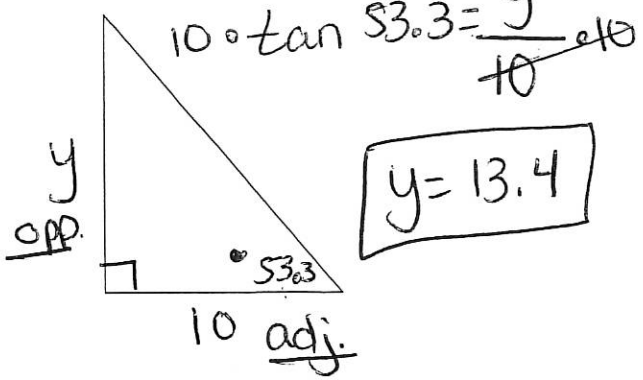
$+5 \quad +5$

$8 = 5x$

$x = \frac{8}{5}$

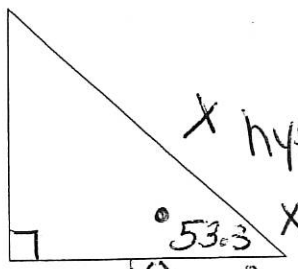
$x = 1.6$

Now let's apply that same knowledge to solve for sides of right triangles.



$10 \cdot \tan 53.3 = \frac{y}{10} \cdot 10$

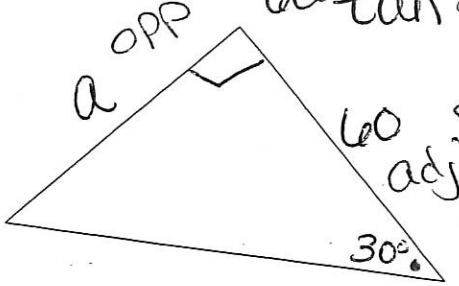
$y = 13.4$



$x \cdot \cos 53.3 = \frac{10}{x} \cdot x$

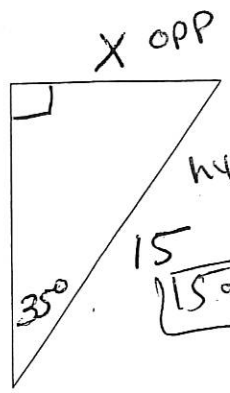
$\frac{x \cdot \cos 53.3}{\cos 53.3} = \frac{10}{\cos 53.3}$

$x = 16.7$



$60 \cdot \tan 30 = \frac{a}{60} \cdot 60$

$a = 34.6$



$15 \cdot \sin 35 = \frac{x}{15} \cdot 15$

$x = 8.6$

8.4

~~7.4~~ Trigonometry – Day 3

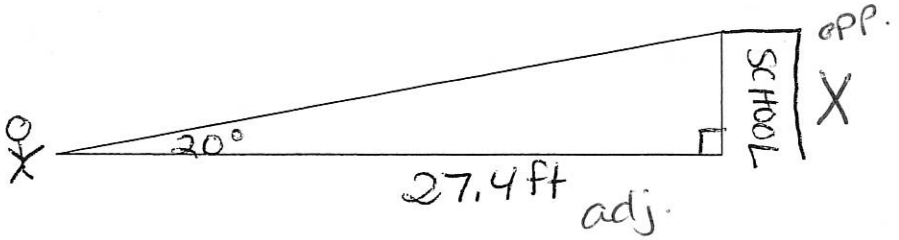
Now that we know how to set up trig ratios we are going to use them to solve for lengths and angles.

Example

Looking at the dimensions from our situation yesterday, find the height of the school building.

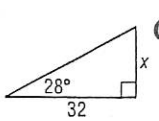
$\tan 20 = \frac{X}{27.4}$

$X = 9.9$



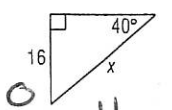
Examples

Find x. Round to the nearest tenth.



$32 \cdot \tan 28 = \frac{X}{32} \cdot 32$

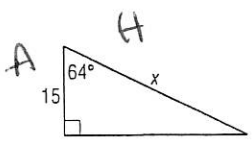
$X = 17.$



$X \cdot \sin 40 = \frac{16}{X} \cdot X$

$\frac{X \cdot \sin 40}{\sin 40} = \frac{16}{\sin 40}$

$X = 24.89$



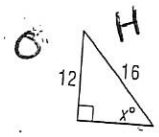
$X \cdot \cos 64 = \frac{15}{X} \cdot X$

$X = \frac{15}{\cos 64}$

$X = 34.2$

Example

Solve for the following angle measure.

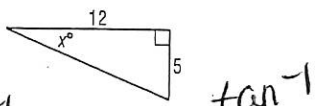


$\sin^{-1}$ ~~$\sin X = \frac{12}{16}$~~

$X = 48.6°$

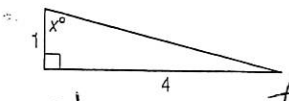
Examples

Solve for the following angle measures.



$\tan^{-1}$ ~~$\tan X = \frac{5}{12}$~~

$22.6°$



$\tan^{-1}$ ~~$\tan X = \frac{4}{1}$~~

$X = 75.9$

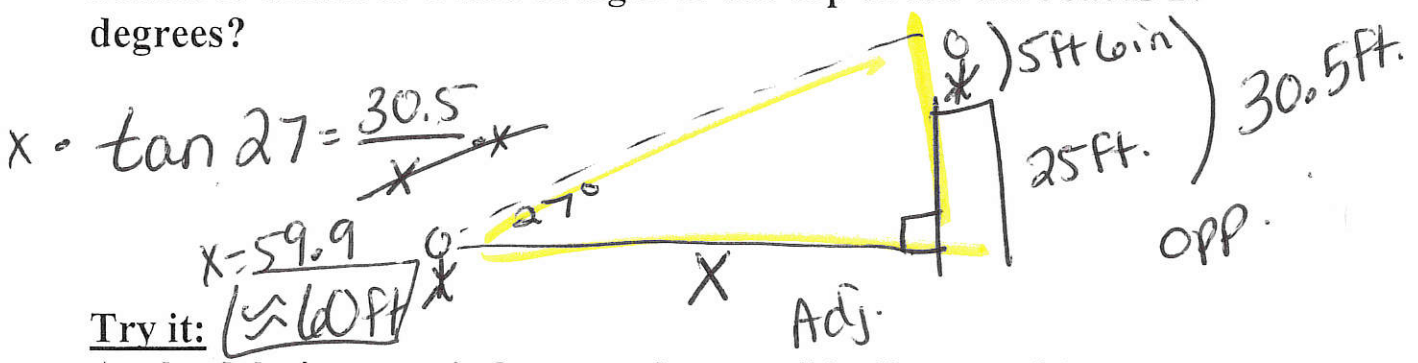
85

~~7.5~~ *Angles of Elevation and Depression*

- Angle of Elevation – the angle between the line of sight and the horizontal when an observer looks upward →

Example

At the circus a person in the audience watches the high-wire routine. A 5 foot 6 inch tall acrobat is standing on a platform that is 25 feet off the ground. How far is the audience member from the base of the platform if the angle of elevation from the audience member's line of sight to the top of the acrobat is 27 degrees?



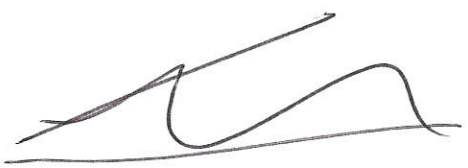
$x \cdot \tan 27 = \frac{30.5}{x}$

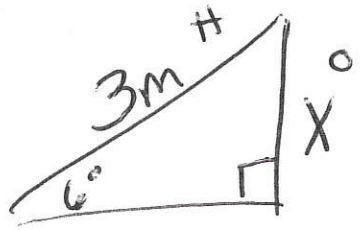
$x = 59.9$ ft

≈ 60 ft

Try it: **A wheelchair ramp is 3 meters long and inclines at 6 degrees. Find the height of the ramp to the nearest tenth centimeter.** 100cm = 1m

$\sin 6 = \frac{x}{3}$

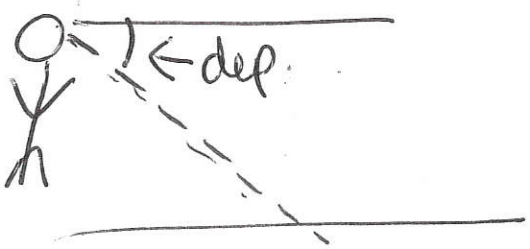




~~x=.032~~

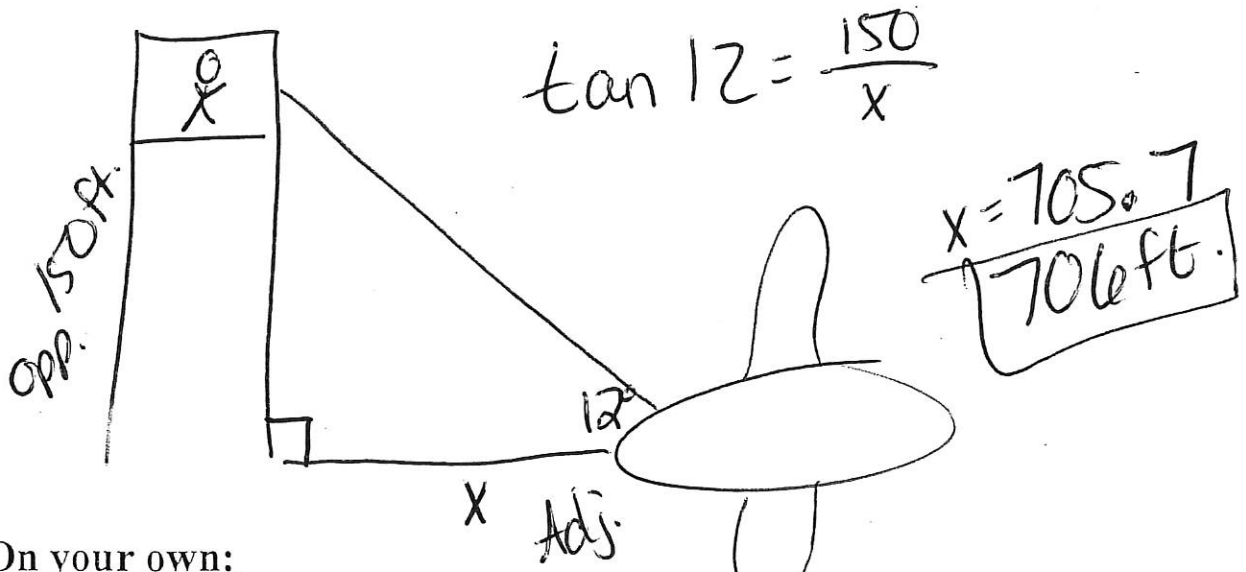
$x = .313$ 31.3 cm

- Angle of Depression – the angle between the line of sight when an observer looks downward and the horizontal.



Example

From the top of a 150-foot high tower an air traffic controller observes an airplane on the runway. To the nearest foot, how far from the base of the tower is the airplane if the angle of depression is 12 degrees?



On your own:

After flying at an altitude of 500 meters, a helicopter starts to descend when its ground distance from the landing pad is 11 kilometers. What is the angle of depression for this part of the flight?