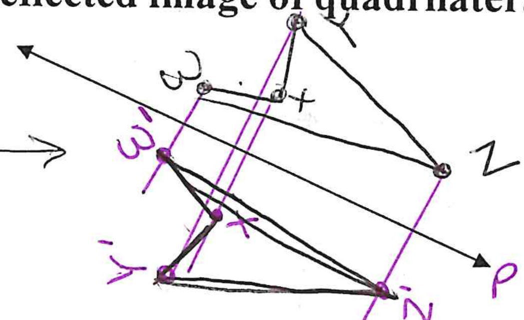
Explore Laptops

9.1 Reflections

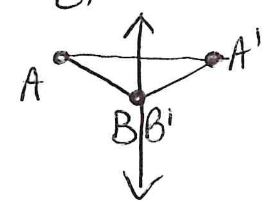
- **Reflection – flip of a figure** "mirror"
  - **Can reflect in a point, line, or plane**

Example
Construct the reflected image of quadrilateral WXYZ in line p.

Prime (Post)

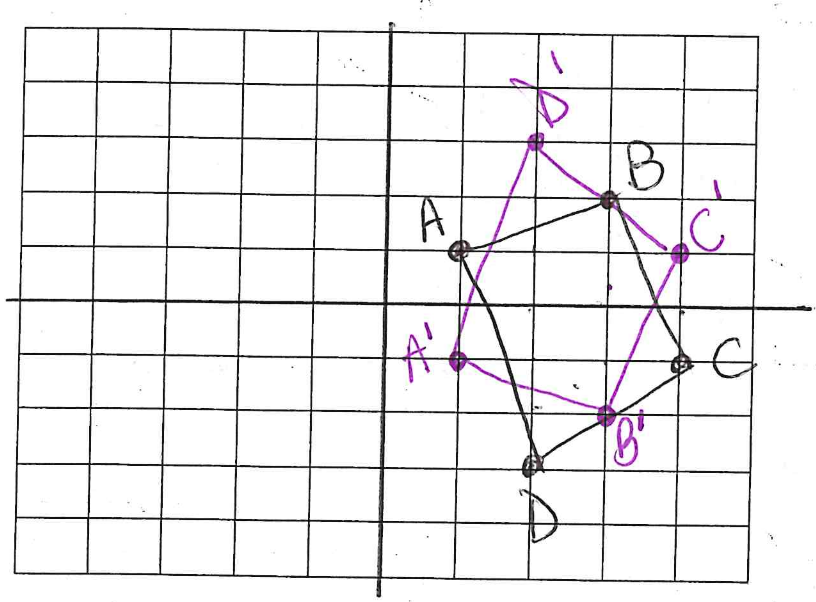


Ex



- **Isometry – preserves distance, angle measure, collinearity, ~~and~~ betweenness of points** "Congruent"

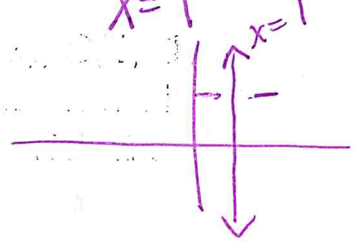
Example
Quadrilateral ABCD has verticies A(1, 1), B(3, 2), C(4, -1) and D (2, -3). Graph ABCD and its image under reflections in the x-axis. Compare the coordinates of each vertex with the coordinates of its image.



x-axis
(x, -y)
A' (1, -1)

origin
(-x, -y)

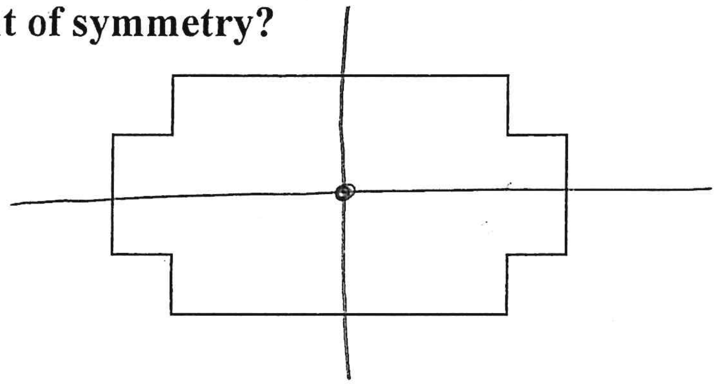
x=1



- Line of symmetry – line of reflection "fold"
- Point of symmetry – common point of reflection

Example

How many lines of symmetry does the figure have? Does it have a point of symmetry?



2 lines

Yes

9.2 Translations

- **Translation – transformation that moves all points of a figure the same distance in the same direction**
  AKA slide, shift, or glide

$\langle -8, 1 \rangle$ left 8 up 1

Rt 2, Down 5

$(x+2, y-5)$

$\langle 2, -5 \rangle$

Example

Parallelogram TUVW has vertices T(-1, 4), U(2, 5), V(4, 3), and W(1, 2). Graph TUVW and its image for the translation (x, y) (x – 4, y – 5)

4 left 5 down

T' (-5, -1)

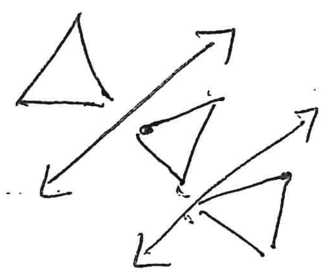
U' (-2, 0)

V' (0, -2)

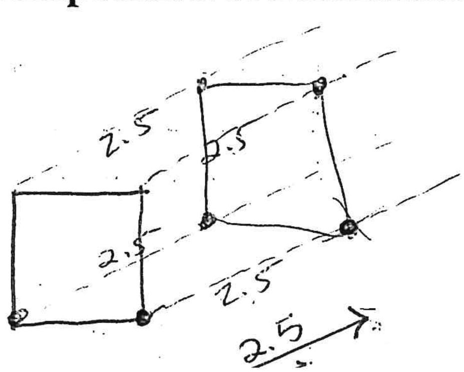
W' (-3, -3)

Example

- **Composition of Reflections – repeated reflection**



Ex



Assume Counterclockwise

Clockwise

9.3 Rotations

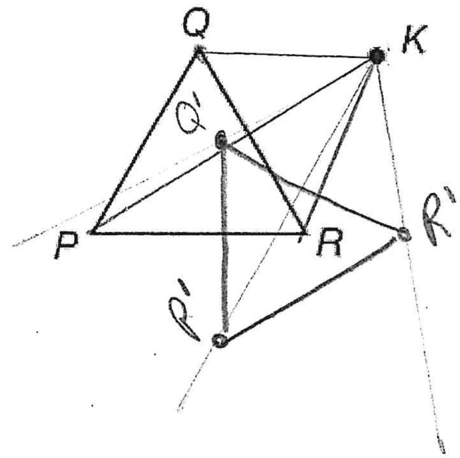
Rotation – transformation that turns every point of an image

Center of rotation – the fixed point

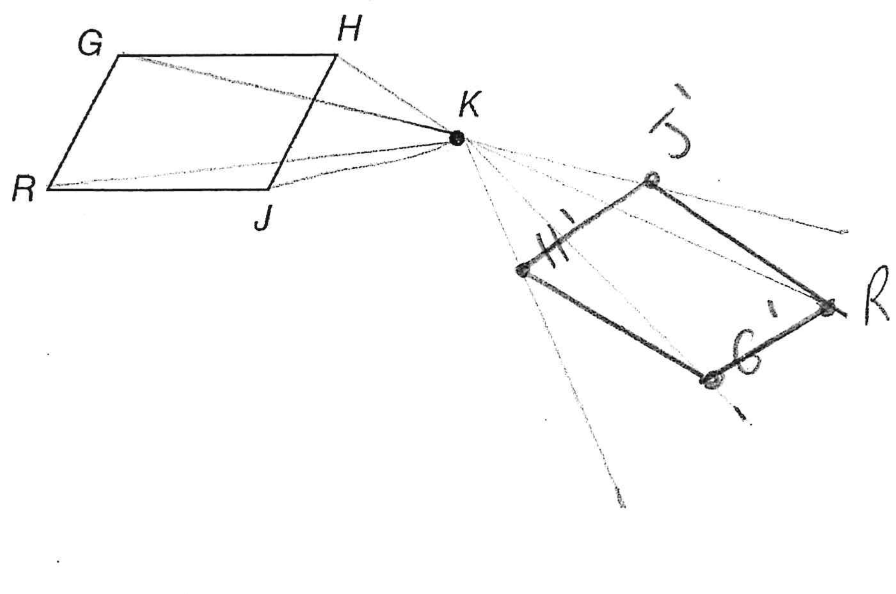
Rotation is an Isometry!

Examples
Rotate the image around point K using the degrees given.

30°



150°

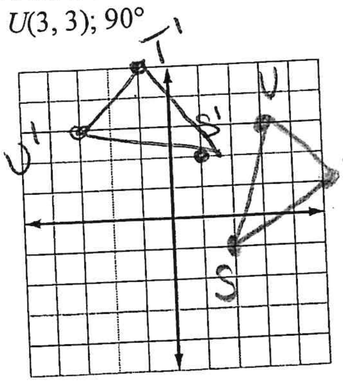


You can also rotate in the coordinate plane! Counterclockwise

90 degrees	$(-y, x)$
180 degrees	$(-x, -y)$
270 degrees	$(y, -x)$

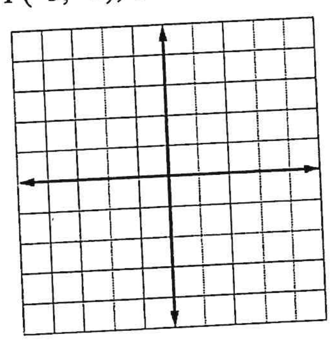
Graph each figure and its image after the specified rotation about the origin.

3. $\triangle STU$ has vertices $S(2, -1)$, $T(5, 1)$ and $U(3, 3)$; 90°

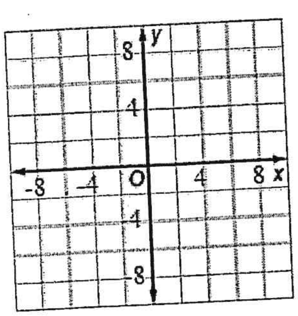


S' (1, 2)
T' (-1, 5)
U' (-3, 3)

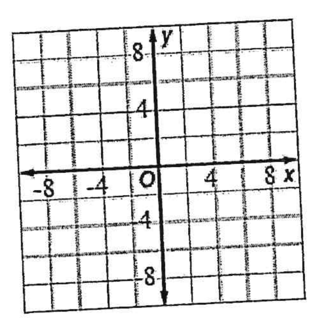
4. $\triangle DEF$ has vertices $D(-4, 3)$, $E(1, 2)$, and $F(-3, -3)$; 180°



5. quadrilateral $WXYZ$ has vertices $W(-1, 8)$, $X(0, 4)$, $Y(-2, 1)$ and $Z(-4, 3)$; 180°



6. trapezoid $ABCD$ has vertices $A(9, 0)$, $B(6, -7)$, $C(3, -7)$ and $D(0, 0)$; 270°



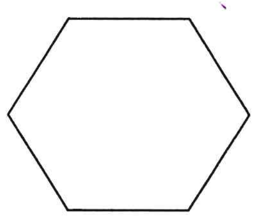
Rotational Symmetry – can rotate an image less than 360 degrees and it is indistinguishable from the pre-image

Order - # of rotations that produce same image

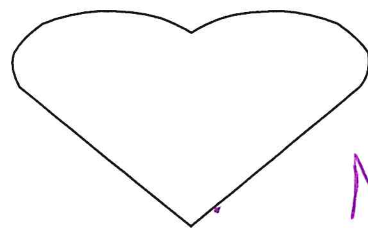
Magnitude – degrees rotated for same image

Example

Do the following have rotational symmetry?
What is the order? Magnitude?



Yes
order 6
magnitude $\frac{360}{6} = 60^\circ$

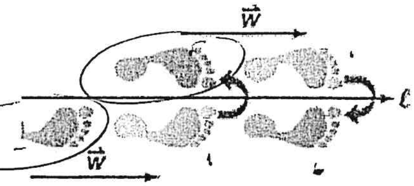


No

9.4 Compositions of Transformations

- Composition of Transformations – when more than 1 transformation applied
- Glide Reflection -

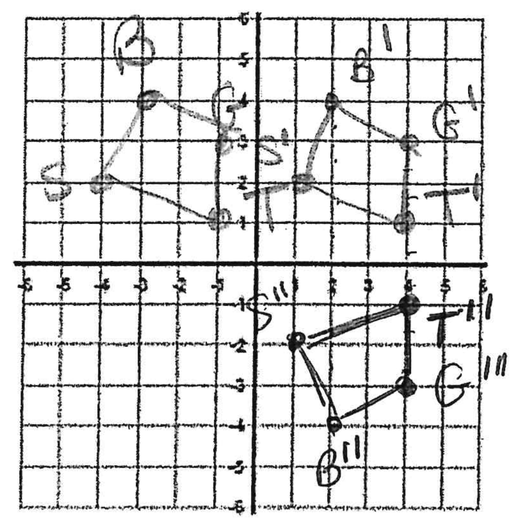
translation followed by reflection



Examples

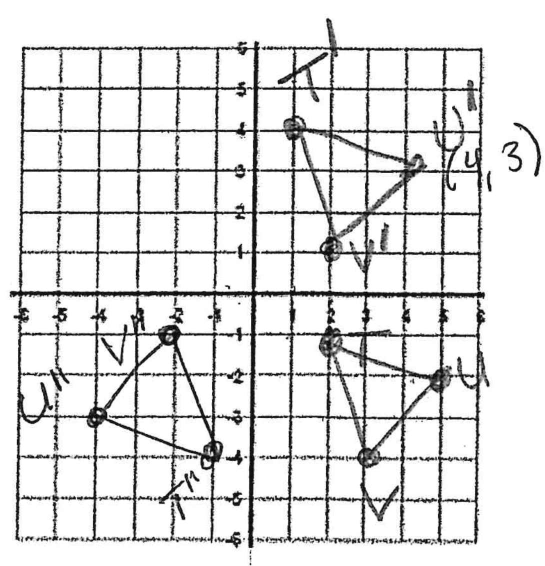
Quadrilateral *BGTS* has vertices *B*(–3, 4), *G*(–1, 3), *T*(–1 , 1), and *S*(–4, 2). Graph *BGTS* and its image after a translation along ⟨5, 0⟩ and a reflection in the *x*-axis.

Rt



Δ*TUV* has vertices *T*(2, –1), *U*(5, –2), and *V*(3, –4). Graph Δ*TUV* and its image after a translation along ⟨–1 , 5⟩ and a rotation 180° about the origin.

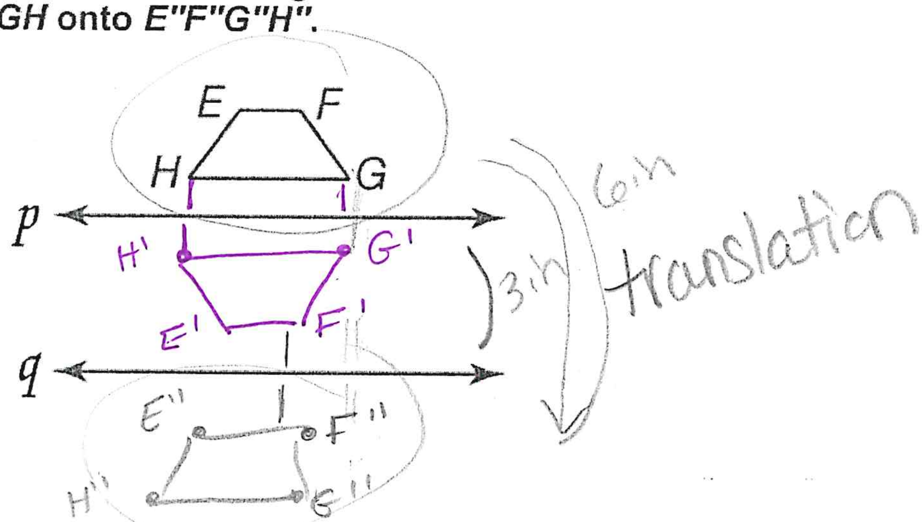
$-x, -y$



- Note: The composition of two or more isometries is an Isometry.

Example

Copy and reflect figure *EFGH* in line *p* and then line *q*. Then describe a single transformation that maps *EFGH* onto *E"F"G"H"*.

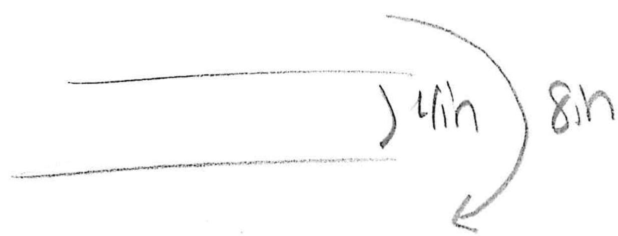


Notice in the picture above the following theorem can be seen:

- The composition of two reflections in parallel lines can be described by a translation vector that is:

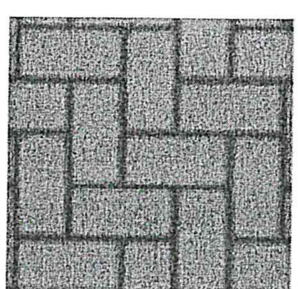
  1) Perpendicular to the two lines and

  * 2) twice the distance between the two lines



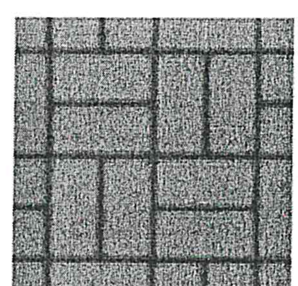
Example – Real World

A. LANDSCAPING Describe the transformations that are combined to create the brick pattern shown.

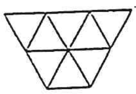


translations & rotations

B. LANDSCAPING Describe the transformations that are combined to create the brick pattern shown.



Rotations



:gular tessellation

9.4 Tessellations - Explore

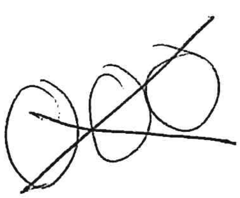
Tessellation – a pattern that covers a plane by transforming the same figure or set of figures so that there are no gaps

Regular Tessellation – made using 1 regular polygon

$\frac{180(n-2)}{n}$ mult of 360

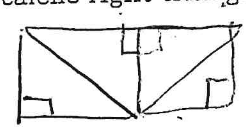
- **Uniform** – tessellation containing the same arrangement of shapes and angles at each vertex
- **Semi-regular tessellation** – a uniform tessellation formed using 2 or more regular polygons

Examples



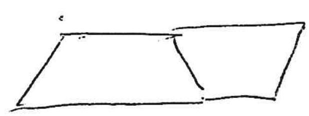
Determine whether each polygon tessellates the plane. If so, draw a sample figure.

1. scalene right triangle



Yes

2. isosceles trapezoid



Yes

Determine whether each regular polygon tessellates the plane. Explain.

3. square

Yes 90°

4. 20-gon

No 162

5. septagon

No 128.5

6. 15-gon

No

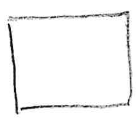
156

9.5 Symmetry

- Symmetry - when a ridged motion maps the figure onto itself.
- Line Symmetry – AKA reflection Symmetry

line of symmetry

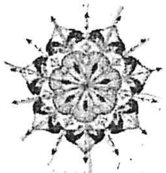
- Rotational Symmetry – AKA radial symmetry
  can be rotated onto itself using ~~degree~~ angle between 0 + 360°



- Order of symmetry (AKA magnitude of symmetry) – # of times a figure maps onto itself.
  mag = 360 ÷ order

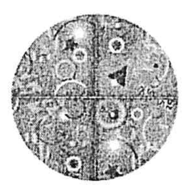
Examples

A. **KALEIDOSCOPES** State whether the object appears to have line symmetry. Write *yes* or *no*. If so, draw all lines of symmetry, and state their number.



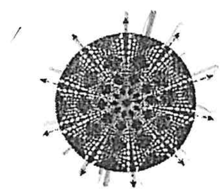
Yes 7

B. **KALEIDOSCOPES** State whether the object appears to have line symmetry. Write *yes* or *no*. If so, draw all lines of symmetry, and state their number.



No

C. **KALEIDOSCOPES** State whether the object appears to have line symmetry. Write *yes* or *no*. If so, draw all lines of symmetry, and state their number.



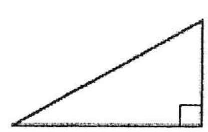
Yes 10

State whether the figure has rotational symmetry. Write *yes* or *no*. If so, copy the figure, locate the center of symmetry, and state the order and magnitude of symmetry.

a.

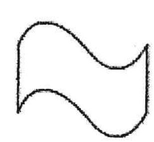
Yes
order 6
mag 60

b.



No

c.



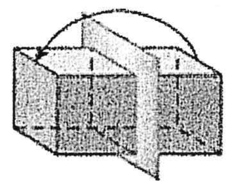
Yes
order 2
mag 180

Three dimensional figures can also have symmetry!

Plane Symmetry

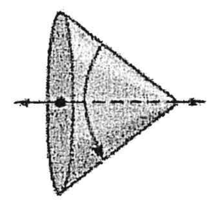
A three-dimensional figure has plane symmetry if the figure can be mapped onto itself by a reflection in a plane.

"slice"



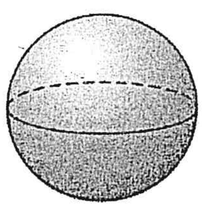
Axis Symmetry

A three-dimensional figure has axis symmetry if the figure can be mapped onto itself by a rotation between 0° and 360° in a line.



Examples

A. State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

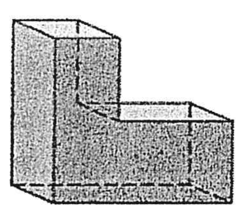


Both

B. State whether the figure has *plane* symmetry, *axis* symmetry, *both*, or *neither*.

Neither

A. State whether the figure has *plane* symmetry, *axis* symmetry, *both* or *neither*.



plane

9.6 Dilations

Dilation – enlarge or reduce a figure proportionally

Scale factor determines what type of dilation (r)

If $|r| > 1$, the dilation is an enlargement
If $0 < |r| < 1$, the dilation is a reduction
If $|r| = 1$, the dilation is a congruence

$r = -\frac{2}{3}$ flipped & red.

Dilation is a similarity transformation (NOT ISOMETRY)

Example
Find the measure of the dilation image or the preimage of segment CD using the given scale factor

a) CD = 15, r = 3 15×3 $C'D' = 45$

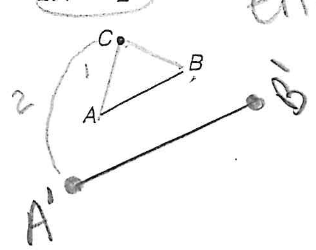
b) C'D' = 7, r = -2/3 $CD = 10.5$ $7 \div 2/3$

➢ **Theorem** – If P(x, y) is the preimage of a dilation centered at the origin with a scale factor r, then the image is (rx, ry)

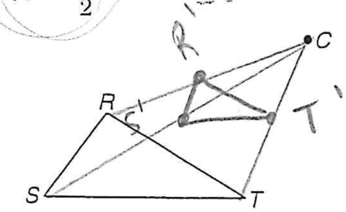
$(2,3)$ $r = 7$ $(14, 21)$

Draw the dilation image of each figure with center *C* and the given scale factor. Describe each transformation as an *enlargement*, *congruence*, or *reduction*.

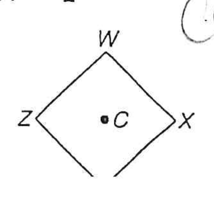
1. $r = 2$ enlargement



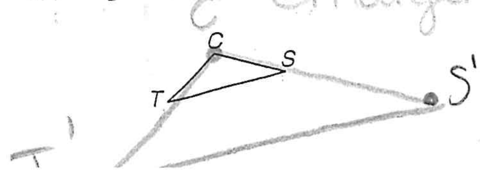
2. $r = \frac{1}{2}$ Red.



3. $r = 1$ congruence



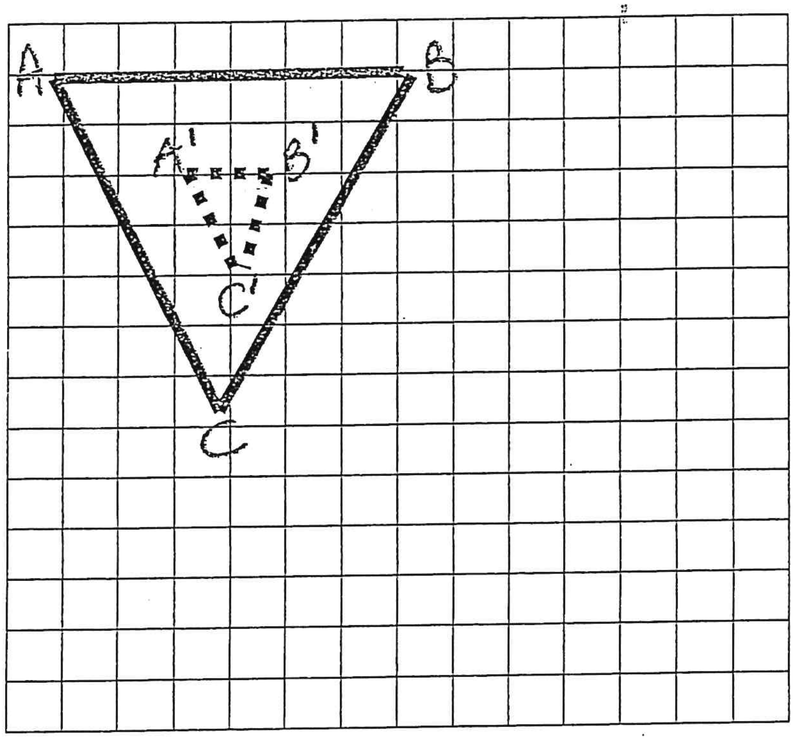
4. $r = 3$ enlargement



Example

Determine the scale factor used for the dilation below with center C.

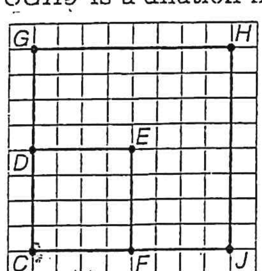
Scale factor = $\dfrac{\textbf{image length}}{\textbf{Preimage length}}$ $\dfrac{2}{6}$ $\dfrac{1}{3}$



Try It:

Determine the scale factor for each dilation with center *C*. Determine whether the dilation is an *enlargement*, *reduction*, or *congruence transformation*.

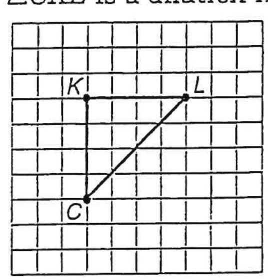
1. *CGHJ* is a dilation image of *CDEF*. (Post: *CGHJ*; Pre: *CDEF*)



Enlargement

2

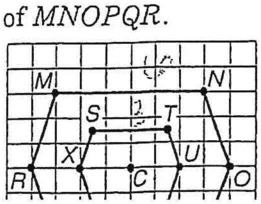
2. $\triangle CKL$ is a dilation image of $\triangle CKL$.



Congr.

1

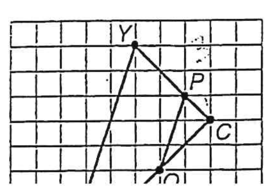
3. *STUVWX* is a dilation image of *MNOPQR*.



Reduction

$\frac{1}{2}$

4. $\triangle CPQ$ is a dilation image of $\triangle CYZ$.



Reduction

$\frac{1}{3}$